

# **Chapter 12. Operational Amplifier Circuits**

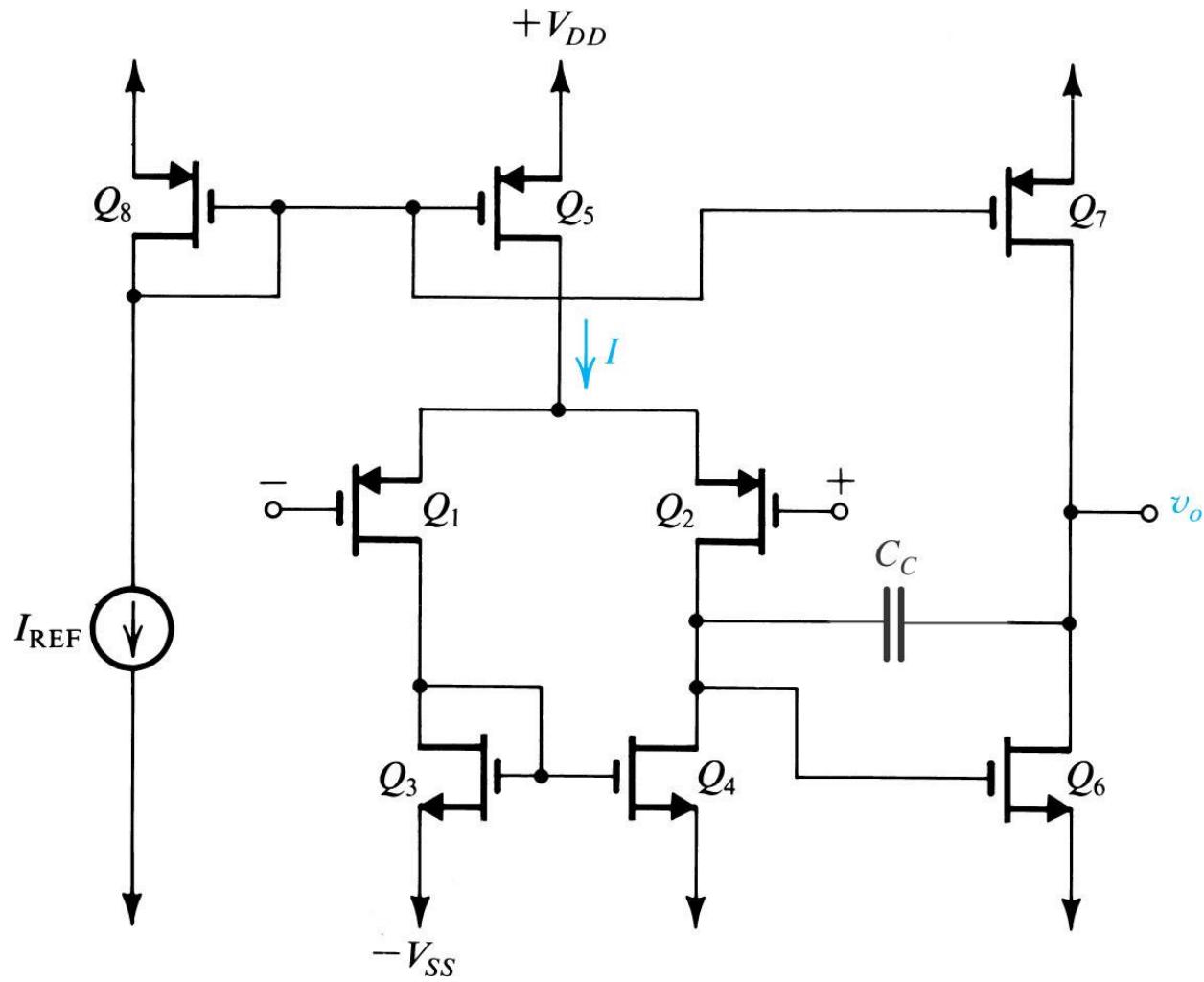
**8.6.1 A Two Stage CMOS Op Amp**

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**12.1 The Two Stage CMOS Op Amp**

**12.2 The Folded Cascode CMOS Op Amp**

## 12.1 The Two Stage CMOS Op Amp

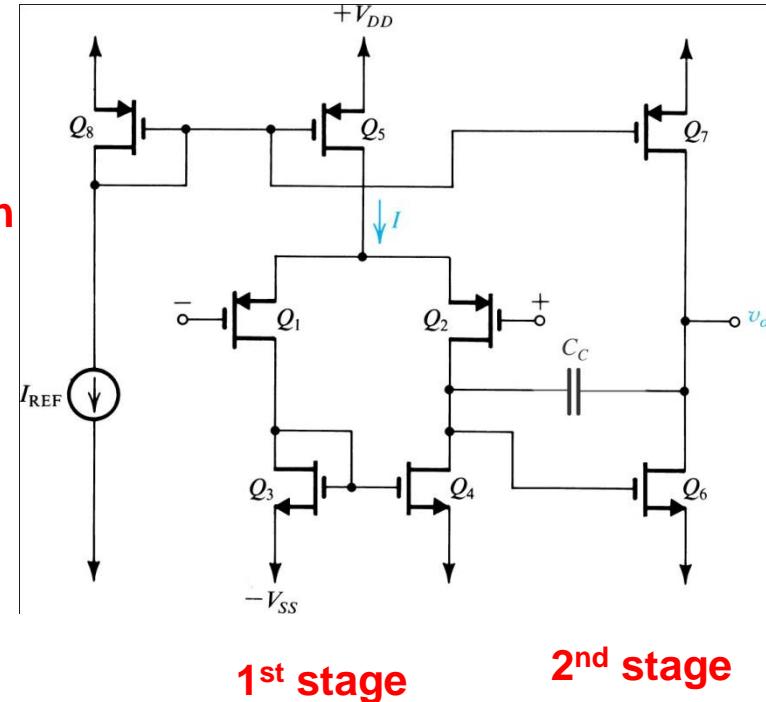


## 12.1.1 The Circuit

Bias generation

- Voltage gain
  - Voltage gain of 1<sup>st</sup> stage:  $A_1 = -g_{m1}(r_{o2} \parallel r_{o4})$
  - Voltage gain of 2<sup>nd</sup> stage:  $A_2 = -g_{m6}(r_{o6} \parallel r_{o7})$
  - DC open-loop gain:  $A_v = A_1 \times A_2$
- Input offset voltage
  - Random offset*: device mismatches as random in nature
  - Systematic offset: due to design technique  $\Rightarrow$  predictable
 
$$\Rightarrow \frac{(W/L)_6}{(W/L)_4} = 2 \frac{(W/L)_7}{(W/L)_5}$$

If this condition is not met, a systematic offset will result.
- $C_C$  is Miller-multiplied by the gain of the second stage to provide the required dominant pole.



1<sup>st</sup> stage

2<sup>nd</sup> stage

## 12.1.2 Input Common-Mode Range

- To keep  $Q_1$  and  $Q_2$  in saturation.

$$V_{ICM} \geq -V_{SS} + V_{tn} + V_{OV3} - |V_{tp}|$$

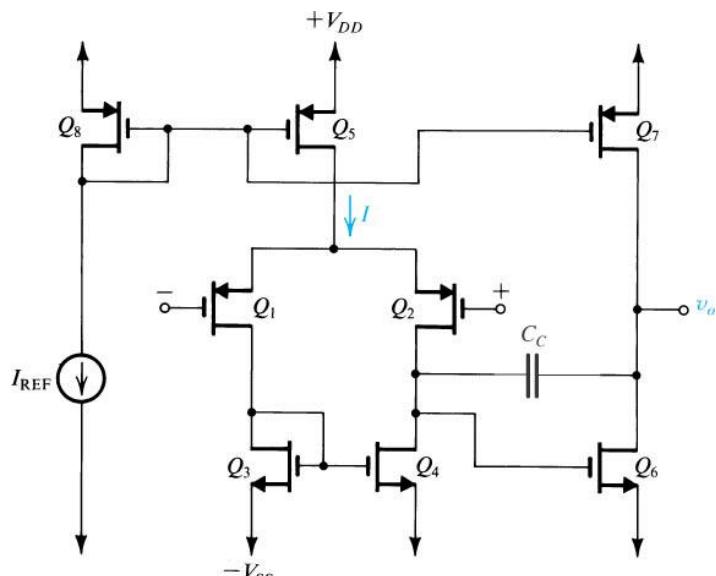
- The highest value of  $V_{ICM}$  ensures  $Q_5$  in saturation.

$$V_{ICM} \leq V_{DD} - |V_{OV5}| - V_{SG1}$$

$$\Rightarrow V_{ICM} \leq V_{DD} - |V_{OV5}| - |V_{tp}| - |V_{OV1}|$$

$$\Leftrightarrow -V_{SS} + V_{OV3} + V_{tn} - |V_{tp}| \leq V_{ICM} \leq V_{DD} - |V_{tp}| - |V_{OV1}| - |V_{OV5}|$$

From the point view of  $V_{ICM}$ , select the values of  $V_{OV}$  as low as possible!!



## Output swing

to keep  $Q_6$  and  $Q_7$  saturated

$$\Leftrightarrow -V_{SS} + V_{OV6} \leq v_O \leq V_{DD} - |V_{OV7}|$$

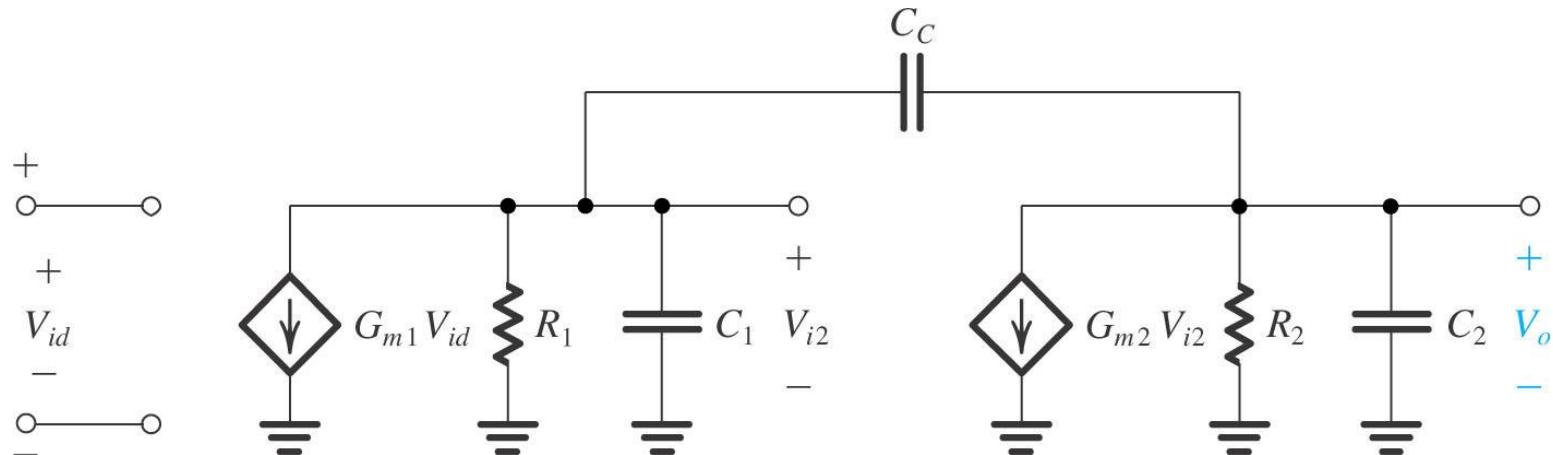
Select the values of  $V_{OV}$  (of  $Q_6$  and  $Q_7$ ) as low as possible!!

