Formal Description & Automated Verification

Embedded Systems
Formal Methods
Lecture 5

Farn Wang
Dept. of Electrical Engineering
National Taiwan University

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Contents

- Timed Automata
- Model-checking of timed automata
- Symbolic algorithm for the model-checking of timed automata
- Linear hybrid automata and symbolic verification procedure

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Real-time system modeling

What is a Real-Time System？

The correctness of a system is defined with both the output values and the time the output appears

- avionics
- battlefield management
- missile guidance
- submarine sonar
- nuclear plant
- life monitor
- chemical reactor
- flight control

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Real-time system verification

Correctness, non-real-time systems

- **For finite computations, defined with relations between precondition and postcondition**
- **For infinite computations, defined with the ordering of events.**
  - Buechi automata
  - mutual exclusion
  - fairness

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Real-Time system modeling

What is a Real-Time System？

The correctness of a system is defined with both the output values and the time the output appears

Example:

synchronous machine guns

- **an untimely output may be worse than no output.**

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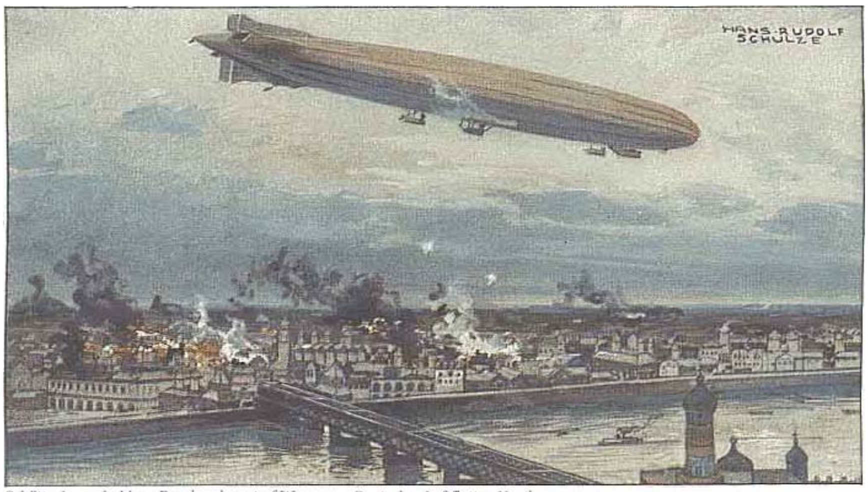
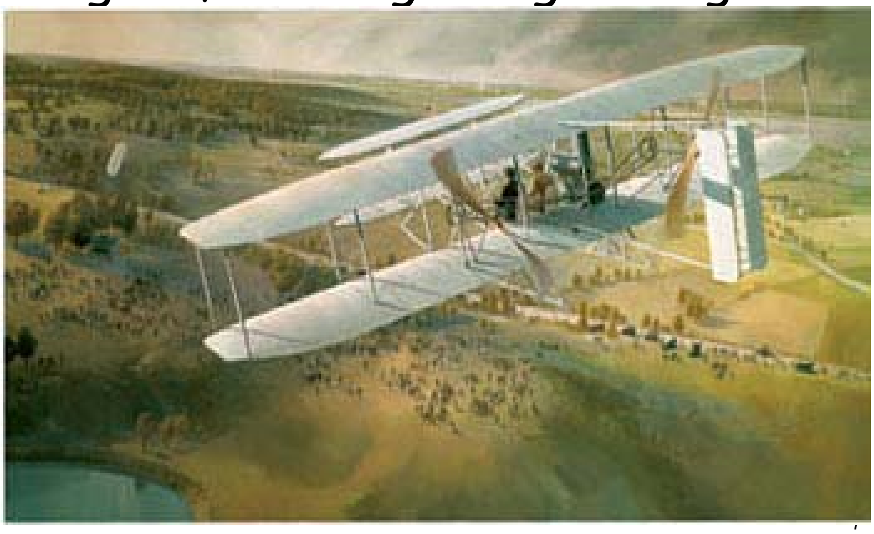
Schütte-Lanz SL2 airship bombing Warsaw



Schütte-Lanz airship – *Bombardment of Warsaw* – Deutscher Luftflotten Verein

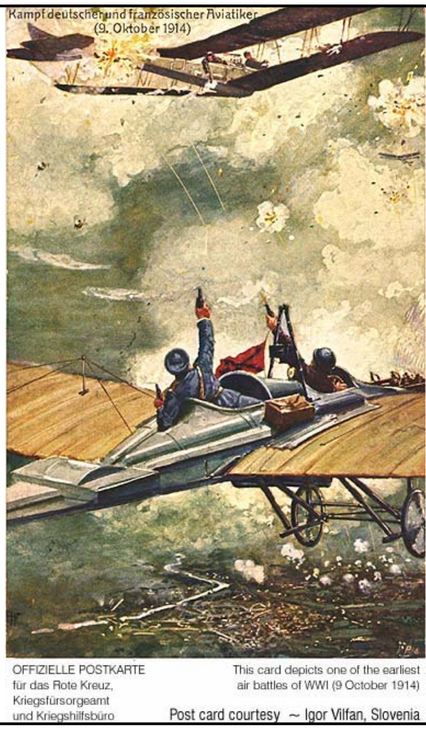
Wrights, the beginning of flight era



The beginning of air-combat

An imaginary solution to air-combat

Difficult to aim in 3D maneuver



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A solution to air-combat

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Another solution to air-combat

)

Aircombat (Dogfight) in WWI

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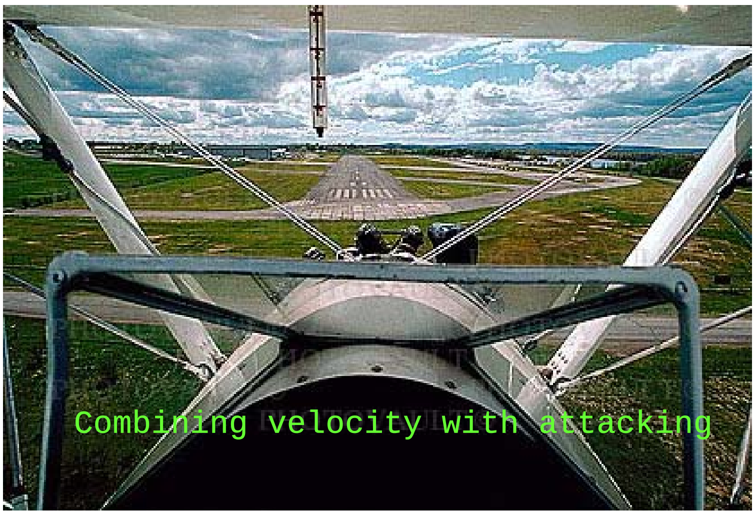
A nice idea to air-combat



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A nice idea to air-combat (cont'd)

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A problem with the idea

If it is not real-time, …

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A solution to air-combat

Synchronous machine gun

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Sopwith Camel

Sopwith Camel, Old Rhinebeck Aerodrome. Photo by Nels Anderson

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SPAD
VII
S.254

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SPAD VII S.254

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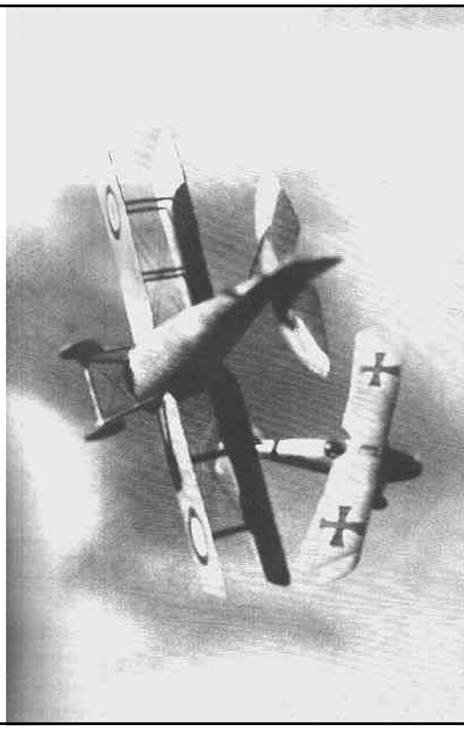
WWI dogfight



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WWI dogfight

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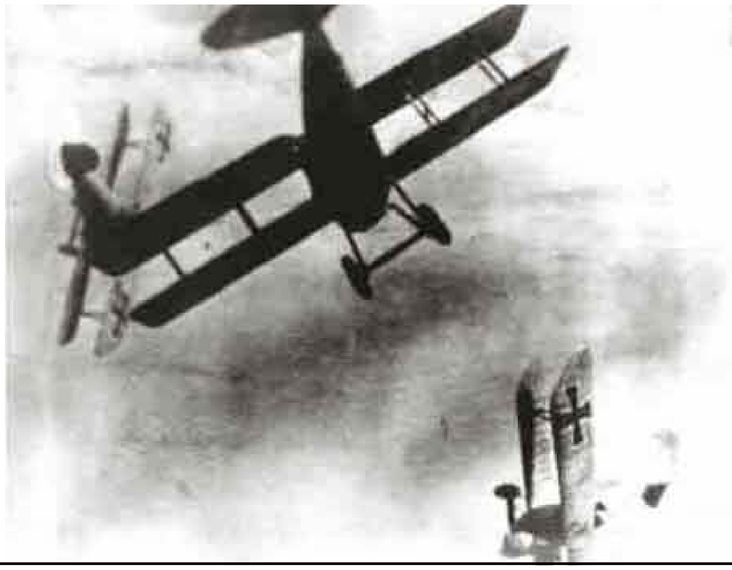
WWI dogfight



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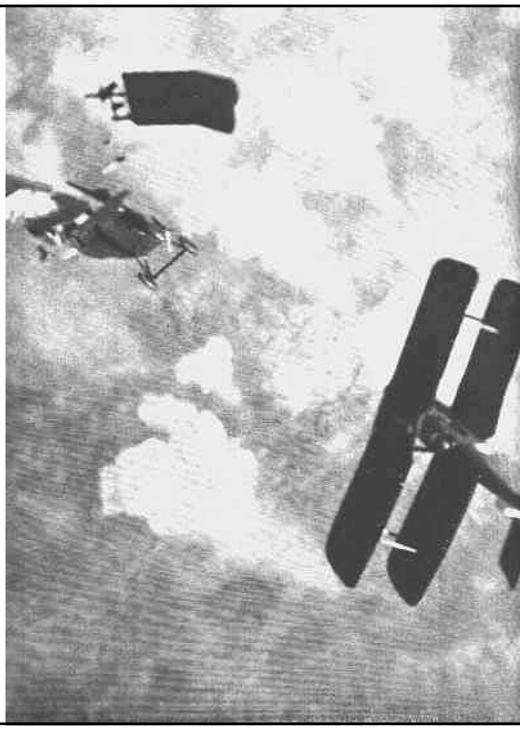
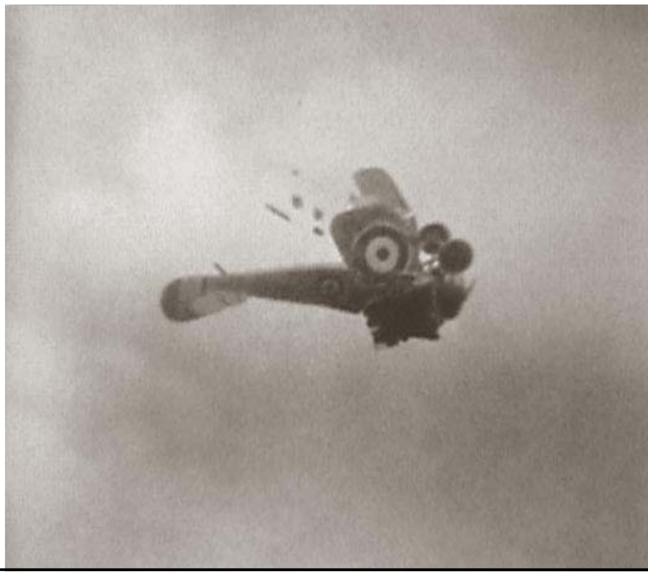
Disintegration in dogfight



Disintegration in dogfight



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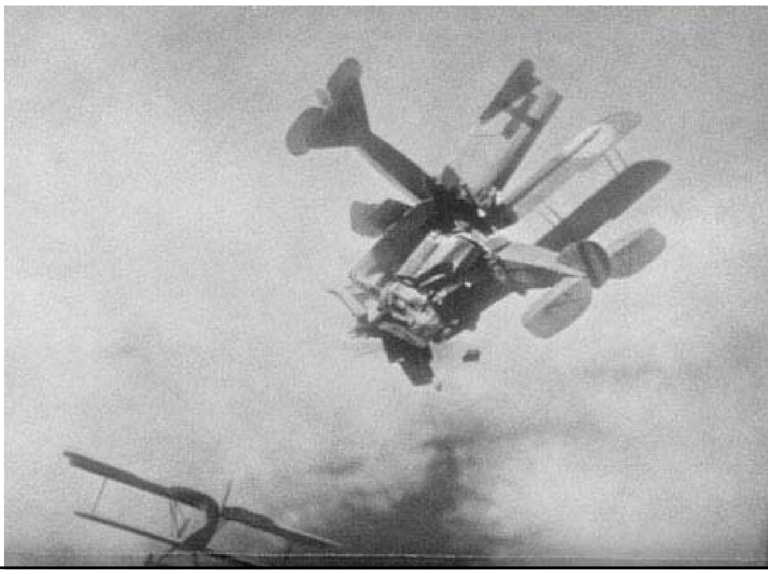
Collision in dogfight



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Pilot falling from burning Albatros

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Why dense-time clock？

Why not discrete clock ?

- discrete clocks can be readily modeled with finite-state automata.
- In engineering, as long as the clock resolution is fine enough, we can have enough timing precision of the models.
- In fact, all clock readings in a digital computer must be recorded as bits.

