Formal Model and Verification Exercise 9: Symbolic on-the-fly safety analysis and liveness analysis

1. We have the following GCM program rules with integer variables x, y, z.

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when (x==1) may x = 0; y = 1; z = y;
when (x==0 && z > 0) may z = z-1;
when (x==0 && y > 0 && z == 0) may y = y-1; z = y;
when (x==0 && y == 0 && z == 0) may y = y+2;
```

The range of the three variables are all [0,10]. The initial condition of the program is x==1 & y==0 & z==0. Please use the on-the-fly least fixpoint algorithm for forward analysis to construct a formula for the forward reachable states from the initial states.

2. Continued from question 1, assume now we have a risk condition y==1&&z==1. Please use the on-the-fly least fixpoint algorithm for backward analysis to check whether the system is safe or not.

3. Continued from question 1, please use the on-the-fly symbolic greatest fixpoint algorithm to construct a propositional formula that characterizes states satisfying $\exists \Box (y < 2)$. According to the formula you constructed, please tell me whether the initial state satisfies $\forall \Diamond (y > = 2)$?