But  $f_T \propto V_{OV}$ ; the high-frequency performance improved with the higher overdrive voltage.

- A substantial allowable range of  $V_{ICM}$  and  $v_O$  needed for an unity-gain application.

## 12.1.3 Voltage Gain

Simplified small-signal equivalent circuit



- $R_{in} = \infty$
- $G_{m1} = g_{m1} = g_{m2} \Rightarrow G_{m1} = \frac{2(I/2)}{V_{OV1}} = \frac{I}{V_{OV1}}$
- $R_1 = r_{o2} \parallel r_{o4}$        $r_{o2} = \frac{|V_{A2}|}{I/2}$        $r_{o4} = \frac{|V_{A4}|}{I/2}$
- Dc gain of 1st stage

$$A_1 = -G_{m1}R_1 = -g_{m1}(r_{o2} \parallel r_{o4}) = -\frac{2}{V_{OV1} \left( \frac{1}{|V_{A2}|} + \frac{1}{|V_{A4}|} \right)}$$

$A_1$  is increased by

①  $Q_1$  and  $Q_2$  with a low overdrive

② Choosing a longer channel length to obtain larger Early voltage,  $|V_A|$ .

Both, however, degrade the frequency response of the amplifier.

- $G_{m2} = g_{m6} = \frac{2I_{D6}}{V_{OV6}}$
- $R_2 = r_{o6} \parallel r_{o7}$        $r_{o6} = \frac{V_{A6}}{I_{D6}}$        $r_{o7} = \frac{|V_{A7}|}{I_{D7}} = \frac{|V_{A7}|}{I_{D6}}$
- Dc gain of 2nd stage

$$A_2 = -G_{m2}R_2 = -g_{m6}(r_{o6} \parallel r_{o7}) = -\frac{2}{V_{OV6} \left( \frac{1}{V_{A6}} + \frac{1}{|V_{A7}|} \right)}$$

- Overall dc gain

$$A_v = A_1 A_2 = G_{m1} R_1 G_{m2} R_2 = g_{m1}(r_{o2} \parallel r_{o4}) g_{m6}(r_{o6} \parallel r_{o7})$$

Generally, 500 ~5000 V/V of  $A_{v,\max}$ .

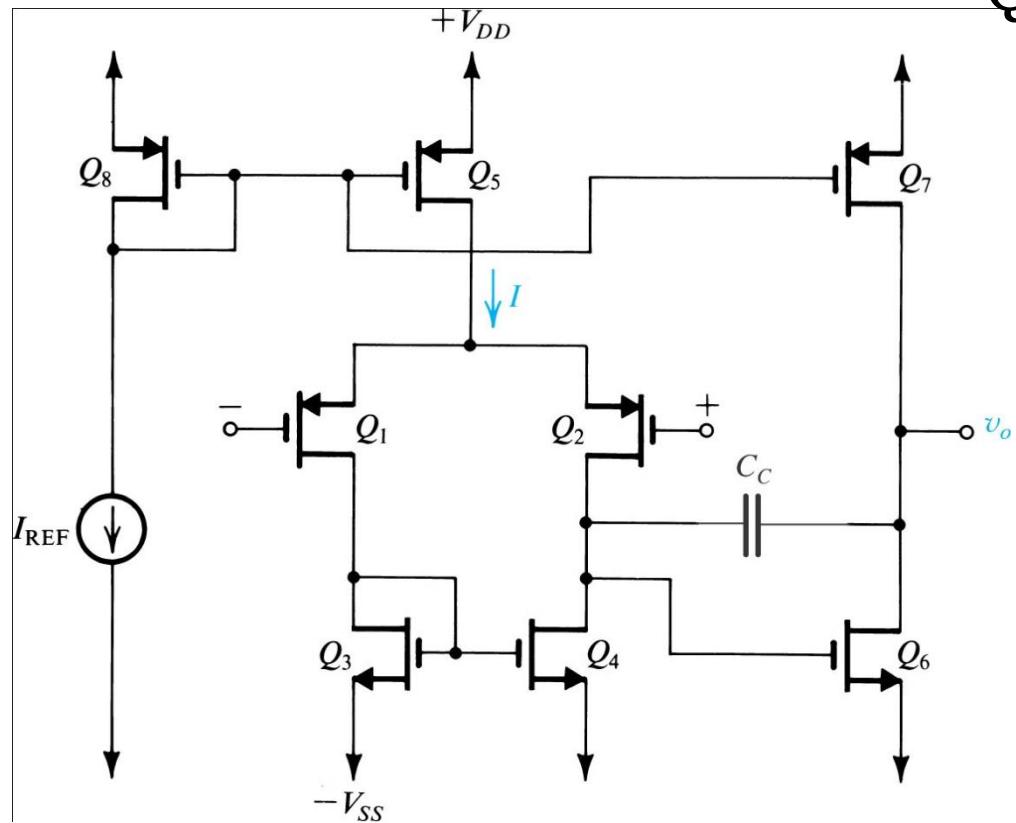
- Output resistance  $R_o = r_{o6} \parallel r_{o7}$

## 12.1.4 Common-Mode Rejection Ratio (CMRR)

**Section 8.5.5; eq. (8.158)**

$$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = [g_m(r_{o2} // r_{o4})][2g_m R_{SS}]$$

$Q_5$ 's output resistance  $R_{SS}$

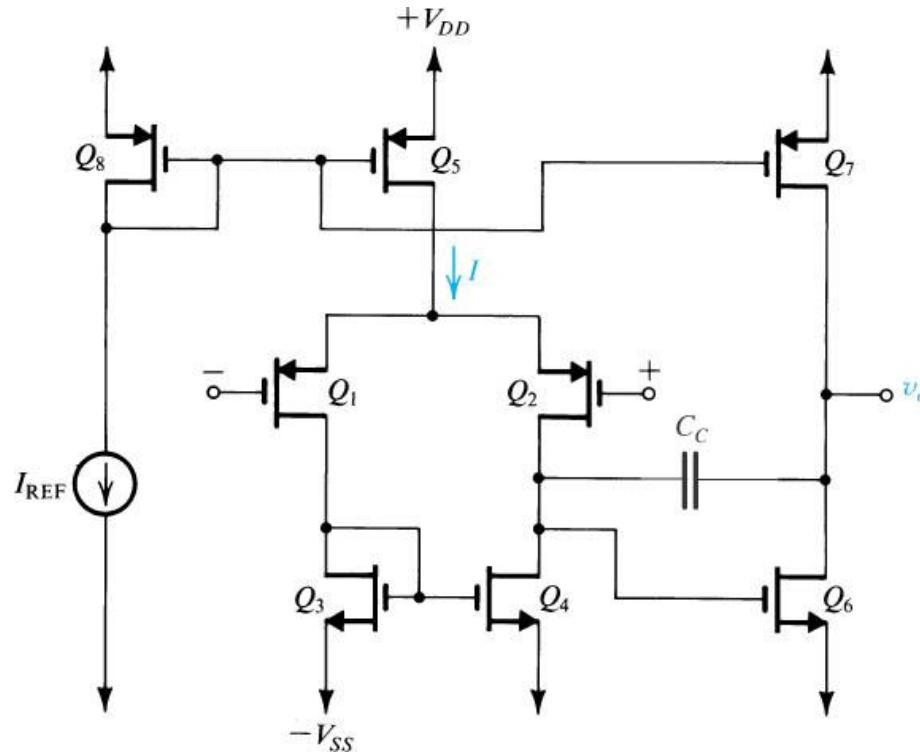


$$CMRR \propto (g_m r_o)^2$$

$$g_m r_o \propto \frac{V_A}{V_{OV}} = \frac{V_A \cdot L}{V_{OV}}$$

$\therefore L \uparrow \Rightarrow CMRR \uparrow$

## 12.1.5 Frequency Response



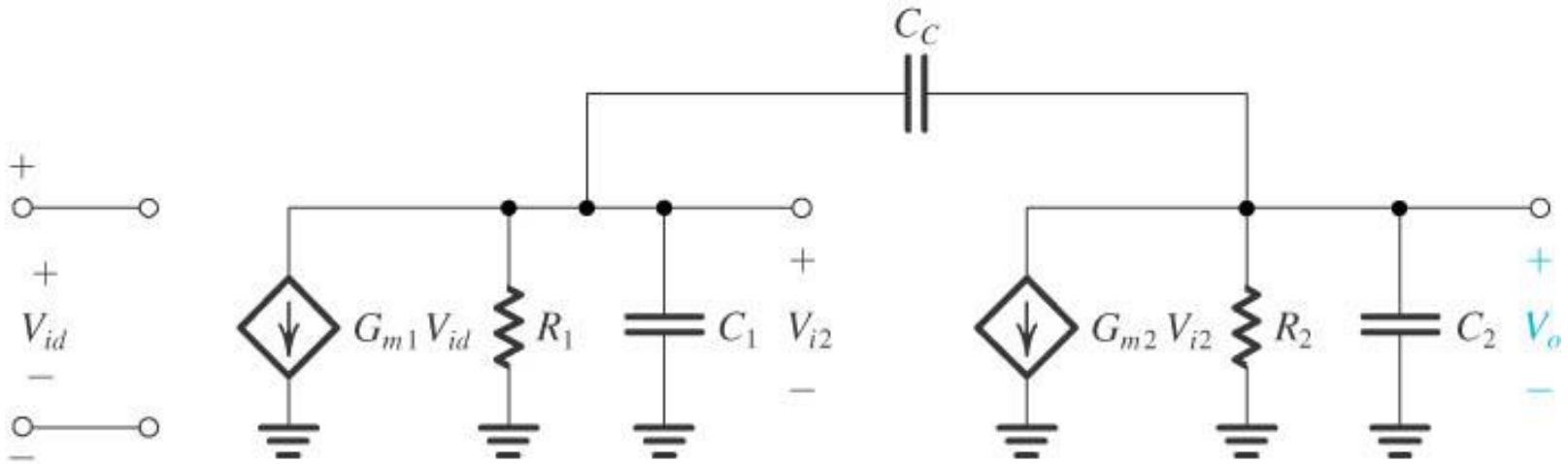
$$C_1 = C_{gd2} + C_{db2} + C_{gd4} + C_{db4} + C_{gs6}$$