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Why dense-time clock ?
- the arguments (1/2)

For model timing precision,

- **distributed clocks – discrete clocks may not increment their readings at the same time.**
  - To check the ordering of clock ticks at many sites engender the same model complexity as dense-time models.
- **How fine is fine enough ?**
  - In theory, there is never a fine enough resolution that guarantees the verification correctness.
  - For safety-critical systems, dense-time models could be a necessity.

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Why dense-time clock ?
- the arguments (2/2)

For verification complexity

- fine time resolution
  → large timing constants
  → high verification complexity
- dense linear constraints → P
  discrete linear constraints → NP-complete

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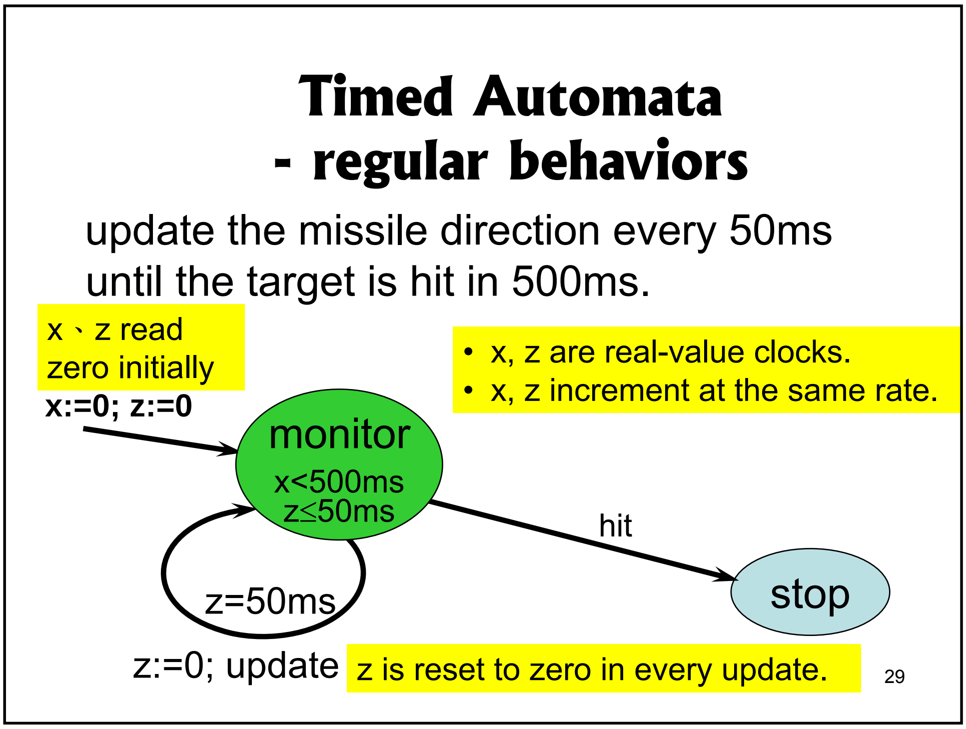
Timed Automata
- regular behaviors
update the missile direction every 50ms
until the target is hit in 500ms.
x、z read
zero initially
x:=0; z:=0
• x, z are real-value clocks.
• x, z increment at the same rate.
monitor
x<500ms
z≤50ms
hit
stop
z=50ms
z:=0; update
z is reset to zero in every update.
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Time
Au
a
ors
• clock readi
ls.
• In between
ck's
rea
• All clo
s at the
same
Sin
e,
• the sta
size and
• state
• No
next states.'
The basic challenge
in verification
How can we check
an infinite state-space
with finite amount
of time and space ?
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Timed automata - syntax

Be sure how to read BNF !

- used for define syntax of context-free language
- important for the definition of
  - automata predicates and
  - temporal logics
- Used throughout the lectures!
- In exam: violate the syntax rules ➔ **no credit.**

A ::= (M) | A1 + A2 | A1 – A2
M ::= (A) | M1 * M2 | M1 / M2

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Timed automata - syntax

State predicates :

Given a proposition set P and a clock set X

$$\eta ::= p \mid x \sim c \mid \neg\eta_1 \mid \eta_1 \vee \eta_2$$

BNF

$$p \in P;\ x \in X;\ c \in N;\ `\sim' \in \{\leq, \geq, <, >, =\}$$

Example:

male ∧ single ∧ age ≤ 30
lonely ∧ male ∧ single ∧ today+2≤nextMonday
monitor ∧ x+5 < ReportPeriod

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Timed automata - syntax

State predicates :

Given a proposition set P and a clock set X

$$\eta ::= p \mid x \sim c \mid \neg\eta_1 \mid \eta_1 \vee \eta_2$$

$$p \in P;\ x \in X;\ c \in N;\ `\sim' \in \{\leq, \geq, <, >, =\}$$

Shorthands:

true	$\equiv$	$x = x$
$\eta_1 \wedge \eta_2$	$\equiv$	$\neg((\neg\eta_1)\vee(\neg\eta_2))$
$\eta_1 \rightarrow \eta_2$	$\equiv$	$(\neg\eta_1)\vee\eta_2$

B(*P*, *X*): **All state-predicates of *P and X***

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Timed automata - syntax

$A=\langle Q, I_0, \mu, E, \tau, \pi\rangle$

- **Q,** ***a finite set of control locations***
- **$I_0 \in$ B(P,X),** **an initial condition**
- **$\mu : Q \rightarrow$ B(*P*,*X*),** **invariance conditions at the locations**
- **$E \subseteq Q \times Q$,** **a set of transitions**
- **$\tau : E \rightarrow$ B(*P*, *X*),** **triggering conditions of the transitions**
- **$\pi : E \rightarrow 2^X$,** **sets of clocks to reset at transitions**

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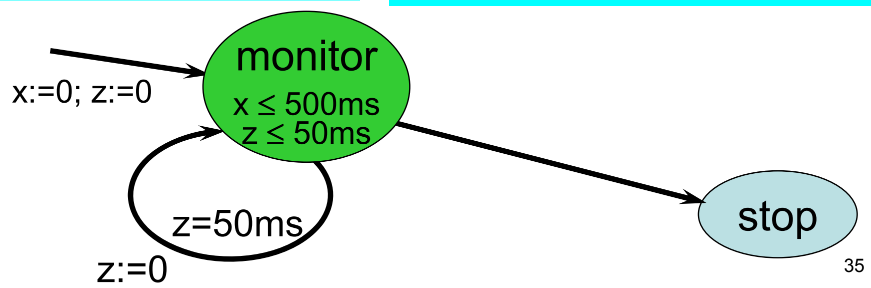
Timed automata - syntax

A=⟨*Q*, I_0, μ, *E*, τ, π⟩
Q ={monitor, stop}
I_0 = monitor∧x=0 ∧z=0
μ(monitor) = *x* ≤ 500∧*z* ≤ 50
μ(stop)= *true*

E={(monitor,monitor),(monitor, stop)}
τ(monitor, monitor)= *z* =50
τ(monitor, stop)= *true*
π(monitor, monitor)= {*z*}
π(monitor, stop)= { }



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Timed automata - semantics

clock valuatoin

defines the values of all Boolean variables.

defines the non-negative real values of all clocks

A=⟨Q, I_0, μ, E, τ, π⟩

states: (*q*, ν)

- ***q*** ∈ **Q**
- ν : ***P*** → **{*true*,*false*}** ∪ ***X*** → ***R***$^+$

★ν ⊨ μ(*q*), **clock values must satisfy invariance condition.**

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Timed automata
- semantics

$A=\langle Q, I_0, \mu, E, \tau, \pi\rangle$

valuations

$\nu \models \eta$, ν *satisfies a* **state predicate** η

$\nu \models p$	iff	$\nu(p) = true$
$\nu \models x \sim c$	iff	$\nu(x) \sim c$
$\nu \models \neg\eta_1$	iff	not $\nu \models \eta_1$
$\nu \models \eta_1 \vee \eta_2$	iff	$\nu \models \eta_1$ or $\nu \models \eta_2$

states $(q,\nu) \models \eta$ iff $\nu \models \eta$

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Timed automata
- semantics

- Given a $\delta \in R^+$, $\nu + \delta$ is a new mapping
  - $(\nu + \delta)(p) = \nu(p)$
  - $(\nu + \delta)(x) = \nu(x) + \delta$
- Given $s = (q,\nu)$, $s + \delta = (q, \nu + \delta)$
- Given a $Y \subseteq X$, νY is a new mapping
  - $(\nu Y)(p) = \nu(p)$
  - $(\nu Y)(x) = 0$ iff $x \in Y$
  - $(\nu Y)(x) = \nu(x)$ iff $x \notin Y$
- Given $s = (q,\nu)$ and $Y \subseteq X$, $sY = (q, \nu Y)$

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Timed automata
- semantics

$A=\langle Q, I_0, \mu, E, \tau, \pi\rangle$, $s = (q,\nu)$, and $s' = (q',\nu')$

$s \rightarrow s'$ *(there is a (discrete) transitions from s to s')*

iff

- $(q, q') \in E$
- $s \models \tau(q, q')$
- $s\pi(q, q') = s'$
- $s' \models \mu(q')$

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Timed automata
- semantics

Intuitively, a (linear) computation of a timed automata should be

- **a mapping**
  from R^+ (non-negative reals) to states
- **dense!**

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Timed automata - semantics

$A=\langle Q, l_0, \mu, E, \tau, \pi\rangle$

s_0-run, a (linear) computation of A

$(s_0,t_0)\ (s_1,t_1)\ (s_2,t_2)\ldots (s_k,t_k)\ldots \ldots \ldots$

- for all $k \geq 0$, $s_k=(q_k, v_k)$
- a nonmonotonically increasing, divergent, non-negative real sequence

  $t_1\ t_2 \ldots t_k \ldots \ldots \ldots$

- for all $k \geq 0$,
  - for all $t\in[0, t_{k+1}-t_k]$, $s_k+t \vDash \mu(q_k)$
  - $s_k+(t_{k+1}-t_k) = s_{k+1}$ or $s_k+(t_{k+1}-t_k) \rightarrow s_{k+1}$

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Parallel Composition of TA

$A_1=\langle Q_1, I_1, \mu_1, E_1, \tau_1, \pi_1\rangle$,

$A_2=\langle Q_2, I_2, \mu_2, E_2, \tau_2, \pi_2\rangle$

$A_1 \parallel A_2 = \langle Q_1\times Q_2, I_1\wedge I_2, \mu, E, \tau_1\wedge\tau_2, \pi_1\cup\pi_2\rangle$

- $\mu(q, q') = \mu_1(q) \wedge \mu_2(q')$
- E: set of interleaved transitions with synchronization transitions identified

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Model of Manufacturing System

- *M* = D-Robot || G-Robot || Station || Box
- Transitions with same labels are identified as one in *M*.
- E.g.: g-put in G-Robot, Station, and Box
- E.g.: d-pick in D-Robot, Station, and Box

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Timed automata - a manufacturing example (1/5)

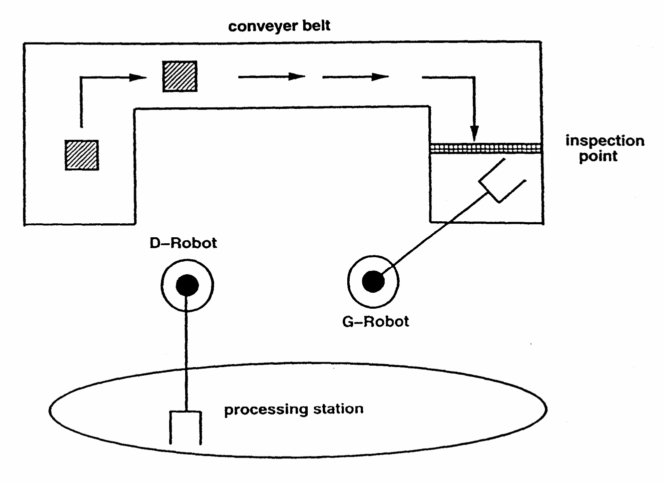

Figure 17.2
A manufacturing example.

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Timed automata - a manufacturing example (2/5)

D-Robot TA

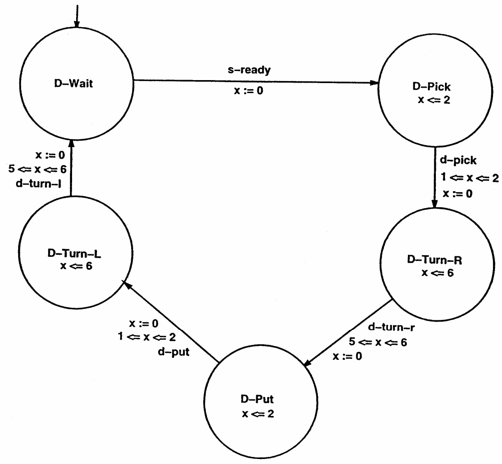

Figure 17.3
Timed automaton for D-Robot.

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Timed automata - a manufacturing example (3/5)

G-Robot TA

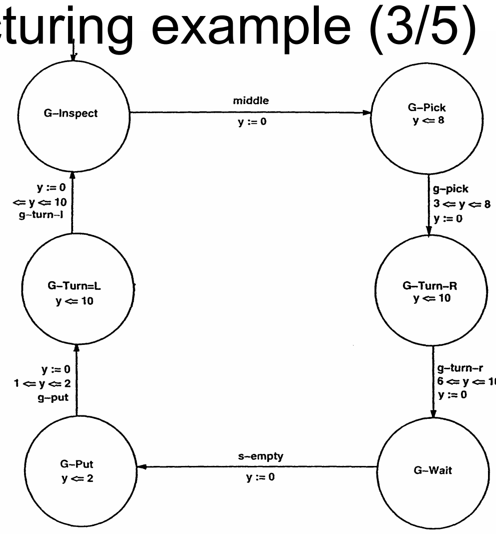

Figure 17.4
Timed automaton for G-Robot.