$$C_2 = C_{db6} + C_{db7} + C_{gd7} + C_L$$

$C_C \gg C_{gd6} \rightarrow C_{gd6}$  is neglected

$$R_1 = r_{o2} \parallel r_{o4}$$

$$R_2 = r_{o6} \parallel r_{o7}$$



$$G_{m1}V_{id} + \frac{V_{i2}}{R_1} + sC_1V_{i2} + sC_C(V_{i2} - V_o) = 0$$

$$G_{m2}V_{i2} + \frac{V_o}{R_2} + sC_2V_o + sC_C(V_o - V_{i2}) = 0$$

$$\frac{V_o}{V_{id}} = \frac{G_{m1}(G_{m2} - sC_C)R_1R_2}{1 + s[C_1R_1 + C_2R_2 + C_C(G_{m2}R_1R_2 + R_1 + R_2)] + s^2[C_1C_2 + C_C(C_1 + C_2)]R_1R_2}$$

$$S_Z = \frac{G_{m2}}{C_C} \Rightarrow \omega_Z = \frac{G_{m2}}{C_C}$$

$$D(s) = \left(1 + \frac{s}{\omega_{P1}}\right)\left(1 + \frac{s}{\omega_{P2}}\right) = 1 + s\left(\frac{1}{\omega_{P1}} + \frac{1}{\omega_{P2}}\right) + \frac{s^2}{\omega_{P1}\omega_{P2}}$$

$$\omega_{P1} \ll \omega_{P2} \Rightarrow D(s) \approx 1 + \frac{s}{\omega_{P1}} + \frac{s^2}{\omega_{P1}\omega_{P2}}$$

$$\omega_{P1} = \frac{1}{C_1R_1 + C_2R_2 + C_C(G_{m2}R_1R_2 + R_1 + R_2)} \quad \omega_{P2} = \frac{C_C G_{m2}}{C_1C_2 + C_C(C_1 + C_2)}$$

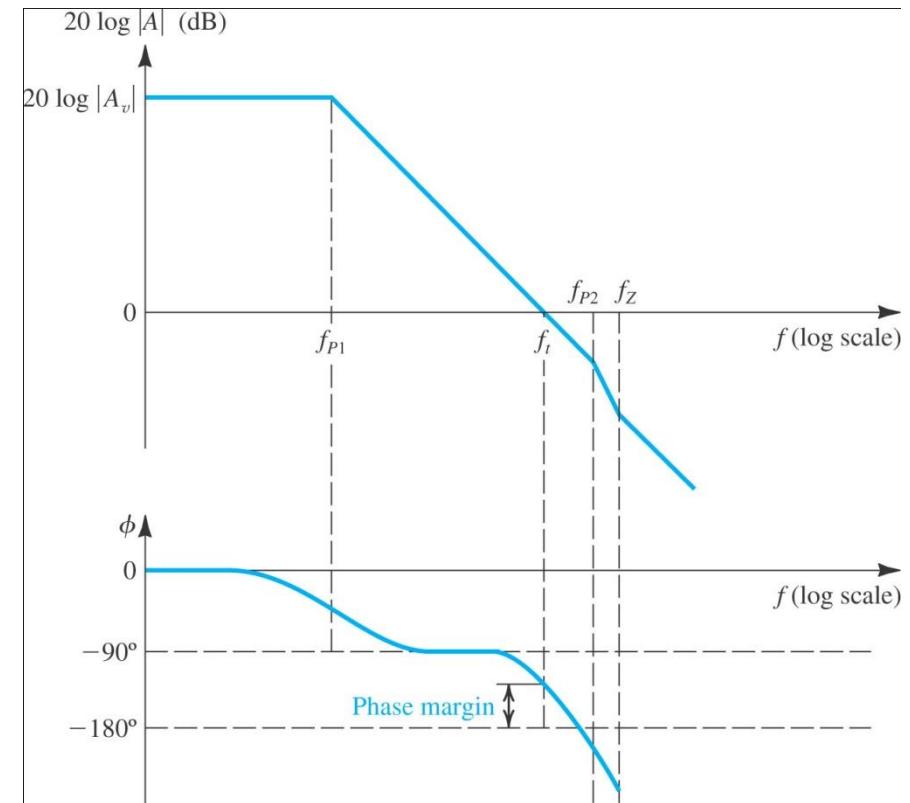
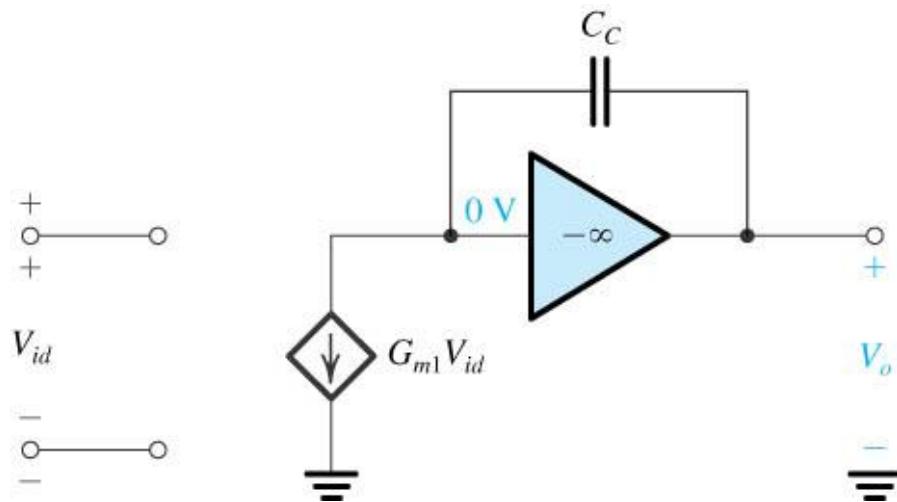
$$\because C_C \gg C_1, C_2 \gg C_1, G_{m2}R_2 \gg 1 \Rightarrow \omega_{P1} \approx \frac{1}{C_C G_{m2} R_1 R_2}, \quad \omega_{P2} \approx \frac{G_{m2}}{C_2}$$

$$\omega_t = A_M \omega_{P1} = G_{m1} G_{m2} R_1 R_2 \frac{1}{C_C G_{m2} R_1 R_2} = \frac{G_{m1}}{C_C}$$

$$\omega_t \ll \omega_{P2}, \omega_z \Rightarrow \frac{G_{m1}}{C_C} < \frac{G_{m2}}{C_2} \text{ and } G_{m1} < G_{m2}$$

When  $f \gg f_{P1}$ , a uniform -20dB/decade gain rolloff down to 0dB

→ Simplified Equivalent Circuit



## Phase margin

$$A(s) = \frac{A_o(1-s/\omega_z)}{(1+s/\omega_{P1})(1+s/\omega_{P2})}$$

$$\omega = \omega_t$$

$$\phi_{P1} = -\tan^{-1}\left(\frac{f_t}{f_{P1}}\right) \approx -90^\circ$$

$$\phi_{P2} = -\tan^{-1}\left(\frac{f_t}{f_{P2}}\right)$$

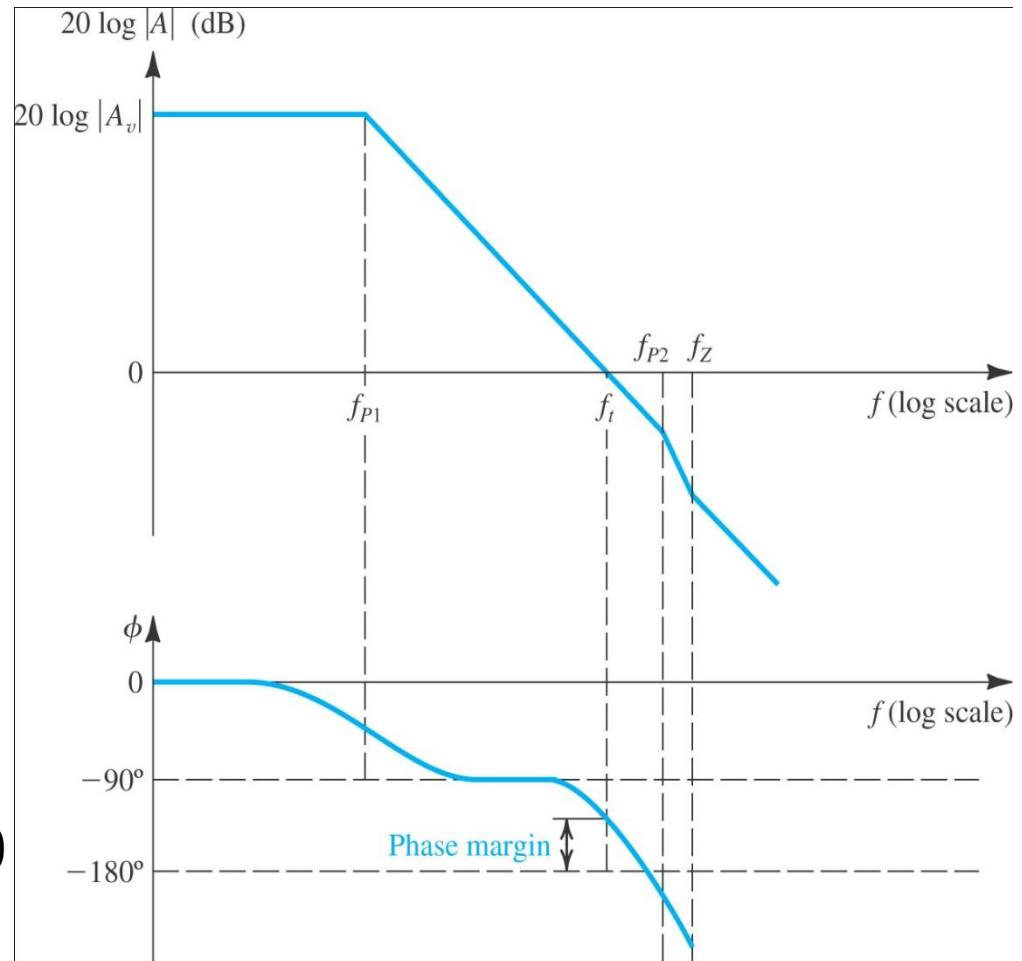
$$\phi_z = -\tan^{-1}\left(\frac{f_t}{f_z}\right)$$

$$\phi_{\text{total}} = 90^\circ + \tan^{-1}(f_t/f_{P2}) + \tan^{-1}(f_t/f_z)$$

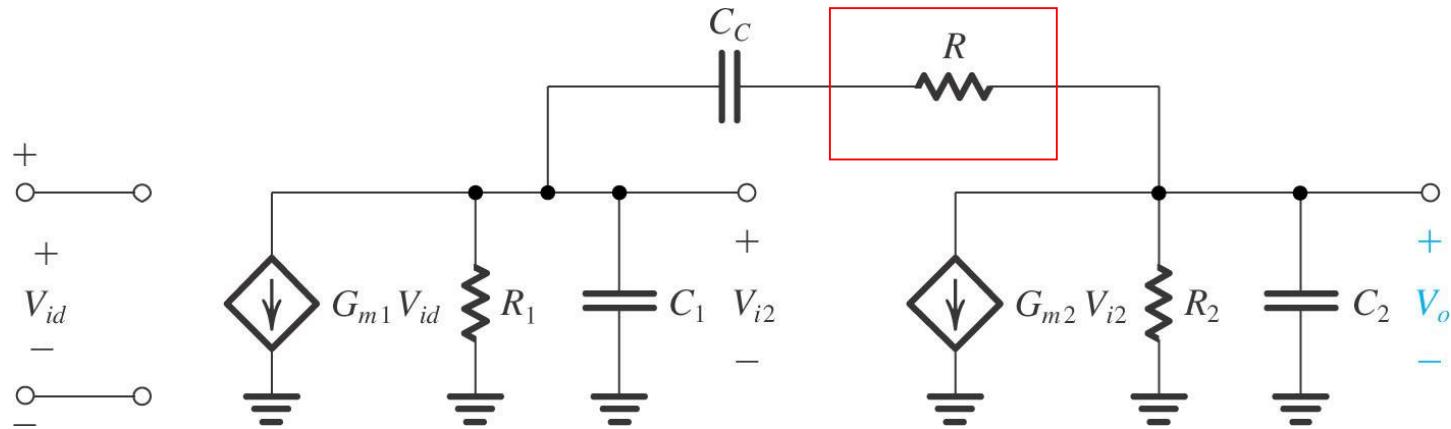
$$\text{Phase Margin(PM)} = 180^\circ - \phi_{\text{total}}$$

$$= 90^\circ - \tan^{-1}(f_t/f_{P2}) - \tan^{-1}(f_t/f_z)$$

If  $f_z \geq 10 f_t$ , PM =  $60^\circ \Rightarrow 60^\circ = 90^\circ - \tan^{-1}(f_t/f_{P2}) - 5.71^\circ$   
 $\Rightarrow f_{P2}/f_t \approx 2.2$



The right-half-plane zero reduces the phase margin.  
How to improve it?



- Find zero:  $V_o=0$

$$\frac{V_{i2}}{R + 1/sC_C} = G_{m2}V_{i2} \Rightarrow s_z = \frac{1}{C_C(1/G_{m2} - R)}$$

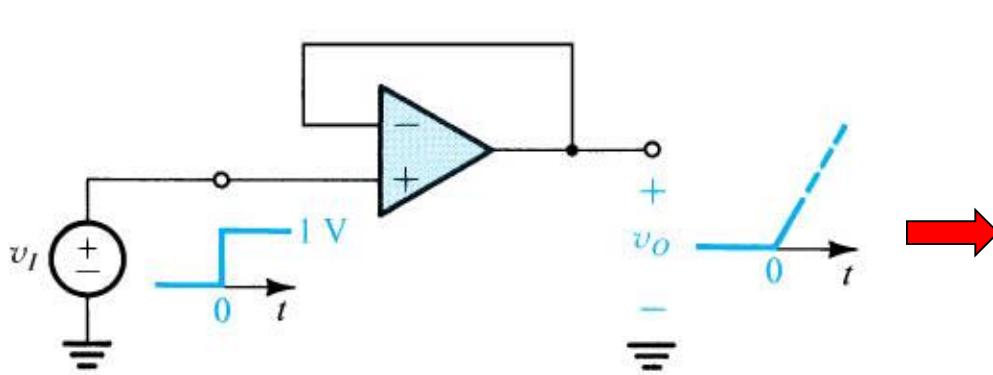
- $R = 1/G_{m2}$ , the zero can be placed at infinite frequency.
- $R > 1/G_{m2}$ , a left-half plane zero improves the phase margin.
- Discussion: The second pole is not very far from  $\omega_t$ . Thus the second pole introduces appreciable phase shift at  $\omega_t$ , which reduces the phase margin.

## 12.1.6 Slew Rate

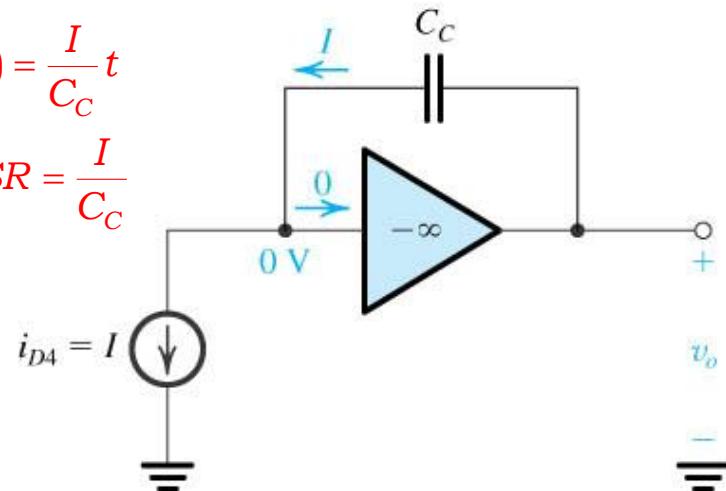
- When a large output signal is present, slew-rate limiting can cause nonlinear distortion.
- Slew rate: the maximum rate of change possible at the output of a real op amp.

$$SR = \left. \frac{dv_o}{dt} \right|_{\max}$$

- A unity-gain follower with a large step input.  
(the output voltage cannot change immediately.)



$$\begin{aligned} v_o(t) &= \frac{I}{C_C} t \\ \Rightarrow SR &= \frac{I}{C_C} \end{aligned}$$



## Relationship Between $SR$ and $f_t$

Slew rate

$$SR = \frac{I}{C_C}$$

$$G_{m1} = g_{m1} = \frac{I}{V_{OV1}}$$

$$\omega_t = \frac{G_{m1}}{C_C}$$

$$\Rightarrow SR = V_{OV1} \omega_t$$

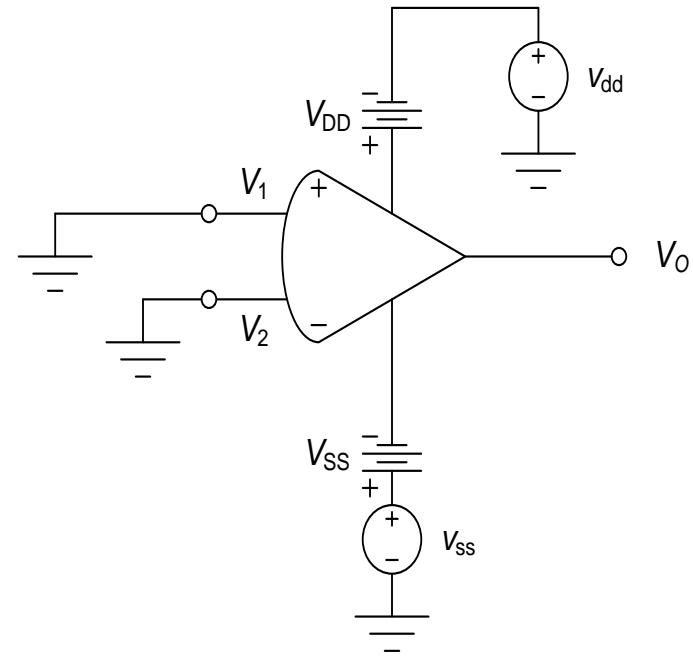
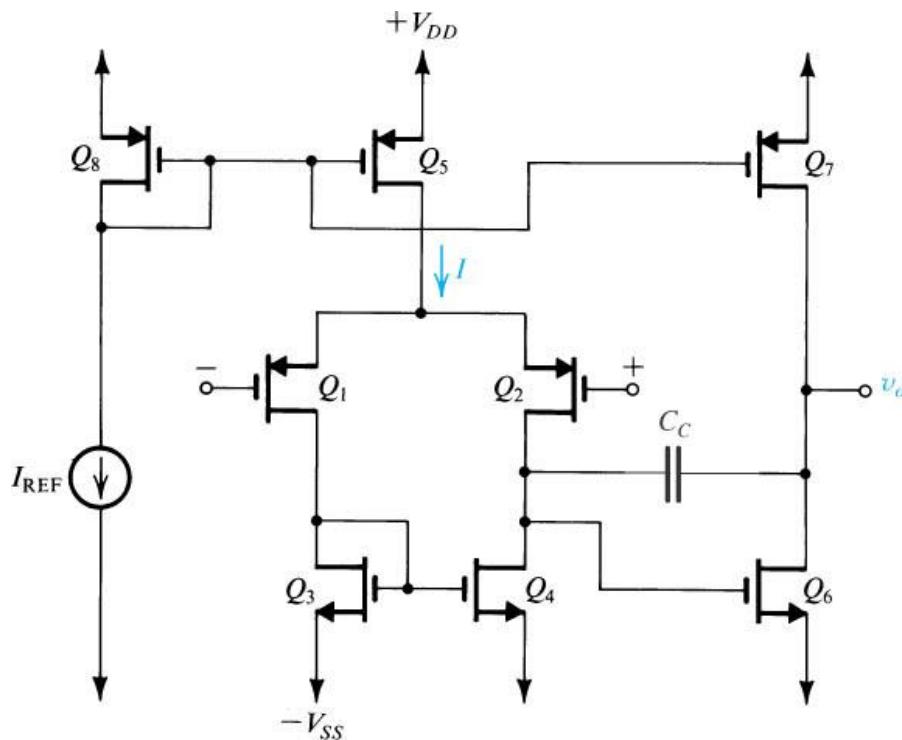
- For a given  $\omega_t$ , the slew rate is determined by the overdrive voltage at which the input transistor are operated.
- $V_{OV} \uparrow \Rightarrow SR \uparrow$ .
  - ❶ For a given bias current  $I$ , a large  $V_{OV}$  is obtained if  $Q_1$  and  $Q_2$  are  $p$ -channel devices. (1<sup>st</sup> stage)
  - ❷ It allows the 2<sup>nd</sup> stage to employ an  $n$ -channel device that has a greater transconductance,  $G_{m2}$ , resulting in a higher second-pole frequency and a corresponding higher  $\omega_t$ .