Timed automata
- a manufacturing example (4/5)

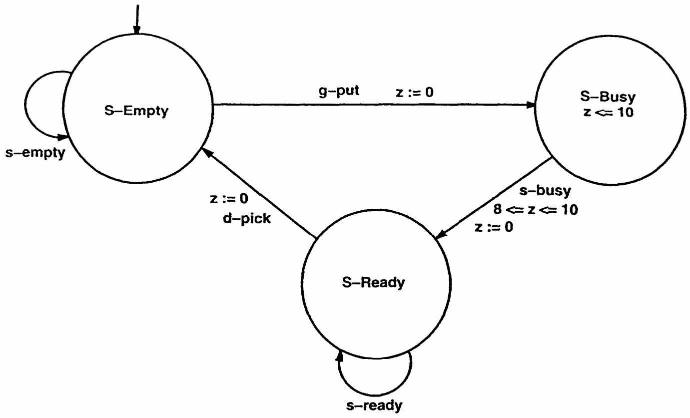

Figure 17.5
Time automaton for processing station.

Station
TA

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Timed automata
- a manufacturing example (5/5)

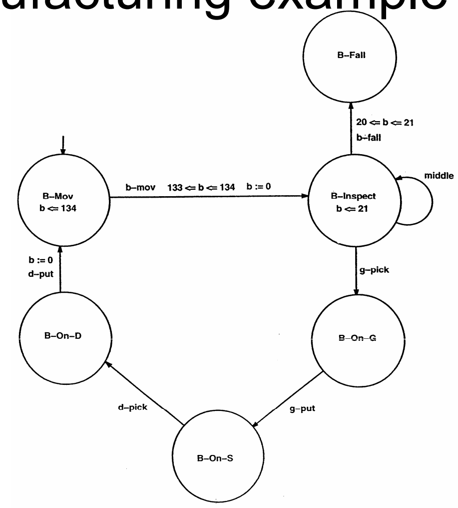

Figure 17.6
Timed automaton for box.

Box TA

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TCTL (Timed CTL)

<u>Example</u>: ***from now on, it is possible that I will get salary on the 7th day.***

$\exists\Diamond_{=7}$ **salary**

<u>Example</u>: ***from now on, I will be surely married in 10 years.***

$\forall\Diamond_{\leq 10}$ **married**

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TCTL, continued

<u>Example</u>: ***When a hostile fighter is found, it is always possible that we take it down in 5 sec.***

$\forall\Box$(HF.Found $\rightarrow \exists\Diamond_{\leq 5}$ HF.TakenDown))

<u>Example</u>: ***When a hostile fighter is found, we will always take it down in 5 sec.***

$\forall\Box$(HF.Found $\rightarrow \forall\Diamond_{\leq 5}$ HF.TakenDown))

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TCTL, continued

Example: **From every state, there is a computation that increment the time divergently.**

$$\forall\Box\exists\Diamond_{=1}\mathit{true}$$

Example: **From every state, time always diverges.**

$$\forall\Box\forall\Diamond_{=1}\mathit{true}$$

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TCTL, continued

Example: ***After the wedding, I could be happy for 5 days.***

$$\forall\Box(\text{wedding}\rightarrow\exists\Box_{<5}\,\mathit{happy})$$

Example: ***After the wedding, I will be happy for 5 days.***

$$\forall\Box(\text{wedding}\rightarrow\forall\Box_{<5}\,\mathit{happy})$$

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TCTL
- syntax

BNF

φ ::= η | $\neg\varphi_1$ | $\varphi_1 \vee \varphi_2$ | $\exists \varphi_1 U_{\sim c} \varphi_2$ | $\exists \Box_{\sim c} \varphi_1$

- $\eta \in$ **B(*P*, *X*)**, the set of state predicates of P and X
- $c \in N$;
- '~'$\in \{\leq, \geq, <, >, =\}$

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TCTL
- syntax

shorthands:

$\varphi_1 \wedge \varphi_2 \equiv \neg((\neg\varphi_1) \vee (\neg\varphi_2))$

$\varphi_1 \rightarrow \varphi_2 \equiv (\neg\varphi_1) \vee \varphi_2$

$\exists \Diamond_{\sim c} \varphi \equiv \exists\, true\, U_{\sim c} \varphi$

$\forall \Box_{\sim c} \varphi \equiv \neg \exists \Diamond_{\sim c} \neg \varphi$

$\forall \varphi_1 U_{\sim c} \varphi_2 \equiv \neg((\exists(\neg\varphi_2) U_{\sim c} \neg(\varphi_1 \vee \varphi_2)) \vee \exists \Box_{\sim c} \neg \varphi_2)$

$\forall \Diamond_{\sim c} \varphi \equiv \forall\, true\, U_{\sim c} \varphi$

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TCTL
- semantics

$A=\langle Q, I_0, \mu, E, \tau, \pi\rangle$

$A,(q,\nu) \vDash \varphi$, state (q,ν) of A satisfies φ.

$A,(q,\nu) \vDash \eta$ *, as defined for state-predicates*

$A,(q,\nu) \vDash \neg\varphi_1$ iff not $A,(q,\nu) \vDash \varphi_1$

$A,(q,\nu) \vDash \varphi_1 \vee \varphi_2$ iff $A,(q,\nu) \vDash \varphi_1$ or $A,(q,\nu) \vDash \varphi_2$

$A,(q,\nu) \vDash \exists\varphi_1 U_{\sim c}\varphi_2$ iff ...

$A,(q,\nu) \vDash \exists\Box_{\sim c}\varphi_1$ iff ...

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TCTL
- semantics of $\exists\varphi_1 U_{\sim c}\varphi_2$

$A=\langle Q, I_0, \mu, E, \tau, \pi\rangle$

$A,(q,\nu) \vDash \exists\varphi_1 U_{\sim c}\varphi_2$ iff there are

- a (q,ν)-run:
  $(s_0,t_0)\ (s_1,t_1)\ (s_2,t_2)\ldots(s_k,t_k)\ldots\ \ldots\ \ldots$, and
- a nonmonotonically increasing, divergent, non-negative real sequence: $t_0\, t_1\, t_2 \ldots\ldots$, and
- $k \geq 0$
  - $t_k - t_0 \sim c$
  - $A, s_k \vDash \varphi_2$
  - for all $k > h \geq 0$ and $t' \in [0, t_{h+1} - t_h]$, $A, s_h + t' \vDash \varphi_1$

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TCTL - semantics of $\exists\Box_{\sim c}\varphi_1$

$A=\langle Q, I_0, \mu, E, \tau, \pi\rangle$

$A,(q,\nu) \models \exists\Box_{\sim c}\varphi_1$ iff there are

- a (q,ν)-run:

  $(s_0,t_0)\ (s_1,t_1)\ (s_2,t_2)\ldots(s_k,t_k)\ldots\ldots\ldots$ and
- a nonmonotonically increasing, divergent, non-negative real sequence: $t_0\, t_1\, t_2\ldots\ldots$

such that for all $k \geq 0$ and $t \in [0, t_{k+1} - t_k]$,

if $t_k + t - t_0 \sim c$, $A, s_k + t \models \varphi_1$

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TCTL - model-checking problem

A $\models \varphi$, ***A* satisfies** φ, ***A is a model of*** φ

Given a TA $A=\langle Q, I_0, \mu, E, \tau, \pi\rangle$

and a TCTL formula φ,

$A \models \varphi$ **iff** $\forall(q,\nu) \models I_0$, **then** $A,(q,\nu) \models \varphi$

TCTL model-checking problem:

Given a TA $A=\langle Q, I_0, \mu, E, \tau, \pi\rangle$

and a TCTL formula φ,

is A a model of φ **?**

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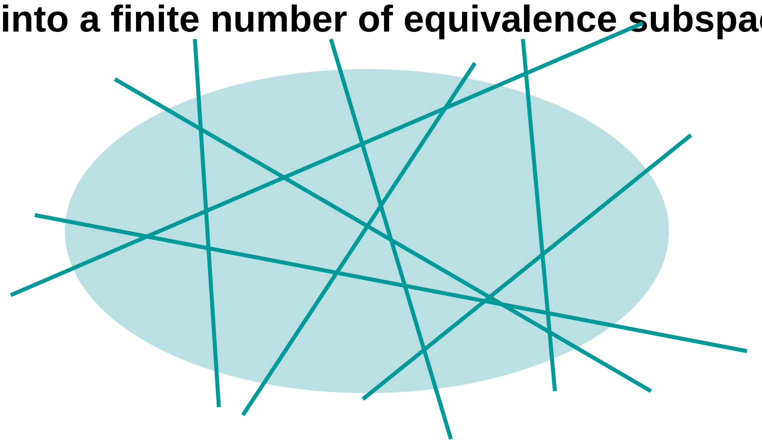
TCTL
- model-checking problem

How to partition the infinite dense state-space into a finite number of equivalence subspaces ?



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Region Equivalence [Alur et al 90]

$C_{A:\varphi}$: the biggest timing constant used in *A and* φ.

$(q, \nu) \equiv (q', \nu')$ if and only if

- q = q'
- for all $p \in P$, $\nu(p) = \nu'(p)$
- for all $x \in X$, *when* $\nu(x) \leq C_{A:\varphi}$ or $\nu'(x) \leq C_{A:\varphi}$,
  their integer parts are the same.
  $$\nu(x) \leq C_{A:\varphi} \vee \nu'(x) \leq C_{A:\varphi} \Rightarrow \lfloor \nu(x) \rfloor = \lfloor \nu'(x) \rfloor$$
- for all $x,y \in X \cup \{0\}$ ，when $\nu(x)$ or $\nu'(x) \leq C_{A:\varphi}$,
  their fractional parts are of the same ordering.
  $$\nu(x) \leq C_{A:\varphi} \wedge \nu'(x) \leq C_{A:\varphi} \Rightarrow$$
  $$(\nu(x) - \lfloor \nu(x) \rfloor \leq \nu(y) - \lfloor \nu(y) \rfloor \Leftrightarrow \nu'(x) - \lfloor \nu'(x) \rfloor \leq \nu'(y) - \lfloor \nu'(y) \rfloor)$$

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Region Eq… ur…

In the same control location

The same sampling of Boolean flags

satisfy the same state-predicates

increment their integer parts in the same order.

$C_{A:\varphi}$: the biggest timing constant used in *A and* …
(q, ν)≡(q',ν') if and only if

- q = q'
- for all *p* ∈ *P*, ν(*p*) = ν'(*p*)
- for all *x* ∈ *X*, *when* ν(*x*) ≤ $C_{A:\varphi}$ or ν'(*x*) ≤ $C_{A:\varphi}$, their integer parts are the same.

$$\nu(x) \le C_{A:\varphi} \vee \nu'(x) \le C_{A:\varphi} \Rightarrow \lfloor \nu(x) \rfloor = \lfloor \nu'(\ldots$$

- for all *x,y* ∈ *X* ∪{0}, their fractional parts are of the same ordering.

$$\nu(x) \le C_{A:\varphi} \wedge \nu'(x) \le C_{A:\varphi} \Rightarrow$$
$$(\nu(x)-\lfloor \nu(x) \rfloor \le \nu(y)-\lfloor \nu(y) \rfloor \Leftrightarrow \nu'(x)-\lfloor \nu'(x) \rfloor \le \nu \ldots$$

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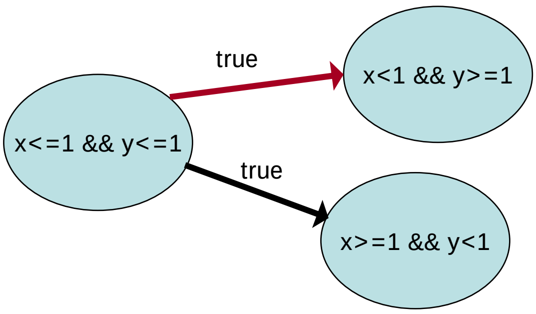
Region equivalence Example (1) of invariances

ν(*x*) = 0.5; ν(*y*) = 0.6;

ν'(*x*)=0.599; ν'(*y*)=0.6;



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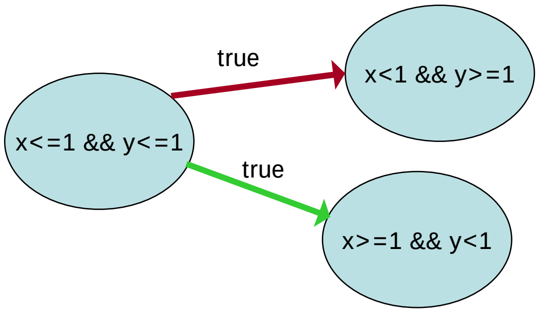
Region equivalence
Example (2) of invariances

$\nu(x) = 0.5$; $\nu(y) = 0.6$;

$\nu'(x)=0.8$; $\nu'(y)=0.6$;



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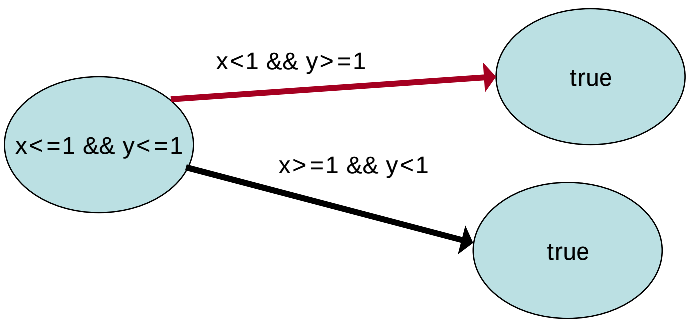
Region equivalence
Example (3) of triggers

$\nu(x) = 0.5$; $\nu(y) = 0.6$;

$\nu'(x)=0.599$; $\nu'(y)=0.6$;



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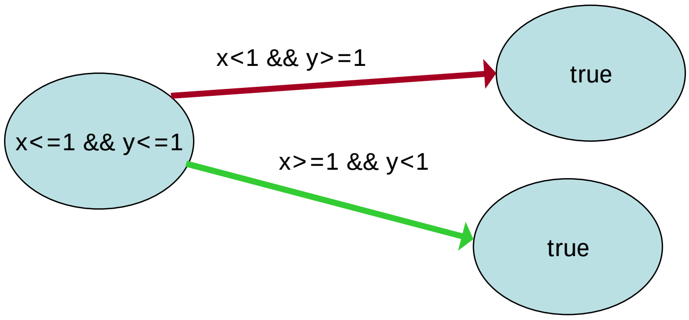
Region equivalence
Example (4) of triggers

$\nu(x)$ = 0.5; $\nu(y)$ = 0.6;

$\nu'(x)$=0.8; $\nu'(y)$=0.6;



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Region Equivalence

$C_{A:\varphi}$**=7,** clocks ***x, y***

(q, ν)	(q, ν')	
$\nu(x)$=3.5 $\nu(y)$=5.5	$\nu'(x)$=3.7 $\nu'(y)$=5.3	**inequivalent**
$\nu(x)$=3.2 $\nu(y)$=5.5	$\nu'(x)$=4.3 $\nu'(y)$=5.8	**inequivalent**
$\nu(x)$=3.1 $\nu(y)$=5.8	$\nu'(x)$=3.5 $\nu'(y)$=5.51	**equivalent**
$\nu(x)$=13.5 $\nu(y)$=8.5	$\nu'(x)$=89 $\nu'(y)$=1003	**equivalent**
$\nu(x)$=5 $\nu(y)$=8.5	$\nu'(x)$=89 $\nu'(y)$=1003	**inequivalent**

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Region Equivalence Sizes

$C_{A:\varphi}$: **the biggest timing con**[…]**ed in *A and*** φ.

x=5?

choices of flag values

Choices of the integer part of a clock value.

$$2 \cdot |Q| \cdot |2^{P}| \cdot |2^{X}| \cdot (C_{A:\varphi} +1)^{|X|} \cdot |X|^{|X|}$$

choices of control locations

Is a clock ≤ $C_{A:\varphi}$?