## 12.1.7 Power-Supply Rejection Ratio (PSRR)

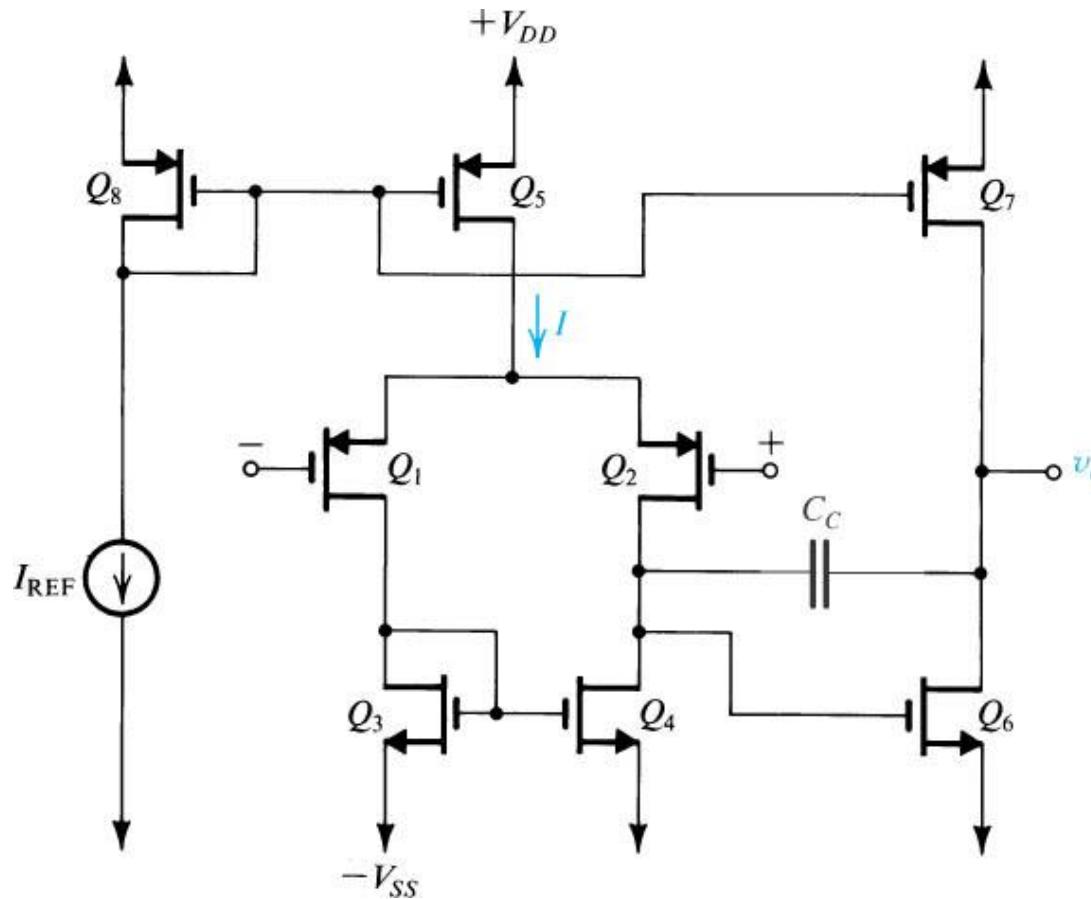
PSRR is defined as the ratio of the amplifier differential gain to the gain experienced by a change in the power-supply voltage ( $v_{dd}$  and  $v_{ss}$ )

$$PSRR^+ = \frac{A_d}{A^+} \text{ and } PSRR^- = \frac{A_d}{A^-}$$

$$A^+ = \frac{v_o}{v_{dd}} \text{ and } A^- = \frac{v_o}{v_{ss}}$$



$PSRR^+$  is high  
since  $Q_5$  and  $Q_7$  are current sources



$PSRR^-$

$$v_o = v_{ss} \frac{r_{o7}}{r_{o6} + r_{o7}}$$

$$A^- = \frac{v_o}{v_{ss}} = \frac{r_{o7}}{r_{o6} + r_{o7}}$$

$$PSRR^- = \frac{A_d}{A^-} = g_{m1}(r_{o2} / r_{o4})g_{m6}r_{o6}$$

$$PSRR^- \propto (g_m r_o)^2$$

$$g_m r_o \propto \frac{V_A'}{V_{OV}} = \frac{V_A' \cdot L}{V_{OV}}$$

$$\therefore L \uparrow \Rightarrow PSRR^- \uparrow$$

## 12.1.8 Design Tradeoffs

1. The length L used for each MOSFET
2. Overdrive voltage  $|V_{OV}|$  for each MOSFET

$$CMRR \propto (g_m r_o)^2$$

$$g_m r_o \propto \frac{V_A}{V_{OV}} = \frac{V'_A \cdot L}{V_{OV}}$$

$\therefore L \uparrow \Rightarrow CMRR \uparrow$

$$g_m = \mu_n C_{ox} \frac{W}{L} V_{OV}$$

$$C_{gs} \approx \frac{2}{3} WLC_{ox}$$

$$PSRR^- \propto (g_m r_o)^2$$

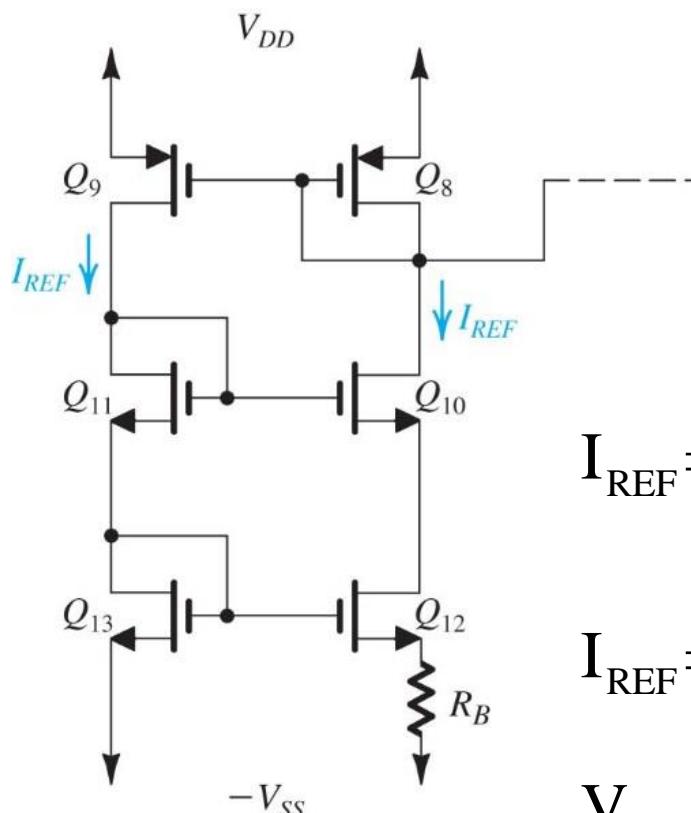
$$g_m r_o \propto \frac{V_A}{V_{OV}} = \frac{V'_A \cdot L}{V_{OV}}$$

$\therefore L \uparrow \Rightarrow PSRR^- \uparrow$

**transition frequency**

$$f_T = \frac{\omega_T}{2\pi} = \frac{g_m}{2\pi(C_{gs} + C_{gd})} \approx \frac{1.5\mu_n V_{OV}}{2\pi L^2}$$

## 12.1.9 A Bias Circuit for the Two-Stage CMOS Op Amp



1. All devices are in the saturation region
2. Q8 and Q9, Q10 and Q11 matched
3. Q12 and Q13 not matched
4. Positive feedback;  $R_B \downarrow I_{D13} \uparrow V_{G13} \uparrow$
5. Need a kick-off circuit

$$I_{\text{REF}} = \frac{\mu_n C_{\text{OX}}}{2} \left( \frac{W}{L} \right)_{12} (V_{GS12} - V_t)^2$$

$$I_{\text{REF}} = \frac{\mu_n C_{\text{OX}}}{2} \left( \frac{W}{L} \right)_{13} (V_{GS13} - V_t)^2$$

$$V_{GS13} = V_{GS12} + I_{\text{REF}} R_B \Rightarrow$$

$$\sqrt{\frac{2I_{\text{REF}}}{\mu_n C_{\text{OX}} (W/L)_{13}}} = \sqrt{\frac{2I_{\text{REF}}}{\mu_n C_{\text{OX}} (W/L)_{12}}} + I_{\text{REF}} R_B$$

$$I_{REF} = \frac{2}{\mu_n C_{OX} (W/L)_{12} R_B^2} \left( \sqrt{\frac{(W/L)_{12}}{(W/L)_{13}}} - 1 \right)^2$$

$$R_B = \frac{2}{\sqrt{2\mu_n C_{OX} (W/L)_{12} I_{REF}}} \left( \sqrt{\frac{(W/L)_{12}}{(W/L)_{13}}} - 1 \right)$$

$$= \frac{2}{g_{m12}} \left( \sqrt{\frac{(W/L)_{12}}{(W/L)_{13}}} - 1 \right)$$

$$\Rightarrow g_{m12} = \frac{2}{R_B} \left( \sqrt{\frac{(W/L)_{12}}{(W/L)_{13}}} - 1 \right)$$

$$\therefore g_{m12} = \sqrt{2\mu_n C_{OX} (W/L)_{12} I_{REF}}$$

for each n-channel transistor in this bias circuit

$$\frac{g_{mi}}{g_{m12}} = \sqrt{\frac{(W/L)_i I_{Di}}{(W/L)_{12} I_{REF}}}$$

for each p-channel transistor in this bias circuit

$$\frac{g_{mi}}{g_{m12}} = \sqrt{\frac{\mu_p (W/L)_i I_{Di}}{\mu_n (W/L)_{12} I_{REF}}}$$