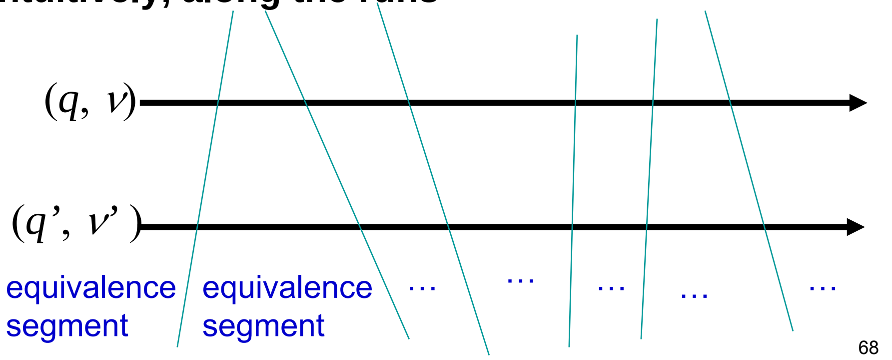
Choices of the ordering of the fractional parts of clock values

TCTL - model-checking problem

Theorem: for all ϕ *with* $C_{A:\phi} \leq C_{A:\varphi}$, *if* $(q, \nu) \equiv (q', \nu')$,

$A, (q, \nu) \models \phi$ iff $A, (q', \nu') \models \phi$

Intuitively, along the runs



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TCTL - model-checking problem

Region graph (V, F)

- **V: the set of equivalence subspace**
  - partition the state-space into a finite number of equivalence subspaces with the equivalence relatoin $s \equiv s'$.
  - $[s]$: the subspace in which all states $\equiv s$
  - region: a maximal equivalence subspace
- **F: transitions between [*s*], [*s'*]**
  - discrete transitions in the timed automata
  - time-progress of the timed automata

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TCTL - model-checking problem

to model-check TCTL formulas, we need an auxiliary clock *z* in the region graph

- ***ν*[*z*] is identical to *ν* except *ν*[*z*](*z*)= 0**
- $A,(q,\nu) \vDash \exists \varphi_1 U_{\sim c} \varphi_2$ iff $A,(q,\nu[z]) \vDash \exists \varphi_1 U_{\sim c} \varphi_2$
- $A,(q,\nu[z]) \vDash \exists \varphi_1 U_{\sim c} \varphi_2$ **iff there exists a region sequence $r_0 r_1 r_2 \ldots r_k$ from $(q,\nu[z])$**
  - $r_0 r_1 r_2 \ldots r_{k-1}$ satisfies φ_1
  - r_k satisfies φ_2 and $z \sim c$ 。

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Region Equivalence How to record a region ?

bits needed for recording a region

$\log (2 \cdot |Q| \cdot |2^P| \cdot |2^X| \cdot (C_{A:\varphi} + 1)^{|X|} \cdot |X|^{|X|})$
$= \log 2 + \log |Q| + |P| \log 2$
$\quad + |X| \log 2 + |X| \log (C_{A:\varphi} + 1) + |X| \log |X|$
$= 1 + \log |Q| + |P| + |X|$
$\quad + |X| \log (C_{A:\varphi} + 1) + |X| \log |X|$

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TCTL - model-checking problem

- **TCTL model-checking problem is PSPACE-complete. We need**
  - **a counter of log|# region| bits and**
  - **a last-region record of log|# region| bits**
- **TCTL satisfiability problem is undecidable.**

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TCTL
- model-checking problem

Workout:

Given a clock set X and a biggest timing constant $C_{A:\phi}$, please derive the complexity upper-bound of region count.

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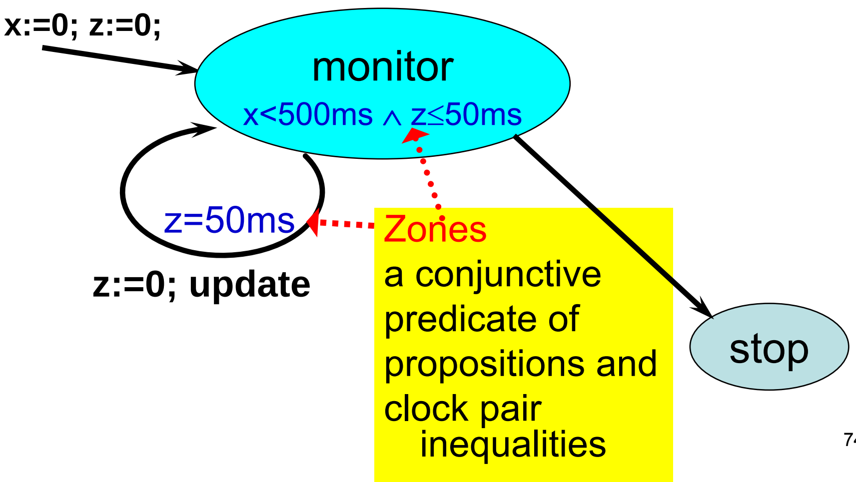
Zones in a timed automata



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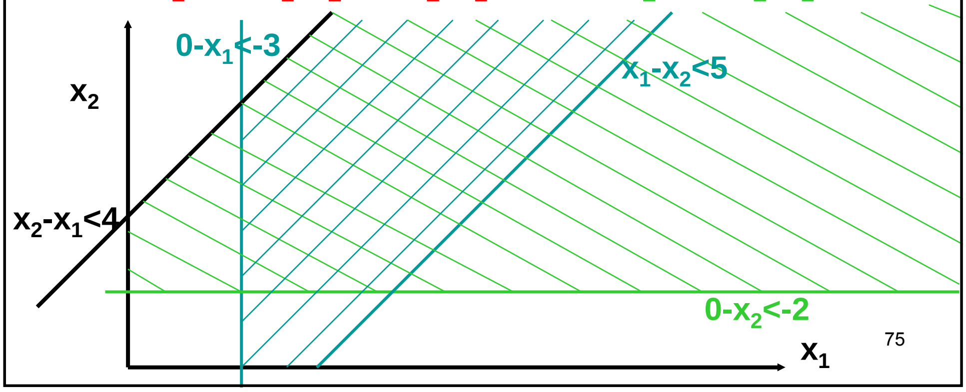
Zones

Two zones

$(0-x_1 \leq -3 \wedge x_1-x_2<5 \wedge x_2-x_1<4) \vee (0-x_2<-2 \wedge x_2-x_1<4)$

Can also be viewed as a union of set of literals,

$\{0-x_1 \leq -3, x_1-x_2<5, x_2-x_1<4\} \cup \{0-x_2<-2, x_2-x_1<4\}$



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Clock Zones

- Finite representation of infinite state-space
- Conjunction of inequalities such as:

$$x < c \quad | \quad x \leq c \quad | \quad x - y < c \quad | \quad x - y \leq c$$

$$x, y \in X, c \in \mathbf{Z}$$

- General form of a clock zone:

$$x_0 = 0 \wedge \bigwedge_{0 \leq i \neq j \leq n} (x_i - x_j \sim c_{ij})$$

$$\sim \in \{<, \leq\}$$

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Clock Zones (Operations)

- Clock zones are closed under 3 operations:
- Let z_1, z_2 be two clock zones, $Y \subseteq X$, $t \geq 0$
- **Intersection**: $z_1 \wedge z_2$ is a clock zone
  (Conjunction of conjunctions)
- **Clock Reset**: $z_1(Y{:=}0)$ is a clock zone
- **Time Elapse**: $z_1 + t$ is a clock zone

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Clock Zones (Time Elapse)

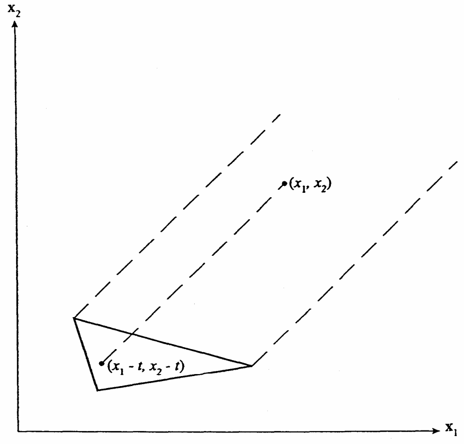

Figure 17.8
The clock zones φ and $\varphi^{\Uparrow}$.

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Zones = Symbolic States

- **Zone: (*s*, *z*) , where *s*: location, *z*: clock zone represents a symbolic state.**
- **Predecessor Clock Zone: *z*' = pred(*z*, *e*), where *z*: clock zone, *e*: transition (predecessor clock zone obtained from *z* before time elapse and executing transition *e*)**
- **Predecessor Zone: (*s*', pred(*z*, *e*)), where *e*: *s*→*s*'**

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Clock Zone (succ(*z*, *e*))

To obtain Predecessor Clock Zone (pred(*z*, *e*))

- **Reset all clocks from $\pi(e)$**
- **Intersect with $\tau(e)$**
- **Intersect with $\mu(s)$**
- **Let time elapse in *s* (operator te())**
- **Intersect *z* with $\mu(s)$**

pred(*z*, *e*) = tbck((*z* [π :=0]) $\wedge$ τ(*e*) $\wedge \mu$ (s)) $\wedge$ μ(s)

Closed under $\wedge$, tbck(), reset, also a clock zone!!!

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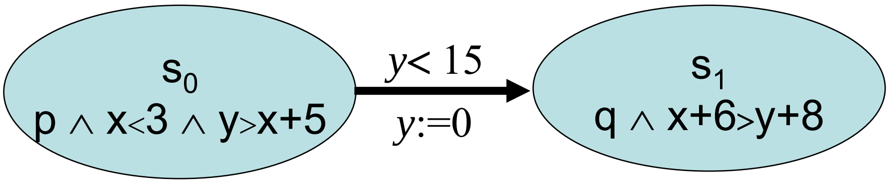
TCTL model-checking
- symbolic algorithms



In s_0,

- **what is the weakest precondition for the transition ?**

  xtion_bck$_e$(η): the weakest precondition through e to η ?

- **what is the weakest precondition to stay in s_0 through time progress $\delta \geq 0$?**

  time_bck(η): the weakest precondition through time progress to η ?

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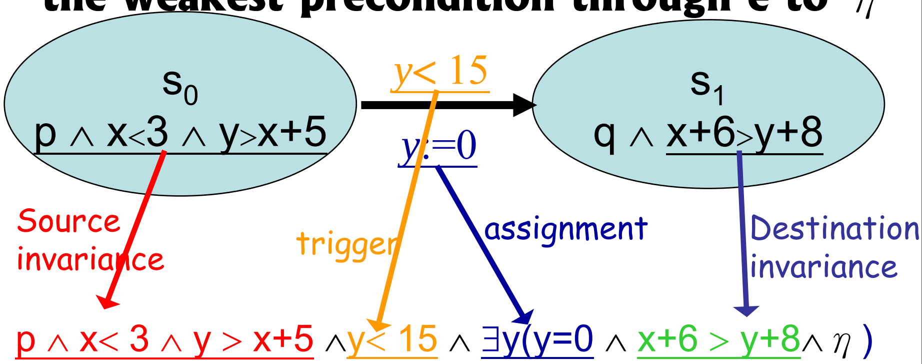
xtion_bck$_e$(η)
- the weakest precondition through e to η



How to get rid of the $\exists y$?