# Ex.12.1 Two-Stage CMOS Op Amp Design

Design a dc gain of 4000 V/V in a 0.5- $\mu\text{m}$  CMOS technology.  $V_{tn} = |V_{tp}| = 0.5\text{V}$ ,  $k'_n = 200 \mu\text{A}/\text{V}^2$ ,  $k'_p = 80 \mu\text{A}/\text{V}^2$ ,  $V'_{An} = |V'_{Ap}| = 20\text{V}/\mu\text{m}$ ,  $V_{DD} = V_{SS} = 1.65\text{V}$ ,  $L = 1\mu\text{m}$  for all devices. Let  $I = 200\mu\text{A}$ ,  $I_{D6} = 0.5\text{mA}$ . If  $C_1 = 0.2\text{pF}$  and  $C_2 = 0.8\text{pF}$ , find the required  $C_C$  and  $R$  to place the transmission zero at  $s = \infty$  for phase margin of 85°.

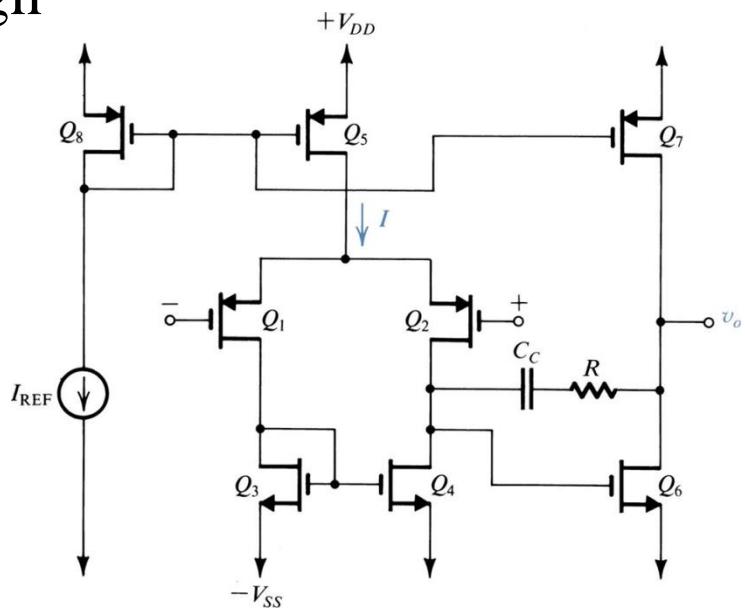
## Solution

- Voltage gain  $A_v = g_{m1}(r_{o2} \| r_{o4})g_{m6}(r_{o6} \| r_{o7}) = \frac{2(I/2)}{V_{OV}} \cdot \frac{1}{2} \cdot \frac{V_A}{(I/2)} \cdot \frac{2I_{D6}}{V_{OV}} \cdot \frac{1}{2} \cdot \frac{V_A}{I_{D6}} = \left(\frac{V_A}{V_{OV}}\right)^2$

$$A_v = 4000 \text{ and } V_A = 20\text{V} \Rightarrow 4000 = \frac{400}{V_{OV}^2} \Rightarrow V_{OV} = 0.316\text{V}$$

- $(W/L)_1$  and  $(W/L)_2$ :

$$I_{D1} = \frac{1}{2}k'_p \left(\frac{W}{L}\right)_1 v_{OV}^2 \Rightarrow 100 = \frac{1}{2} \cdot 80 \left(\frac{W}{L}\right)_1 \cdot 0.316^2 \Rightarrow \left(\frac{W}{L}\right)_1 = \frac{25\mu\text{m}}{1\mu\text{m}} = \left(\frac{W}{L}\right)_2$$



- $(W/L)_3$  and  $(W/L)_4$ :
 
$$I_{D1} = \frac{1}{2} k_n \left( \frac{W}{L} \right)_3 v_{OV}^2 \Rightarrow 100 = \frac{1}{2} \cdot 200 \left( \frac{W}{L} \right)_3 \cdot 0.316^2 \Rightarrow \left( \frac{W}{L} \right)_3 = \frac{10 \mu m}{1 \mu m} = \left( \frac{W}{L} \right)_4$$
- $(W/L)_5$ :
 
$$I_{D5} = \frac{1}{2} k_p \left( \frac{W}{L} \right)_5 v_{OV}^2 \Rightarrow 200 = \frac{1}{2} \cdot 80 \left( \frac{W}{L} \right)_5 \cdot 0.316^2 \Rightarrow \left( \frac{W}{L} \right)_5 = \frac{50 \mu m}{1 \mu m}$$
- $(W/L)_6$  and  $(W/L)_7$ :
 
$$\left( \frac{W}{L} \right)_7 = 2.5 \left( \frac{W}{L} \right)_5 = \frac{125 \mu m}{1 \mu m} \quad 500 = \frac{1}{2} \cdot 200 \left( \frac{W}{L} \right)_6 \cdot 0.316^2 \Rightarrow \left( \frac{W}{L} \right)_6 = \frac{50 \mu m}{1 \mu m}$$
- Select  $I_{REF} = 20 \mu A$ , thus  $\left( \frac{W}{L} \right)_8 = 0.1 \left( \frac{W}{L} \right)_5 = \frac{5 \mu m}{1 \mu m}$
- Input CM range:  $-1.33 \text{ V} \leq V_{ICM} \leq 0.52 \text{ V}$   

$$(-V_{SS} + V_{OV3} + V_{tn} - |V_{tp}| \leq V_{ICM} \leq V_{DD} - |V_{tp}| - |V_{OV1}| - |V_{OV5}|)$$
- Max. signal swing:  $-1.33 \text{ V} \leq v_o \leq 1.33 \text{ V}$
- Input resistance:  $R_i = \infty$
- Output resistance:  $R_o = r_{o6} \parallel r_{o7} = 20 \text{ k}\Omega$

$$\text{CMRR} = [g_m(r_{o2} // r_{o4})][2g_{m3}R_{SS}]$$

- $R_{SS} = r_{o5} = V_A / I$

$$\begin{aligned}\text{CMRR} &= \frac{2(I/2)}{V_{OV}} \cdot \frac{1}{2} \cdot \frac{V_A}{(I/2)} \cdot 2 \cdot \frac{2(I/2)}{V_{OV}} \cdot \frac{V_A}{I} \\ &= 2\left(\frac{V_A}{V_{OV}}\right)^2 = 2\left(\frac{20}{0.316}\right)^2 = 8000\end{aligned}$$

$$\text{CMRR} = 20 \log(8000) = 78 \text{dB}$$

- $PSRR^- = g_{m1}(r_{o2} // r_{o4})g_{m6}r_{o6}$

$$\begin{aligned}&= \frac{2(I/2)}{V_{OV}} \cdot \left(\frac{1}{2} \cdot \frac{V_A}{(I/2)}\right) \cdot \frac{2I_{D6}}{V_{OV}} \cdot \frac{V_A}{I_{D6}} \\ &= 2\left(\frac{V_A}{V_{OV}}\right)^2 = 2\left(\frac{20}{0.316}\right)^2 = 8000\end{aligned}$$

$$PSRR^- = 20 \log(8000) = 78 \text{dB}$$

- Determine  $f_{p2}$ :

$$G_{m2} = g_{m6} = \frac{2I_{D6}}{V_{OV}} = \frac{2 \times 0.5}{0.316} = 3.2 \text{mA/V} \quad f_{p2} \approx \frac{G_{m2}}{2\pi C_2} = \frac{3.2 \times 10^{-3}}{2\pi \cdot 0.8 \times 10^{-12}} = 637 \text{MHz}$$

- To move the transmission zero to  $s = \infty$ , we select the value of  $R$  as

$$R = \frac{1}{G_{m2}} = \frac{1}{3.2 \times 10^{-3}} = 316\Omega$$

- For a phase margin of  $85^\circ$ , the phase shift due to  $f_{p2}$  at  $f = f_t$  must be  $5^\circ$ , that is

$$\tan^{-1} \frac{f_t}{f_{p2}} = 5^\circ$$

$$\Rightarrow f_t = 637 \times \tan 5^\circ = 55.7 \text{ MHz}$$

- Determine  $C_C$ :

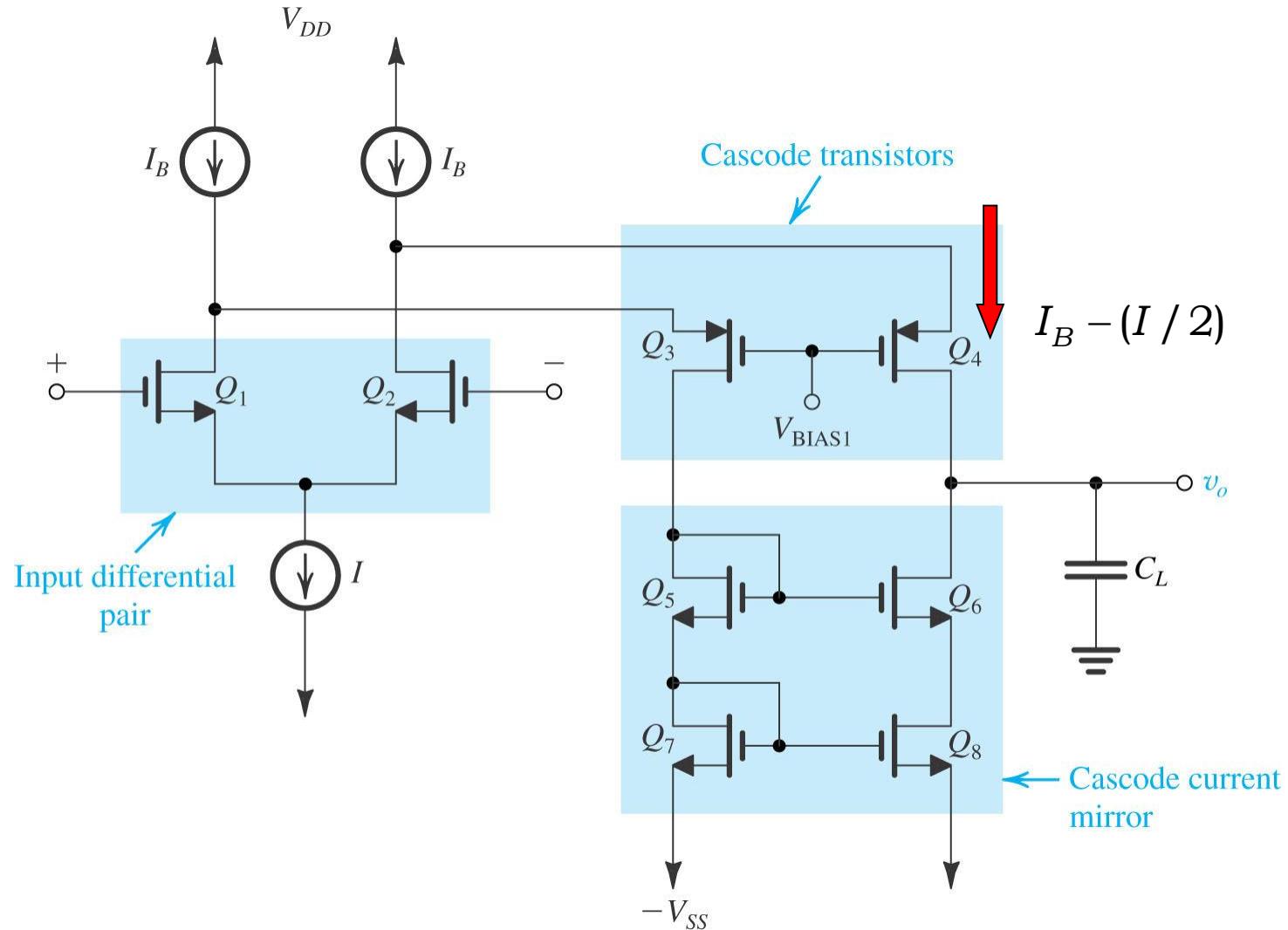
$$C_C = \frac{G_{m1}}{2\pi f_t} \quad \text{where } G_{m1} = g_{m1} = \frac{2 \cdot 100 \mu A}{0.316 V} = 0.63 \text{ mA/V}$$

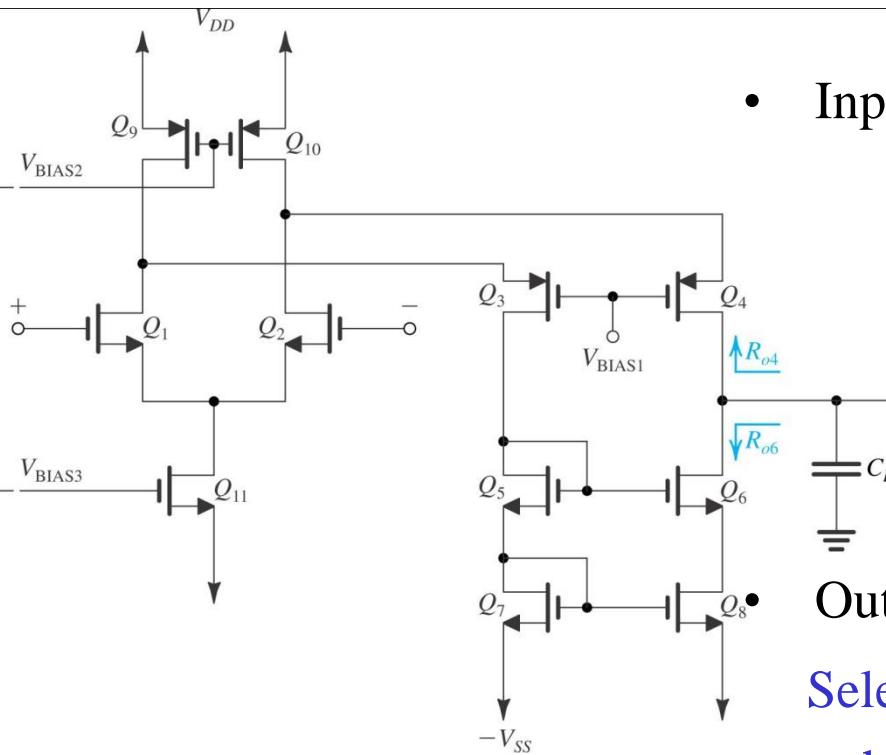
$$\therefore C_C = \frac{0.6 \times 10^{-3}}{2\pi \cdot 55.7 \times 10^6} = 1.8 \text{ pF}$$

- Slew rate: [Eq. (12.46)]

$$SR = 2\pi f_t V_{OV} = 2\pi \cdot 55.7 \times 10^6 \times 0.316 = 111 \text{ V/\mu s}$$

## 12.2 The Folded Cascode CMOS Op Amp





- Input common-mode range:

$$V_{ICM\max} = V_{DD} - |V_{OV9}| + V_{tn}$$

$$V_{ICM\min} = -V_{SS} + V_{OV11} + V_{OV1} + V_{tn}$$

$$-V_{SS} + V_{OV11} + V_{OV1} + V_{tn}$$

$$\leq V_{ICM} \leq V_{DD} - |V_{OV9}| + V_{tn}$$

Output voltage swing:

Select  $V_{BIAS1}$  so that  $Q_{10}$  operates at the edge of saturation:

$$V_{BIAS1} = V_{DD} - |V_{OV10}| - V_{SG4}$$

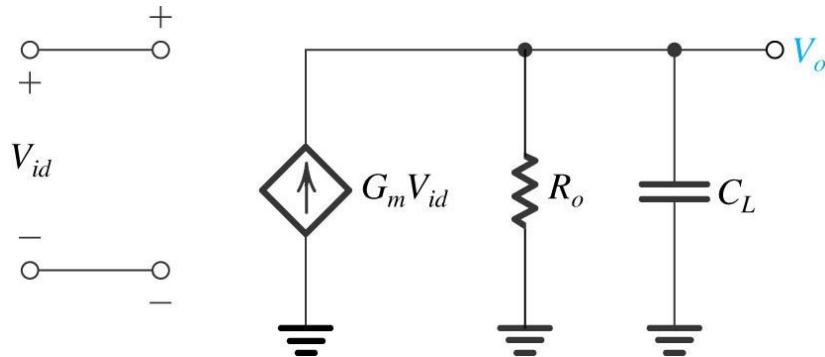
$$\Rightarrow v_{o\max} = V_{DD} - |V_{OV10}| - |V_{OV4}|$$

$v_{o,min}$  is obtained when  $Q_6$  reaches the edge of saturation.

$$v_{o\min} = -V_{SS} + V_{OV7} + V_{OV5} + V_{tn}$$

- Voltage gain

Small-signal equivalent circuit:



- Transconductance

$$G_m = g_{m1} = g_{m2} \quad G_m = \frac{2(I/2)}{V_{OV1}} = \frac{I}{V_{OV1}}$$

- Output resistance

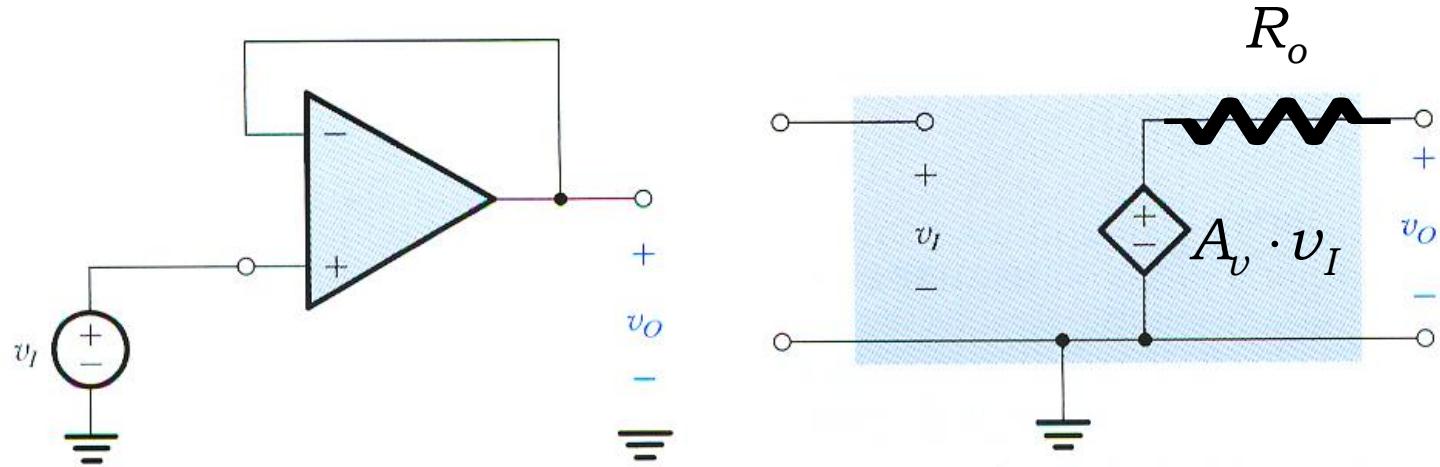
$$R_o = R_{o4} \parallel R_{o6} \quad R_{o4} \approx (g_{m4} r_{o4})(r_{o2} \parallel r_{o10}) \quad R_{o6} \approx g_{m6} r_{o6} r_{o8}$$

$$\Rightarrow \quad R_o = [g_{m4} r_{o4} (r_{o2} \parallel r_{o10})] \parallel (g_{m6} r_{o6} r_{o8})$$

- Open-loop gain

$$A_v = G_m R_o = g_{m1} \left\{ [g_{m4} r_{o4} (r_{o2} \parallel r_{o10})] \parallel (g_{m6} r_{o6} r_{o8}) \right\}$$

- Unity-gain buffer (voltage follower):



- Output resistance

$$R_{of} = \frac{R_o}{1 + A_v} \approx \frac{R_o}{A_v} = \frac{R_o}{G_m R_o} = \frac{1}{G_m} = \frac{1}{g_{m1}}$$

- It is not very small for a single-stage op amp (OTA, operational transconductance amplifier).
- An ideal op amp has an zero output resistance!