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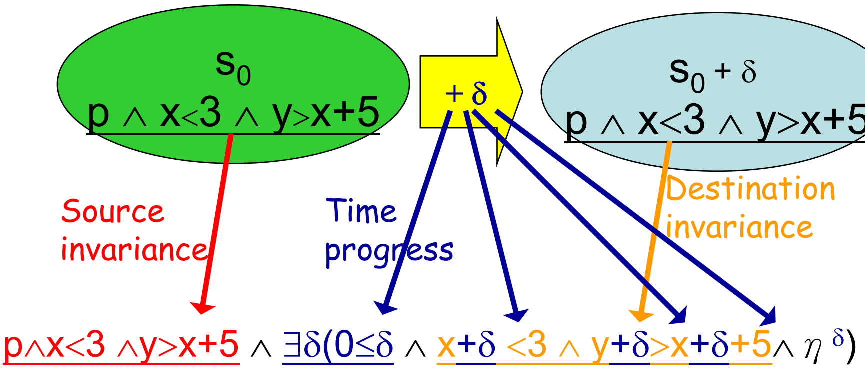
time_bck(η)
- the weakest precondition through time progress to η



How to get rid of the ∃δ ?

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How to get rid of the ∃δ ?

$$\exists\delta\ (2\text{-}x < \delta \wedge \delta < 15\text{-}y) \equiv 2\text{-}x<15\text{-}y$$

- Density of reals!
- Projections in geometry.

Joseph Fourier, 1824!
(1768-1830)

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How to get rid of the $\exists\delta$?

Assume that $\eta = 2\text{-}x < 0 \wedge 0 < 15\text{-}y$

s_0: $p \wedge x<3 \wedge y>x+5$ → $+\delta$ → s_0: $p \wedge x<3 \wedge y>x+5$

pairwise matching, eliminate δ with transitivity

$p \wedge y > x + 5 \wedge \exists\delta\,(0 \leq \delta \wedge 2\text{-}x < \delta \wedge \delta < 15\text{-}y \wedge \delta < 3\text{-}x)$

independent of δ — ***δ to the right*** — ***δ to the left***

$0 \leq \delta$ $\quad$ $\delta < 15\text{-}y$ $\quad$ ⇒ $\quad$ $0 < 15\text{-}y$, $0 < 3\text{-}x$

$2\text{-}x < \delta$ $\quad$ $\delta < 3\text{-}x$ $\quad$ $2\text{-}x < 15\text{-}y$, $2\text{-}x < 3\text{-}x$

thus we get $p \wedge y > x + 5 \wedge y<15 \wedge x< 3 \wedge y\text{-}x<13$

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TCTL model-checking - symbolic algorithms

The weakest precondition of time-progress in s_0: $x\text{-}2 > 0 \wedge y< 15 \wedge p \wedge x< 3 \wedge y > x+5$?

$s_0 + \delta$: $p \wedge x<3 \wedge y>x+5$ → $+\delta$ → s_0: $x\text{-}2>0 \wedge y<15 \wedge p \wedge x<3 \wedge y>x+5$

With the invariance condition of s_0,

$p \wedge y>x + 5 \wedge y<15 \wedge x<3 \wedge y\text{-}x<13 \wedge$ $\boldsymbol{p \wedge x<3 \wedge y>x+5}$

we still get

$p \wedge y > x + 5 \wedge y<15 \wedge x< 3 \wedge y\text{-}x<13$

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TCTL model-checking - symbolic algorithms

Safety analysis: Given initial condition I and risk condition $\neg\eta$,

How to evaluate if **$\neg\eta$ can happen ?**

S:= $\neg\eta$ ***; S':=false;***

while S $\neq$ ***S' {***

S':=S;

S:= S $\vee$ *time_bck* ($\vee_{e \in T}$ *xtion_bck* (*S, e*)) ;

}

return I $\wedge$ ***S*** **;**

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Zone Graph

- **Zone Graph is a transition system *Z*(*A*)**
- **States = zones of *A***
- **Initial state = (s, [X := 0])**
- **For each transition *e* of A:**

  a transition: (*s*, *z*) → (*s*', succ(*z*, *e*))
- **Zone reachability ➔State reachability**

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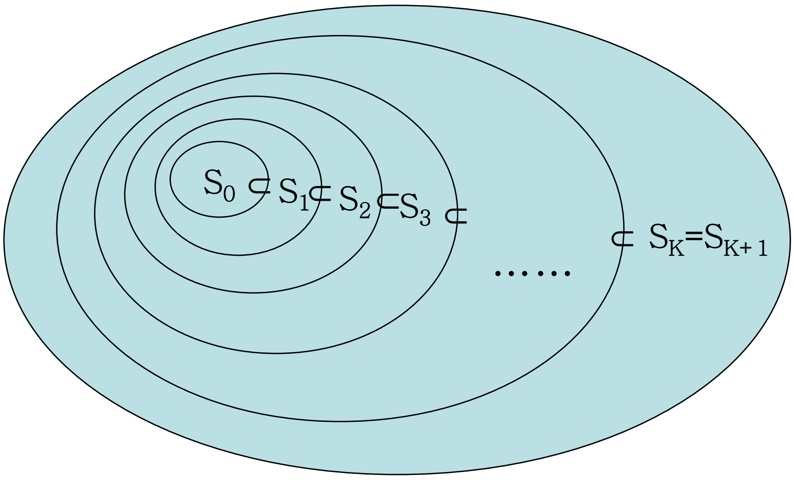
Backward Reachability Analysis
- *Least fixpoint calculation*



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Compute $\exists x(\ f_1(x_1, ..., x_n)<x \wedge ...\wedge f_h(x_1, ..., x_n)<x$
$\wedge \ x< g_1(x_1, ..., x_n)\wedge...\wedge x< g_k(x_1, ..., x_n))$

x in the upper-bound

An important technique:

with transitivity, pairwise match the inequalities with x

$f_1(x_1, ..., x_n)< x \wedge f_2(x_1, ..., x_n)< x \wedge ...\wedge f_h(x_1, ..., x_n)< x$
$\wedge \ x< g_1(x_1, ..., x_n)\wedge x< g_2(x_1, ..., x_n)\wedge...\wedge x< g_k(x_1, ..., x_n)$

x in the lower-bound

The condition for a solution to x

$\forall \ 1\leq i \leq k \ \forall \ 1\leq j \leq k \ (f_i(x_1, ..., x_n)< g_j(x_1, ..., x_n))$

i、j pairwise match

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Compute $\exists x(\ f_1(x_1, \dots, x_n)<x \wedge \dots \wedge f_h(x_1, \dots, x_n)<x \wedge x< g_1(x_1, \dots, x_n) \wedge \dots \wedge x< g_k(x_1, \dots, x_n))$

An important technique:

with transitivity, pairwise match the inequalities with x

$f_1(x_1, \dots, x_n)< x \wedge f_2(x_1, \dots, x_n)< x \wedge \dots \wedge f_h(x_1, \dots, x_n)< x$
$\wedge\ x< g_1(x_1, \dots, x_n) \wedge x< g_2(x_1, \dots, x_n) \wedge \dots \wedge x< g_k(x_1, \dots, x_n)$

In geometry, this is the projection of a convex hull in n+1 dimension space of $x, x_1, \dots, x_n$ **on the n dimension space of** $x_1, \dots, x_n$

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Compute $\exists x(\ f_1(x_1, \dots, x_n)<x \wedge \dots \wedge f_h(x_1, \dots, x_n)<x \wedge x< g_1(x_1, \dots, x_n) \wedge \dots \wedge x< g_k(x_1, \dots, x_n))$

In symbolic computation, an equivalence space is described with combinational inequalities.

Example: $x< 3 \wedge y > x+5 \wedge z \le x+5$

This is a convex hull surounded by the planes of $x=3$, $y = x+5$, **and** $z = x+5$ **in the 3-dimension space.**

Example: $x< 3 \wedge y > x+5 \wedge z \le w +5$

This is a convex hull surounded by the three planes of $x=3$, $y = x+5$, $z = w+5$ **in the 4-dimension space.**

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Zones

$(0-x_1 \le -3 \wedge x_1-x_2<5 \wedge x_2-x_1<4) \vee (0-x_2<-2 \wedge x_2-x_1<4)$

Normal forms

- *closure form:* ***all-pair shortest-path form***

  $(0-x_1 \le -3 \wedge 0-x_2<2 \wedge x_1-x_2<5 \wedge x_2-x_1<4)$

  $\vee (0-x_2<-2 \wedge 0-x_1<2 \wedge x_2-x_1<4)$

  - ***always the most number of constraints***

- *reduced form:* ***minimum number of constraints***

  $(0-x_1 \le -3 \wedge x_1-x_2<5 \wedge x_2-x_1<4)$

  $\vee (0-x_2<-2 \wedge x_2-x_1<4)$

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Data-structures for Zones - Difference Bound Matrix (DBM)

- DBM is a matrix to represent a clock zone

	0	x_1	$\dots$	x_n	
0		$< c$			$0 - x_1 < c$, i.e., $x_1 > c$
x_1	$< \infty$			$\le r$	$x_1 < \infty$ $x_1 - x_n \le r$
$\dots$					
x_n	$\le d$				$x_n \le d$

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Data-structures for Zones - DBM (Example)

- $x_1 - x_2 < 2 \quad \wedge \quad 0 < x_2 \leq 2 \quad \wedge \quad 1 \leq x_1$

	0	x_1	x_2
0	≤ 0	≤ -1	< 0
x_1	$< \infty$	≤ 0	< 2
x_2	≤ 2	$< \infty$	≤ 0

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Data-structures for Zones - DBM (Uniqueness)

- A zone is not uniquely represented by a DBM
- Zone: $x_1 - x_2 < 2 \quad \wedge \quad 0 < x_2 \leq 2 \quad \wedge \quad 1 \leq x_1$
- $x_1 - x_2 < 2$ and $x_2 \leq 2 \quad \Rightarrow \quad x_1 < 4$

	0	x_1	x_2
0	≤ 0	≤ -1	< 0
x_1	$< \infty$	≤ 0	< 2
x_2	≤ 2	$< \infty$	≤ 0

⇨

	0	x_1	x_2
0	≤ 0	≤ -1	< 0
x_1	< 4	≤ 0	< 2
x_2	≤ 2	$< \infty$	≤ 0

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Data-structures for Zones - DBM (Canonical Form)

- **Canonical (unique) form of DBM for a zone**
- **Tightening operation:**

$x_i - x_j \sim_{ij} d_{ij}$ **and** $x_j - x_k \sim_{jk} d_{jk}$ ➔ $x_i - x_k \sim_{ik} d_{ik}$

where $d_{ik} = d_{ij} + d_{jk}$

$\sim_{ik} = \leq$ **if both** $\sim_{ij}$ **and** $\sim_{jk}$ **are** $\leq$

$<$ **otherwise**

- **Apply tightening operations to a DBM until no more change is possible!**

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Data-structures for Zones - DBM (Emptiness)

- **Check if all elements on main diagonal are (≤ 0)**
  - Yes ➔ nonempty
  - No ➔ empty or unsatisfiable
- **Empty or unsatisfiable ➔ At least 1 negative entry on main diagonal**
- **E.g.** $x_i - x_i \leq -1$ ➔ $0 \leq -1$ ➔ **FALSE!!!**

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Data-structures for Zones
- DBM (3 operations: Intersection)

- **Intersection:** $D = D_1 \wedge D_2$ **(all DBMs)**
- **Let** $D_1(i, j) = \sim_1 c_1$ **and** $D_2(i, j) = \sim_2 c_2$

$$D(i, j) = (\min(c_1, c_2), \sim)$$

where $\sim = \sim_1$ **if** $c_1 < c_2$

$\sim = \sim_2$ **if** $c_2 < c_1$

$\sim = \sim_1$ **if** $c_1 = c_2$ **and** $\sim_1 = \sim_2$

$\sim = <$ **if** $c_1 = c_2$ **and** $\sim_1 \neq \sim_2$

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Data-structures for Zones
- DBM (3 operations: Clock Reset)

- **Clock Reset:** $D' = D\,[Y := 0],\ \ Y \subseteq X$
  defined as follows:
  - D'(i, j) = ($\leq$ 0) if $x_i, x_j \in Y$
  - D'(i, j) = D(0, j) if $x_i \in Y$ and $x_j \notin$ Y
  - D'(i, j) = D(i, 0) if $x_j \in Y$ and $x_i \notin$ Y
  - D'(i, j) = D(i, j) if $x_j \notin Y$ and $x_i \notin$ Y

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Data-structures for Zones - DBM (3 operations: Time Elapse)

- **Time Elapse**: $D' = \mathrm{te}(D)$ defined as follows:
- $D'(i, 0) = (< \infty)$, for any $i \neq 0$
- $D'(i, j) = D(i, j)$, for $i = 0$ or $j \neq 0$

	0	x_1	x_2
0	≤ 0	≤ -1	< 0
x_1	< 4	≤ 0	< 2
x_2	≤ 2	$< \infty$	≤ 0

⇒

	0	x_1	x_2
0	≤ 0	≤ -1	< 0
x_1	$< \infty$	≤ 0	< 2
x_2	$< \infty$	$< \infty$	≤ 0

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Data-structures for Zones - DBM (3 operations)

- All 3 operations can be **efficiently** implemented
- DBM must be **canonalized** (using tightening) before any of the 3 operations (intersection, clock reset, and time elapse)
- After any of the 3 operations, a DBM might **no longer** be **canonical**!
- **Last step: Reduce to canonical form!!!**

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Data-structures for Zones - DBM (Zone Graph Construction)

- **Clock zones: represented by DBM**
- **Succesor clock zones succ(*z*, *e*): computed by the 3 operations: intersection, reset, and time elapse on DBM in**

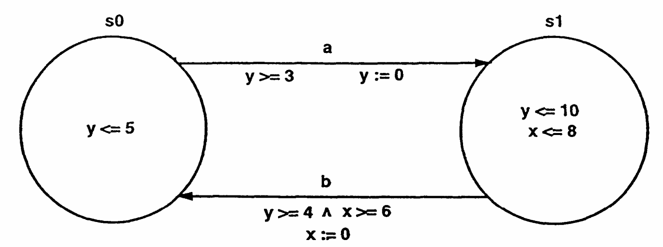

Figure 17.1
A simple timed automaton.

Data-structures for Zones - DBM (Zone Graph Construction)

- **Initial state: (s0, Z0), Z0: $x = 0 \wedge y = 0$**

	0	x	y
0	≤ 0	≤ 0	≤ 0
x	≤ 0	≤ 0	≤ 0
y	≤ 0	≤ 0	≤ 0

Zone D_0

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Data-structures for Zones - DBM (Zone Graph Construction)

- **Invariant μ (s0) is $0 \leq x \wedge 0 \leq y \leq 5$**

	0	x	y
0	≤ 0	≤ 0	≤ 0
x	$< \infty$	≤ 0	$< \infty$
y	≤ 5	≤ 5	≤ 0

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Data-structures for Zones - DBM (Zone Graph Construction)

- **Step 1: Intersection D_0 with μ (s0) gives D_0**
- **Step 2: Let time elapse: te($D_0 \wedge \mu$ (s0))**

	0	x	y
0	≤ 0	≤ 0	≤ 0
x	≤ 0	≤ 0	≤ 0
y	≤ 0	≤ 0	≤ 0

⇨

	0	x	y
0	≤ 0	≤ 0	≤ 0
x	$< \infty$	≤ 0	≤ 0
y	$< \infty$	≤ 0	≤ 0

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Data-structures for Zones
- DBM (Zone Graph Construction)

- **Step 3: Intersect with μ (s0) again**

	0	x	y
0	≤ 0	≤ 0	≤ 0
x	$< \infty$	≤ 0	≤ 0
y	$< \infty$	≤ 0	≤ 0

⇨

	0	x	y
0	≤ 0	≤ 0	≤ 0
x	≤ 5	≤ 0	≤ 0
y	≤ 5	≤ 0	≤ 0

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Data-structures for Zones
- DBM (Zone Graph Construction)

- **Trigger τ(a) = $y \geq 3$**

	0	x	y
0	≤ 0	≤ 0	≤ -3
x	$< \infty$	≤ 0	$< \infty$
y	$< \infty$	$< \infty$	≤ 0

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Data-structures for Zones - DBM (Zone Graph Construction)

- **Step 4: Intersect with trigger τ(a)**

	0	x	y
0	≤ 0	≤ 0	≤ 0
x	≤ 5	≤ 0	≤ 0
y	≤ 5	≤ 0	≤ 0

⇒

	0	x	y
0	≤ 0	≤ -3	≤ -3
x	≤ 5	≤ 0	≤ 0
y	≤ 5	≤ 0	≤ 0

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Data-structures for Zones - DBM (Zone Graph Construction)

- **Step 5: Reset clock *y* in DBM**

	0	x	y
0	≤ 0	≤ -3	≤ -3
x	≤ 5	≤ 0	≤ 0
y	≤ 5	≤ 0	≤ 0

⇒

	0	x	y
0	≤ 0	≤ -3	≤ -3
x	≤ 5	≤ 0	≤ 5
y	≤ 0	≤ -3	≤ 0

$Z1 = 3 \le x \le 5 \wedge 3 \le x - y \le 5 \wedge y = 0$

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Data-structures for Zones - DBM (Zone Graph Construction)

- **Successor of (s0, Z0) is (s1, Z1)**
- **Repeat the same 5 steps:**
- **(s0, $4 \le y \le 5 \wedge 4 \le y - x \le 5 \wedge x = 0$)**
- **(s1, $0 \le x \le 1 \wedge 0 \le x - y \le 1 \wedge y = 0$)**
- **(s0, $5 \le y \le 8 \wedge 5 \le y - x \le 8 \wedge x = 0$)**
- **(s1, $x = 0 \wedge y = 0$),**
  which is contained in the 2nd zone, thus no more new zones can be generated!!!
  No more change! Stop! Zone Graph!

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Data-structures for Zones - CRD: Clock-Restriction Diagram

- **A BDD-like data-structure**
- **Recording device for (zone) DBM set**
- **variables like x-x'**
- **Arc values like (<, d), d $\in[-C_A, C_A]\cup\{\infty\}$; or**
  (≤, d), d $\in[-C_A, C_A]$
- **Default value on arcs: (<, ∞)**
  – No constraint!

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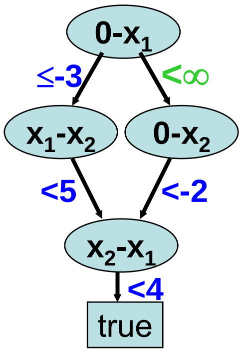
Data-structures for Zones - CRD Example

$(0-x_1 \le -3 \wedge x_1-x_2<5 \wedge x_2-x_1<4) \vee (0-x_2<-2 \wedge x_2-x_1<4)$



- Each path represents a zone, which is convex.
- The whole CRD represents a concave state-space.
- No canonical anymore.
- <∞ can be used sometimes.

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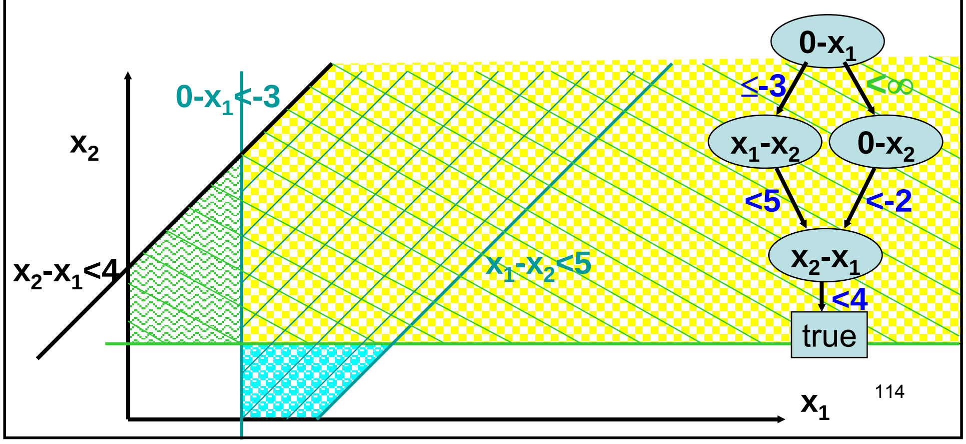
Data-structures for Zones - *Representation fragmentation of CDD*

Two zones

$(0-x_1 \le -3 \wedge x_1-x_2<5 \wedge x_2-x_1<4) \vee (0-x_2<-2 \wedge x_2-x_1<4)$



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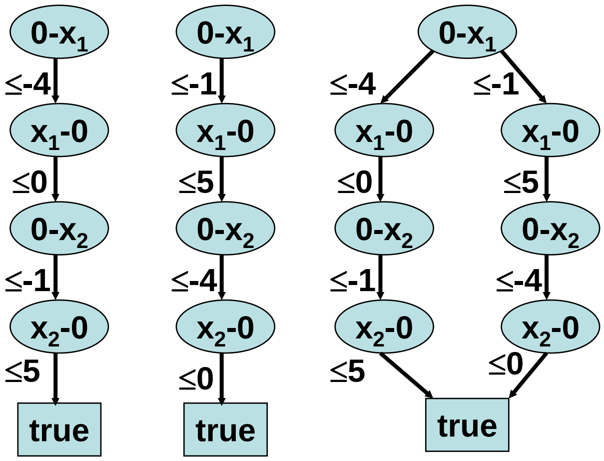
Data-structures for Zones
- *CRD union is simple.*

Example: $(0 \le x_1 \le 4 \wedge 1 \le x_2 \le 5) \vee (1 \le x_1 \le 5 \wedge 0 \le x_2 \le 4)$



- ***Linear to the sizes of components***
- ***Preserve zone structures***

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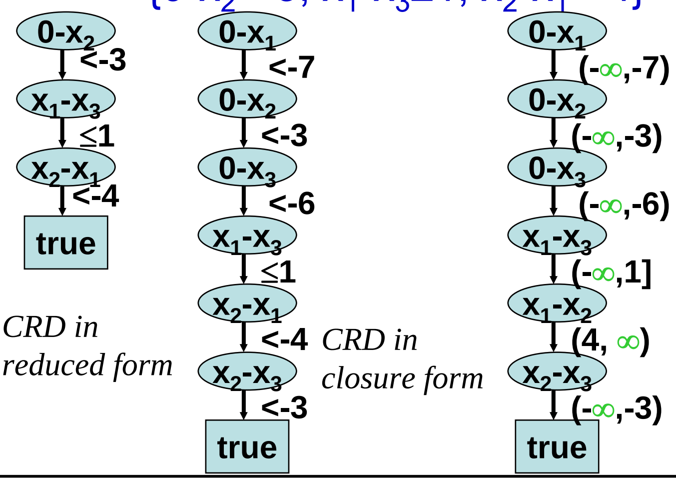
Data-structures for Zones
- *CRD is not canonical!*

In general, reduced CRDs have much fewer nodes along each path!

$\{0-x_2<-3,\ x_1-x_3 \le 1,\ x_2-x_1<-4\}$



CRD in reduced form

CRD in closure form

CDD in closure form

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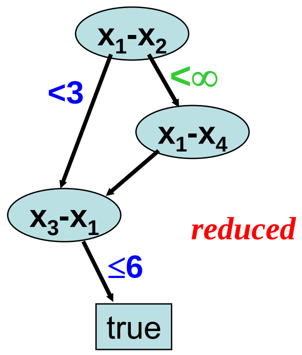
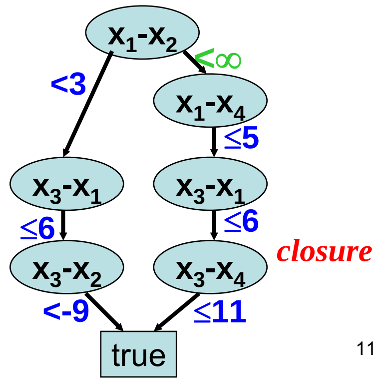
Data-structures for Zones - *Sensitivity to choice of normal forms*

Reduced CRDs are less likely to interfere data-sharing!

(x1-x2<3∧ x3-x1≤6)
∨ (x1-x4≤5∧ x3-x1≤6)



(x1-x2<3 ∧ x3-x1≤6 ∧ x3-x2≤6)
∨(x1-x4≤5 ∧ x3-x1≤6 ∧ x3-x4≤11)



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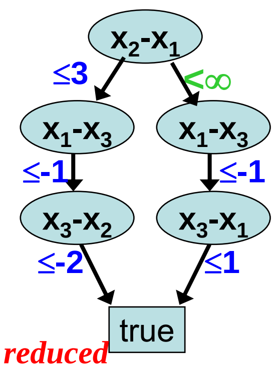
Data-structures for Zones - *Sensitivity to choice of normal forms*

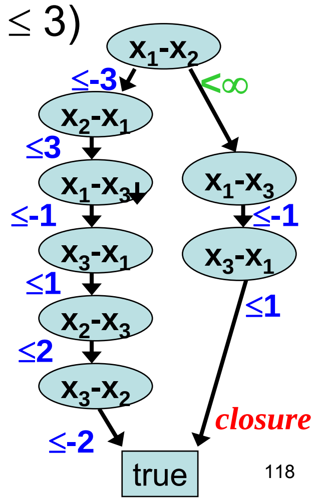
Reduced form makes it difficult to detect zone-containment.

$(x_1-x_3 \leq -1 \wedge x_3-x_2 \leq -2 \wedge x_2-x_1 \leq 3)$

$\vee\ (x_1-x_3 \leq -1 \wedge x_3-x_1 \leq 1)$



Note, it is $O(n^3)$ to deduce the all-pair shortest-path relation.



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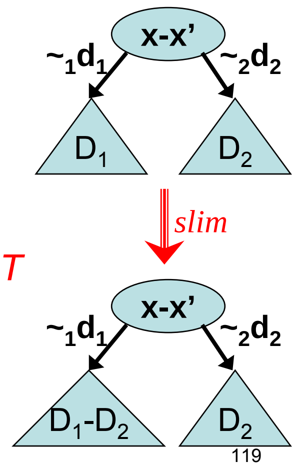
Data-structures for Zones
- *Contained zone-path elimination*

- A node with single outgoing arc labeled <∞ can be bypassed.
- Given two arcs, when
  - $\sim_1 d_1$ more restrictive than $\sim_2 d_2$
  - $D_1 \subseteq D_2$

  then D1 can be removed.



★ *The operation MAY or MAY NOT lead to smaller CRD sizes.*

★ *Don't know how to do this with CDD.*

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Data-structures for Zones
- *Set-oriented manipulations on CRDs*

Given two CRDs D_1:{ζ_1, ζ_2} and D_2:{ζ_2, ζ_3},

- $D_1 \cap D_2$ is the CRD for {ζ_2} $O(|D_1|\times|D_2|)$
- $D_1 \cup D_2$ is the CRD for {$\zeta_1, \zeta_2, \zeta_3$} $O(|D_1|\times|D_2|)$
- D_1 - D_2 is the CRD for {ζ_1} $O(|D_1|\times|D_2|)$

Space-intersection: $D_1 \wedge D_2$

- For every $\zeta_1(x,x') = (\sim_1, d_1)$ and $\zeta_2(x,x') = (\sim_2, d_2)$

$$\zeta_1 \wedge \zeta_2(x,x') = \begin{cases} (\sim_1, d_1) & \text{if } d_1 < d_2 \vee (d_1 == d_2 \wedge \sim == \text{"<"}) \\ (\sim_2, d_2) & \text{otherwise} \end{cases}$$

- $D_1 \wedge D_2 = \{\zeta_1 \wedge \zeta_2 \mid \zeta_1 \in D_1; \zeta_2 \in D_2\}$ $O(|D_1|^2\times|D_2|^2)$

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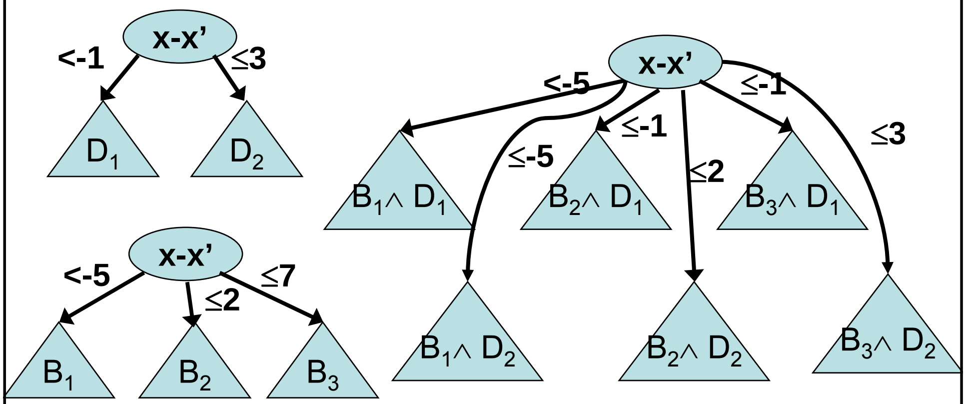
Data-structures for Zones

- *How to translate the mathematics to procedures on CRDs ?*

Space-intersection: $D \wedge B = \{\zeta_d \wedge \zeta_b \mid \zeta_d \in D; \zeta_b \in B\}$



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Data-structures for Zones

- Style of CRD manipulating algorithm

set Ψ; /* database of already-processed cases */

$\cup$(B,D) { $\Psi = \emptyset$; return rec$\cup$(B,D); }

rec$\cup$(B, D) with B=(x_B-x_B',(β_i, B_i)$_{1\leq i\leq n}$),D =(x_D-x_D',(α_j, D_j)$_{1\leq j\leq m}$) {

  if B==true, return true; else if D is true, return true;

  else if $\exists$H,(B,D,H)$\in\Psi$, return H; /* efficiency on common structures */

  else if x_B-x_B' precedes x_D-x_D', H:=(x_B-x_B' , (β_i, rec$\cup$(B_i,D))$_{1\leq i\leq n}$);

  else if x_D-x_D' precedes x_B-x_B', H:=(x_D-x_D' , (α_j, rec$\cup$(B, D_j))$_{1\leq j\leq m}$);

  else {

    for (i=n, j= m, H=false; i$\geq$1$\wedge$j $\geq$ 1, do {

      if β_i == α_j , { H= H $\cup$ (x_B-x_B', (β_i , rec$\cup$(B_i,D_j))); i--; j--; }

      else if β_i< α_j, { H= H $\cup$ (x_B-x_B', (α_j, D_j)); j--; }

      else if β_i> α_j, { H= H $\cup$ (x_B-x_B', (β_i , B_i)); i--; }

    }

    if I $\geq$ 1, H= H $\cup$ (x_B-x_B', (β_h , B_h)$_{1\leq h\leq i}$);

    if j $\geq$ 1, H= H $\cup$ (x_B-x_B', (α_h , D_h)$_{1\leq h\leq j}$);

  }

  $\Psi = \Psi \cup$ {(B,D,H)}; return H; /* saving results for common structures */

}

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Data-structures for Zones - *BDD+CRD*

Can combine BDD with CRD in the same data-structure.

- $D_1 \cap D_2$ is like $D_1 \wedge D_2$
- $D_1 \cup D_2$ is like $D_1 \vee D_2$
- $D_1 - D_2$ is like $D_1 \wedge \neg D_2$

with integrated evaluation ordering.

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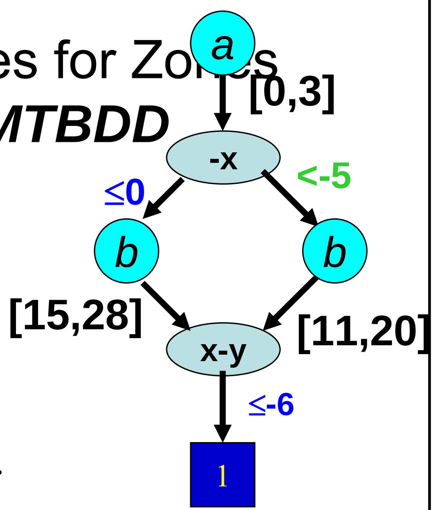
Data-structures for Zones - *CRD+MTBDD*

- Case analysis in the procedures for MTBDD and CRD
- One integrated variable ordering for MTBDD variables and CRD variables.



discrete a, b: 0..30;

clock x, y;

$0 \le a \le 3 \wedge (-x \le 0 \wedge 15 \le b \le 28 \vee -x < -5 \wedge 11 \le b \le 20) \wedge x-y \le -6$

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Data-structures for Zones

- *Exercise: Costruction of a CRD+MTBDD*

discrete a, b:0..100.

clock x_1, x_2, x_3;

$(a = 3 \wedge x_1 - x_3 \leq -1 \wedge x_3 - x_2 \leq -2$

$\wedge\, b \in [10,20] \wedge x_2 - x_1 \leq 3)$

$\vee\, (a \in [1,5] \wedge b \in [16,20] \wedge x_1 - x_3 \leq -1 \wedge x_3 - x_1 \leq 1)$

Assume the variable ordering:

$a \angle x_1 - x_3 \angle x_3 - x_2 \angle b \angle x_2 - x_1 \angle x_3 - x_1$

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Workout *for symbolic verification of timed automata*

Given transition (q,q'): when *x* $\leq$ *11* may *y:= 0*;

μ(q): *4 $\leq$ y $\wedge$ y < 25,*

and *ϕ = 9 < x $\wedge$ x-y $\leq$ 7 $\wedge$ y $\leq$ 5*

please calculate

- ***xtion_bck* (ϕ, (q, q'))**
- ***time_bck* (*xtion_bck* (ϕ, (q,q')))**

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TCTL inevitability analysis

- *Safety properties:* ∀□φ
  - Negation: reachability properties: ∃◊φ
    - **Least fixpoint evaluation**
  - Heavily researched for efficient evaluation
  - nonZeno requirement, not very necessary
- *Inevitability properties:* ∀◊φ
  - Kind of parallel to *liveness* properties in LTL
  - Negation: ∃□φ
    - **Greatest fixpoint evaluation**
  - Not very much researched for efficient evaluation
  - nonZeno requirement necessary

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Model checking with Non-Zeno requirement (1)

- Basic procedures
  - Xtion_bck(η ,e)
    - weakest precondition of discrete transitions
  - Time_bck(η)
    - backward time-progression
- Reachable-bck(η_1, η_2) $\equiv$ lfpY.($\eta_2 \vee (\eta_1 \wedge$time_bck($\eta_1 \wedge \bigvee_{e \in T}$xtion_bck(Y,e))))

The lfp (least fixpoint) of F(Y) is the minimal solution to Y=F(Y).

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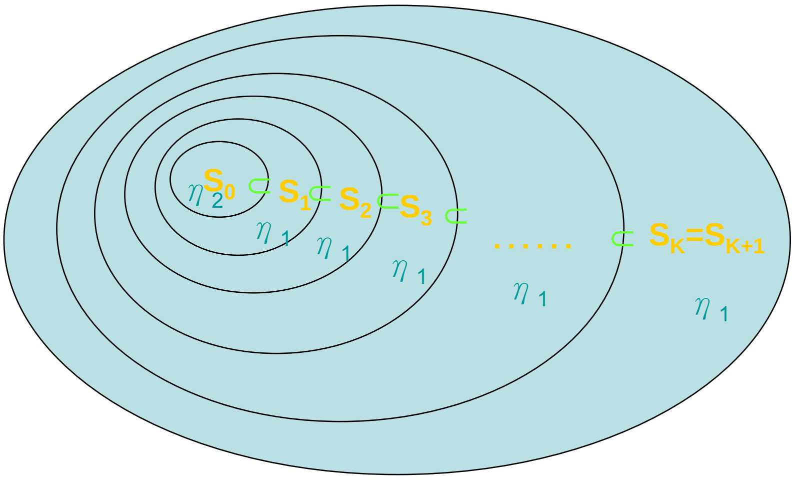
Reachable-bck(η_1, η_2)
- *Least fixpoint calculation*



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Model checking with Non-Zeno requirement (2)

- Lemma: given d≥1, A,

  ν ⊨∃□ η iff there is a finite run ρ
  - from ν
  - of duration ≥d ≥1
  - along ρ every state satisfies η and
  - ρ ends at a state satisfying ∃□ η
- ∃□ η ≡gfp Y.(ZC.reachable-bck(η ,Y∧ZC≥d))

The gfp (greatest fixpoint) of F(Y) is the maximal solution to Y=F(Y).

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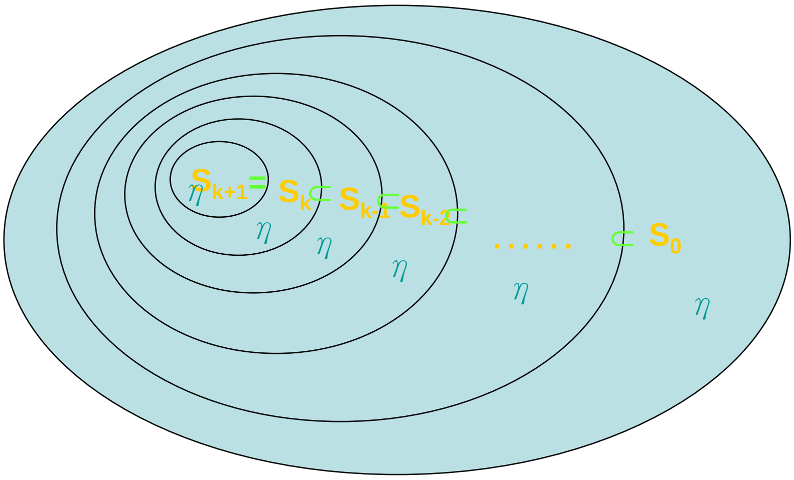
∃□ η
- *Greatest fixpoint calculation*



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Gfp procedure

∃□ η ≡gfp Y.(ZC.reachable-bck(η ,Y∧ZC≥d))

```
gfp( η ){
    Y:= η ; Y':=true;
    Repeat until Y=Y',{
      Y':=Y;
      Y:=Y∧∃ZC  ( ZC=0∧reachable-bck( η ,Y∧ZC≥d));
    }
    return Y;
 }
```

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EDGF
- early decision on GFP evaluation

- Observation
  - The state space shrinks iteratively
- Basic idea
  - Stop at a gfp iteration if already no target states are in the gfp.
- Cost
  - Small extra computation

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EDGF
- early decision on GFP evaluation

Example:

$$\textbf{TargetIdentified} \rightarrow \forall\Diamond\textbf{TargetHit}$$

- **After negation,**

$$\textbf{TargetIdentified} \wedge \exists\Box \neg \textbf{TargetHit}$$

- **Can quit evaluation iff**

the intersection is already empty!!!

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Linear hybrid automata

FSA + continuous variables

- **Continuous variables: rationals, reals**
  - temperature, volume, velocity, distance, voltage, …
- **Continuous variables + their derivatives for system state descriptions/transitions**
  - $T > 100℃ \wedge V < 400l \wedge dV/dt < 10l/s$
  - $dT/dt = 6℃/s \wedge V < 400l$

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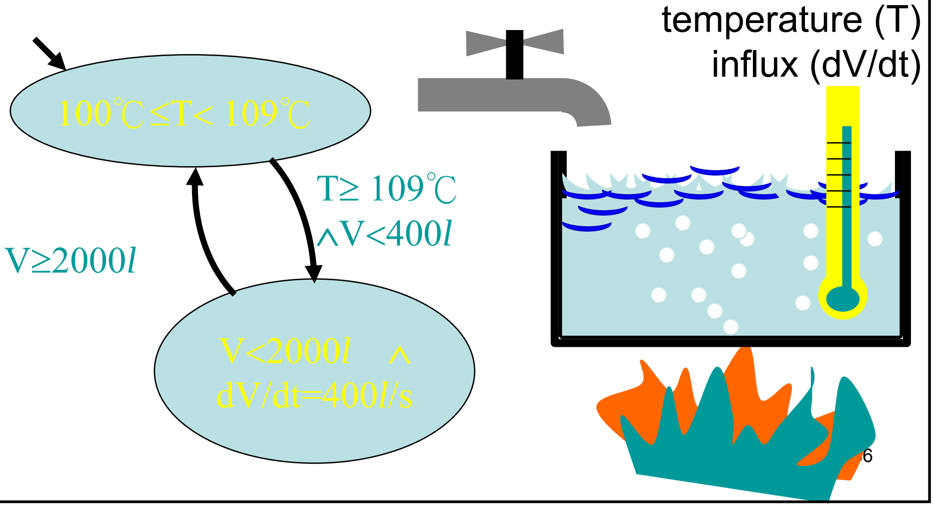
Linear hybrid automata - example

A boiler control system with
water volume (V)
temperature (T)
influx (dV/dt)



6

Linear hybrid automata - verification

- **linear state-predicates, e.g.,**

  $6x + y + 5 < 2z + 12$

  and their Boolean combinations
  - *reachability problem is undecidable.*
- **In timed automata with constraints like $5 < y$, $x < 5$**

  model-checking problem is in PSPACE.

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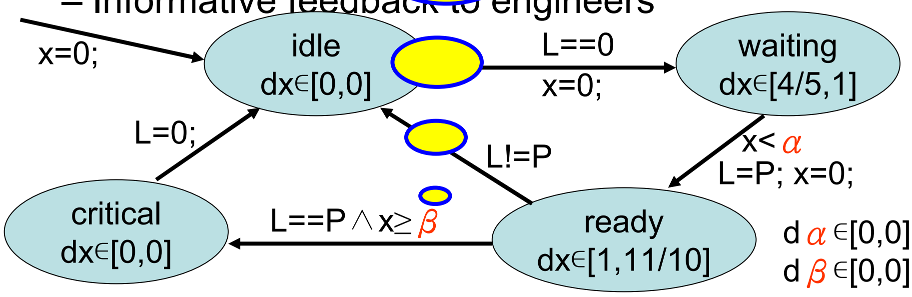
Linear hybrid automata - p alysis

- **Expre tems**
- **Genera ric analysis**
  - Informative feedback to engineers

What values of α and β make the system safe ?



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Linear hybrid automata

- parametric safety analysis (PSA)

Given an LHA A and a safety predicate η ,

``What is the constraint on parameters that make A safe w.r.t. η ?''

i.e., η is satisfied in all states along all computations of A.

- Decision support
- Informative feedback to engineers
- An undecidable problem

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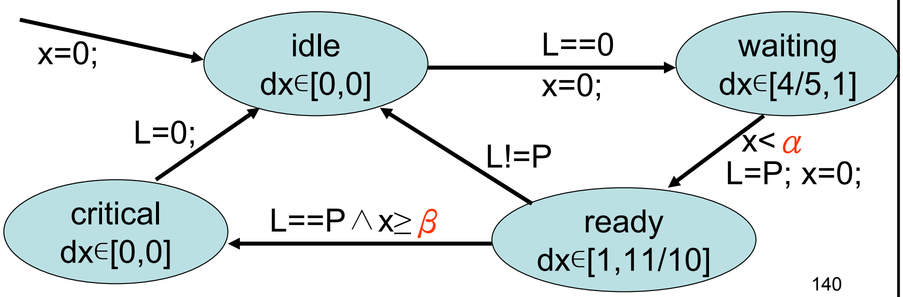
Linear hybrid automata

- parametric safety analysis (PSA)

Given an LHA A and a safety predicate η ,

"What is the constraint on parameters that make A safe w.r.t. η ?''

Safety solution: **$-\alpha<0 \wedge -11\alpha+8\beta<0$**



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Linear hybrid automata
- How to get rid of ∃δ ?

Example:

x-3y<-5Ù 4x+2y <9, dx/dt ∊ [1,3], dy/dt ∊[-10,-9],

what is the weakest precondition of time-progress ?

Techniques:

present copies of variables are *x', y'*

future copies (after δ time units) are *x, y*

Solution:

∃x' ∃y' ∃δ (x'-3y'<-5∧ 4x'+2y' <9

∧ 0≤δ ∧ δ≤ *x-x'*≤3δ ∧ -10δ ≤ *y-y'* ≤-9δ

)

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Linear hybrid automata
- related works

The technology for LHA parametric analysis

- **Non-algorithmic in general**
- **State-space analysis te**
  - Convex polyhed
  - Frames
  - Zones for time
    - DBM (Differe
    - DDD, CDD, CRD (D

Can LHA analysis benefit from BDD-technology ?

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Linear hybrid automata - symbolic verification

Convex polyhedra:

Given system variables $x_1, x_2, \ldots, x_n$,

$$\bigwedge_k a_{1k} x_1 + a_{2k} x_2 + \ldots + a_{nk} x_n \sim c_k$$

- $a_{1k}, a_{2k}, a_{nk}, c_k \in Z$
- $\sim$ is $<, \leq, =, \geq, >$

Example: $11\alpha - 8\beta < 0 \wedge 4\alpha - 5x_1 + 9x_2 < 0$

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Parametric safety analysis

$\neg \exists (X\text{-}H)$
$(I \wedge \text{lfp } Z.(\text{time}(\neg \eta, q_f) \vee \bigvee_{e=(q,q') \in E} \text{time}(\text{xtion}(Z,e),q)))$

- **H: the set of static parameters**
- η **: safety predicate (in location q_f)**
- **I: initial predicate**
- **time(η',q): WPC through time passage**
- **xtion(η',e): WPC through discrete transition e**

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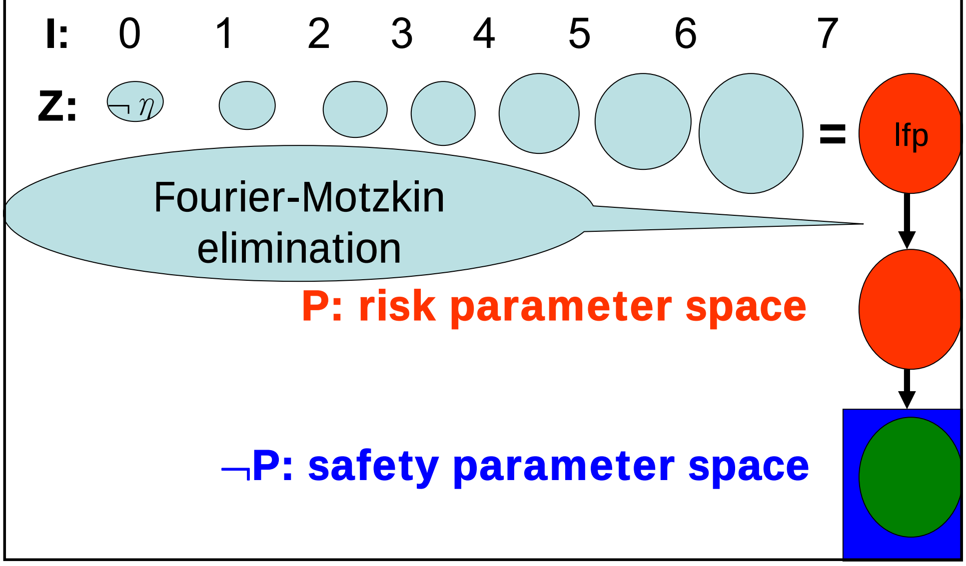
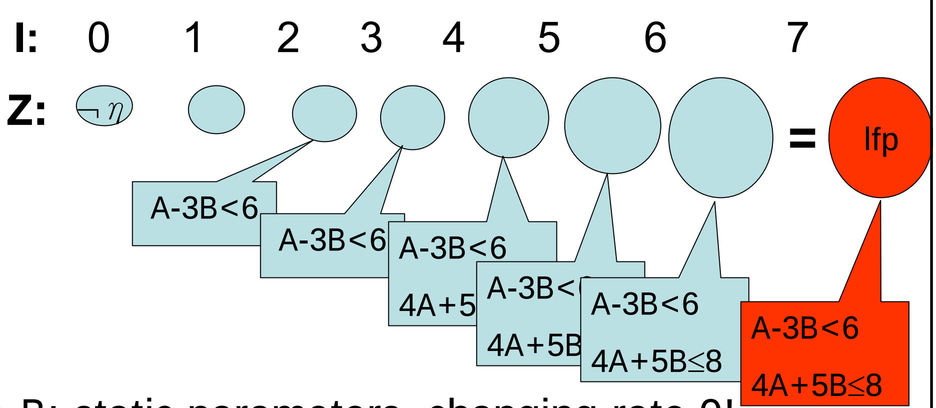
Animation of parametric safety analysis



Observation for speedup



A,B: static parameters, changing rate 0!

Static parameter constraints are stable!

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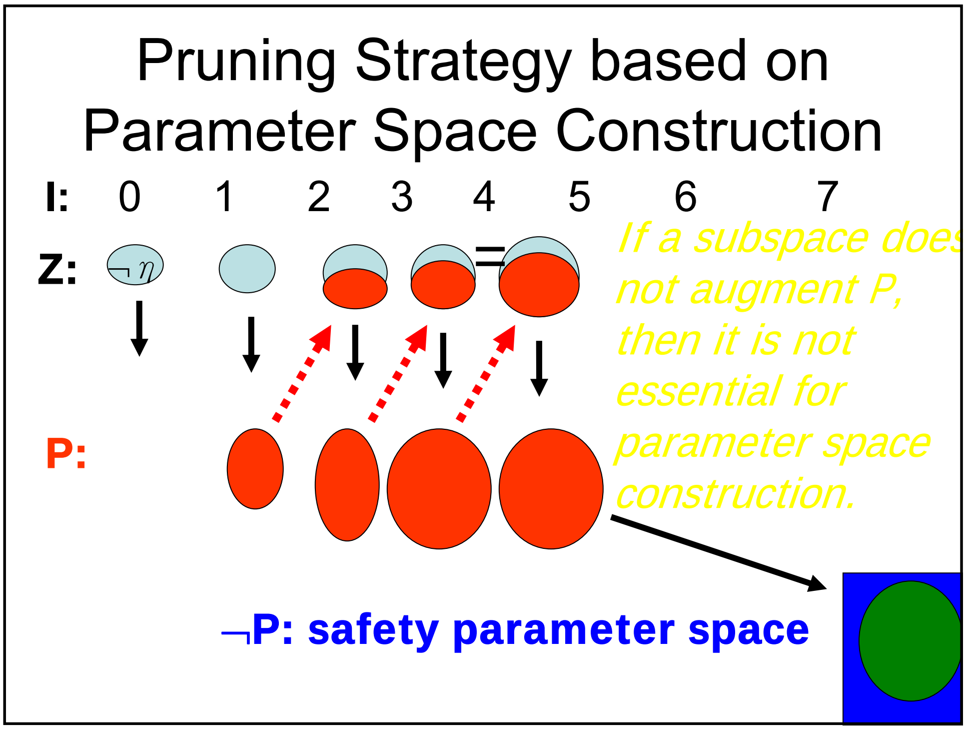
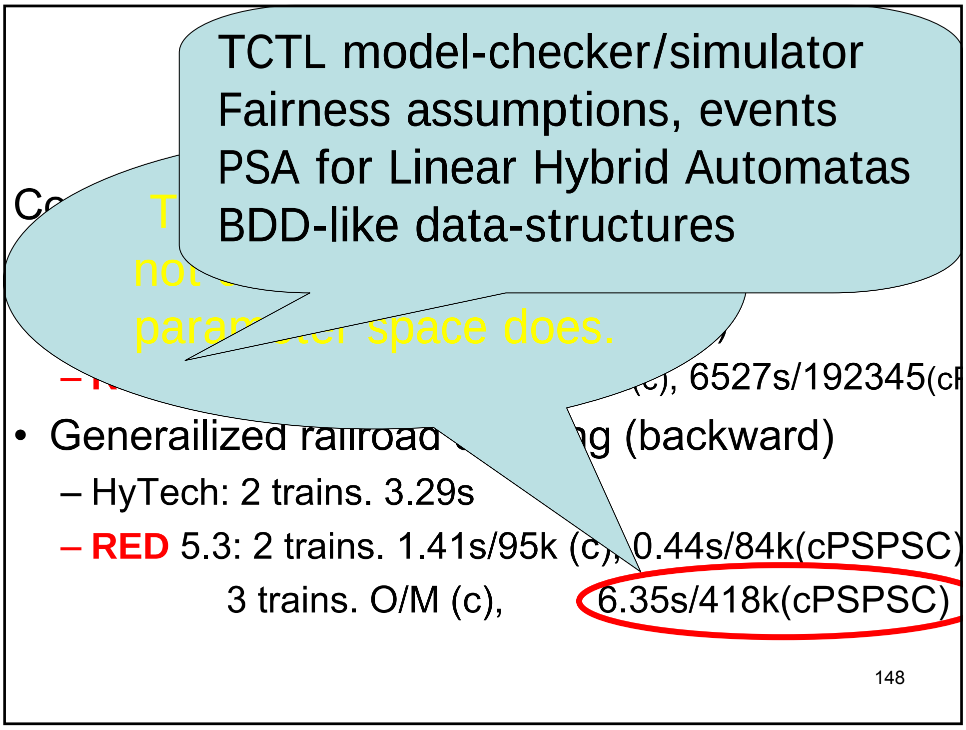

Pruning Strategy based on Parameter Space Construction
I: 0 1 2 3 4 5 6 7
Z:
¬η
P:
If a subspace does not augment P, then it is not essential for parameter space construction.
¬P: safety parameter space


TCTL model-checker/simulator
Fairness assumptions, events
PSA for Linear Hybrid Automatas
BDD-like data-structures
parameter space does.
(c), 6527s/192345
• Generailized railroad
ng (backward)
– HyTech: 2 trains. 3.29s
– RED 5.3: 2 trains. 1.41s/95k (c), 0.44s/84k(cPSPSC)
3 trains. O/M (c), 6.35s/418k(cPSPSC)
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Experiment

When PSPSC does not perform, it charges a constant ration overhead.

– …7k (m), 0.66s/105k(cPS

6 senders … (m), 40.58s/3513k(…)

- CSMA/CD (forward)
  - HyTech: 4 proc. 68.75s
  - **RED** 5.3: 4proc. 17.49s/378k (c), 27.03s/378k(cPSPS
    6proc. 3123s/11525k (c), 4163s/11552k(cPSP

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Experiment

Comparison with HyTech 2.4.5

- Fischer’s (backward)
  - HyTech: 4 proc. 28.04s
  - **RED** 5.3: 4 proc. 12.38s/215k (m), 5.14s/163k(cPSPS
    6 proc. 1485s/4000k (m), 168.6s/1170k(…)
- Fischer’s (forward)
  - HyTech: 3 proc. 37.89s
  - **RED** 5.3: 3proc. 19.18s/654k (m), 5.59s/538k(cPSPSC

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Experiment

Comparison with HyTech 2.4.5
- Nuclear reactor (backward)
  - HyTech: 6 rods. 647.8s
  - **RED** 5.3: 6 trains. 839.3s/8191k (m)
    461.8s/6941k(cPSPSC)

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Experiment

Comparison with TReX 1.3

Parameters,
Clocks,
Lossy channels

- Fischer's (forward)
  - TReX: 2 proc. 1.12s
  - **RED** 5.3: 4 proc. 197s/2714k (m),
    5 proc. 752.7s/5254k(cPSPSC)
- Fischer's (backward)
  - TReX: 2 proc. 8.96s
  - **RED** 5.3: 6 proc. 567.1s/4341k (m),
    170.3s/2798k(cPSPSC)

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Experiment

Comparison with TReX 1.3

- Nuclear reactor (backward)
  - TReX: 2 rods. O/M
  - **RED** 5.3: 6 rods. 839.3s/8191k (m),
    461.8s/6941k(cPSPSC)

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