- Frequency response:

Since the primary purpose of CMOS op amps is to feed capacitive loads,  $C_L$  is usually large, and the pole at the output becomes dominant.

- Transfer function  $\frac{V_o}{V_{id}} = \frac{G_m R_o}{1 + sC_L R_o}$

- Dominant pole  $f_p = \frac{1}{2\pi C_L R_o}$

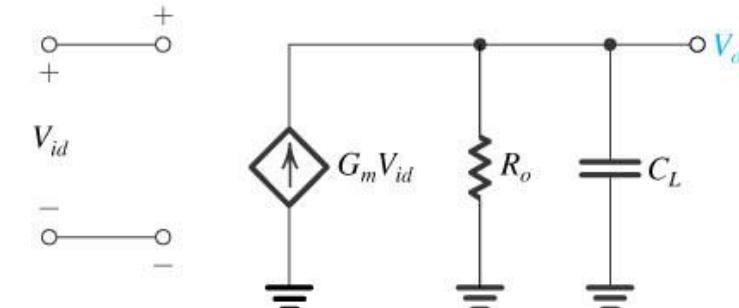
- Unity-gain frequency

$$f_t = A_v f_p = G_m R_o f_p = \frac{G_m}{2\pi C_L}$$

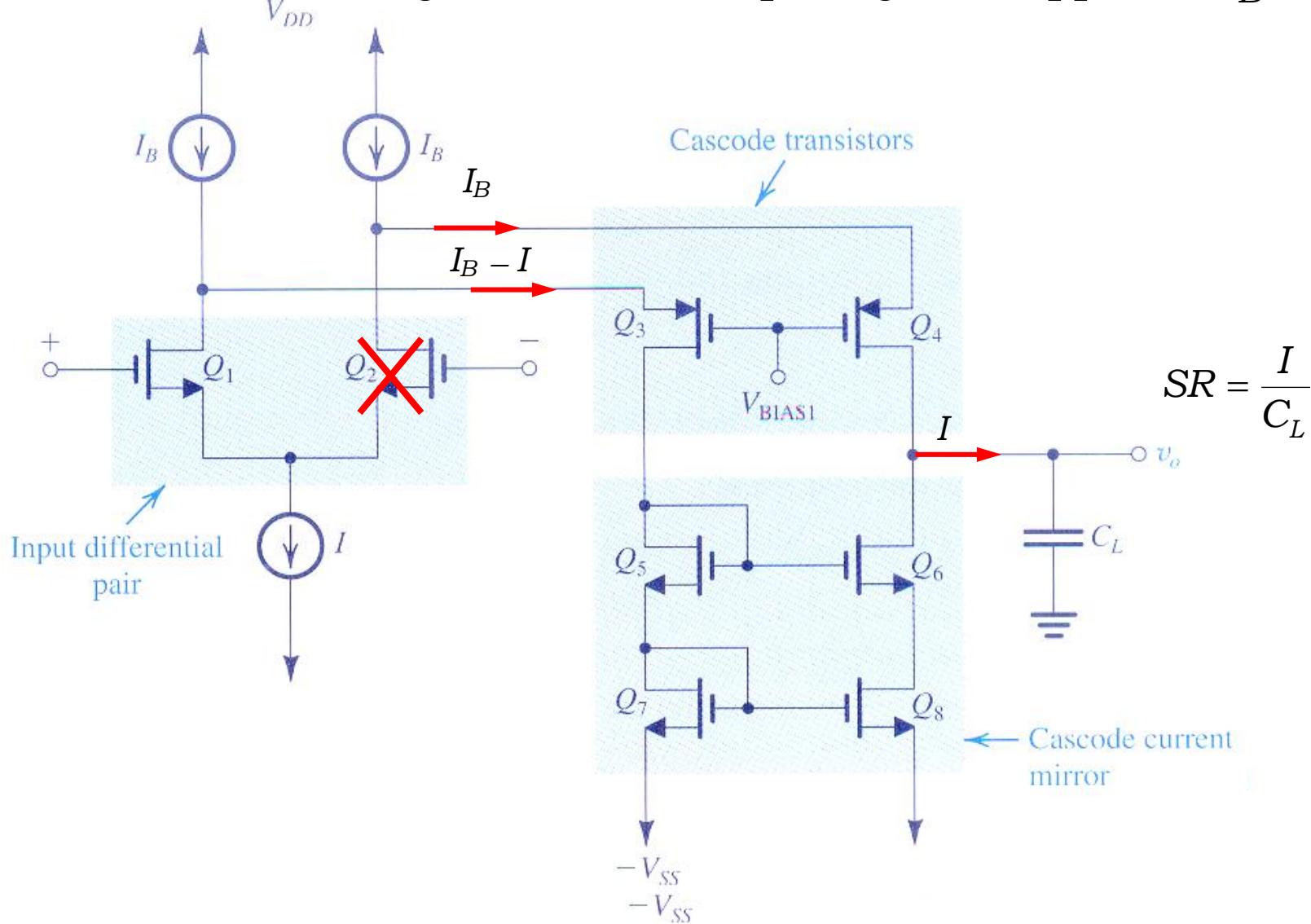
- Discussion

Folded-cascode op amp:  $C_L \uparrow \Rightarrow f_{p1}, f_t \downarrow \Rightarrow$  phase margin  $\uparrow$

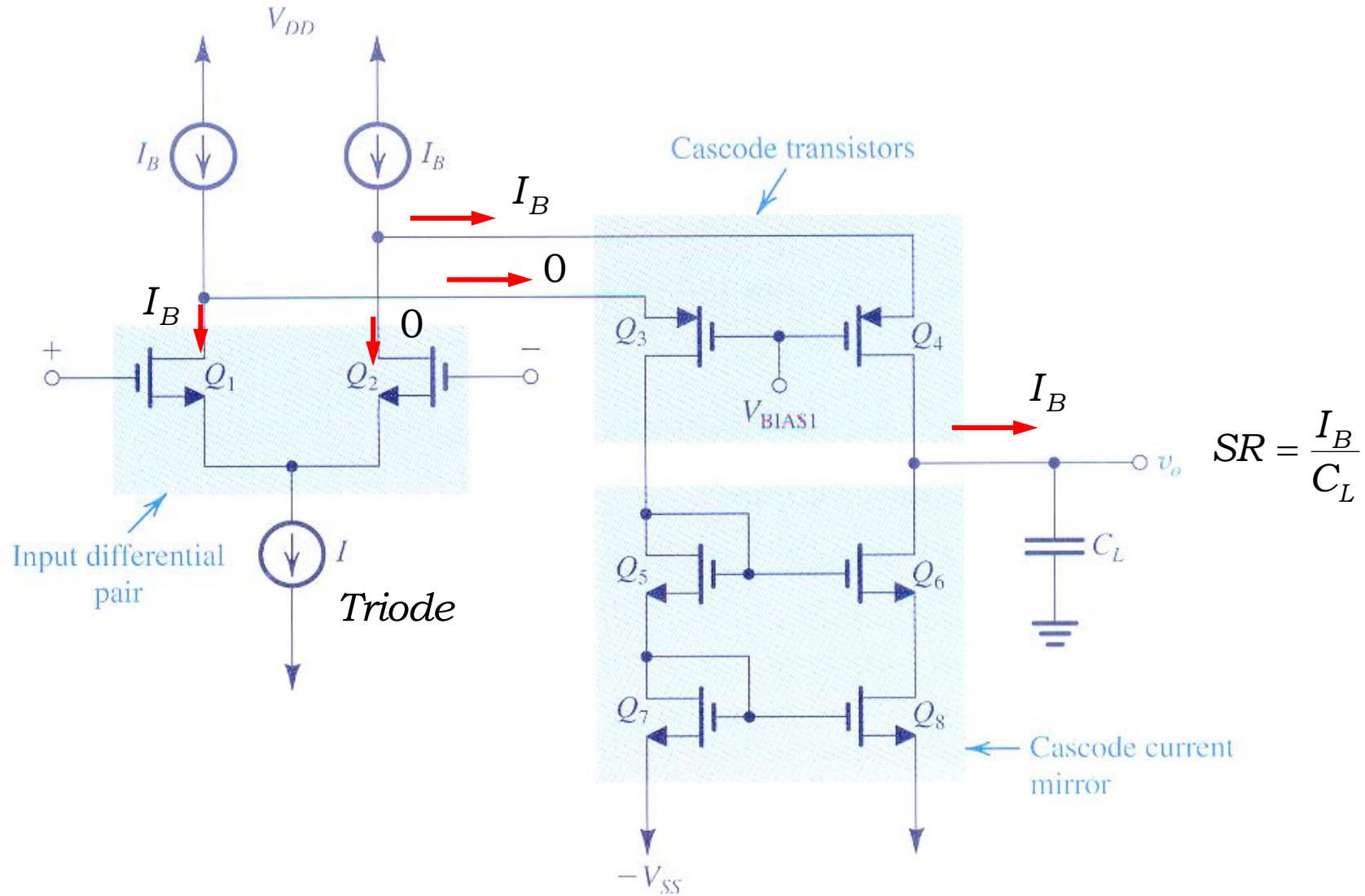
Two-stage op amp:  $C_L \uparrow \Rightarrow f_{p2} \downarrow \Rightarrow$  phase margin  $\downarrow$



- Slew Rate: a large differential input signal is applied. ( $I_B > I$ )



- Slew Rate: a large differential input signal is applied. ( $0.5I < I_B < I$ )



## Ex 12.2: Design of a folded-cascode op amp.

$I = 240\mu\text{A}$ ,  $I_B = 150\mu\text{A}$ , and  $|V_{OV}| = 0.25\text{V}$  for all transistors.  $V_{DD} = V_{SS} = 2.5\text{V}$   
 $K_n' = 100\mu\text{A/V}^2$ ,  $K_p' = 40\mu\text{A/V}^2$ ,  $|V_A'| = 20\text{V}/\mu\text{m}$  and  $|V_t| = 0.75\text{V}$ .  $L = 1\mu\text{m}$ ,  
 $C_L = 5\text{pF}$ .

- Find  $I_D$ ,  $g_m$ ,  $r_o$  and  $W/L$  for all transistors.

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2I_D}{0.25}$$

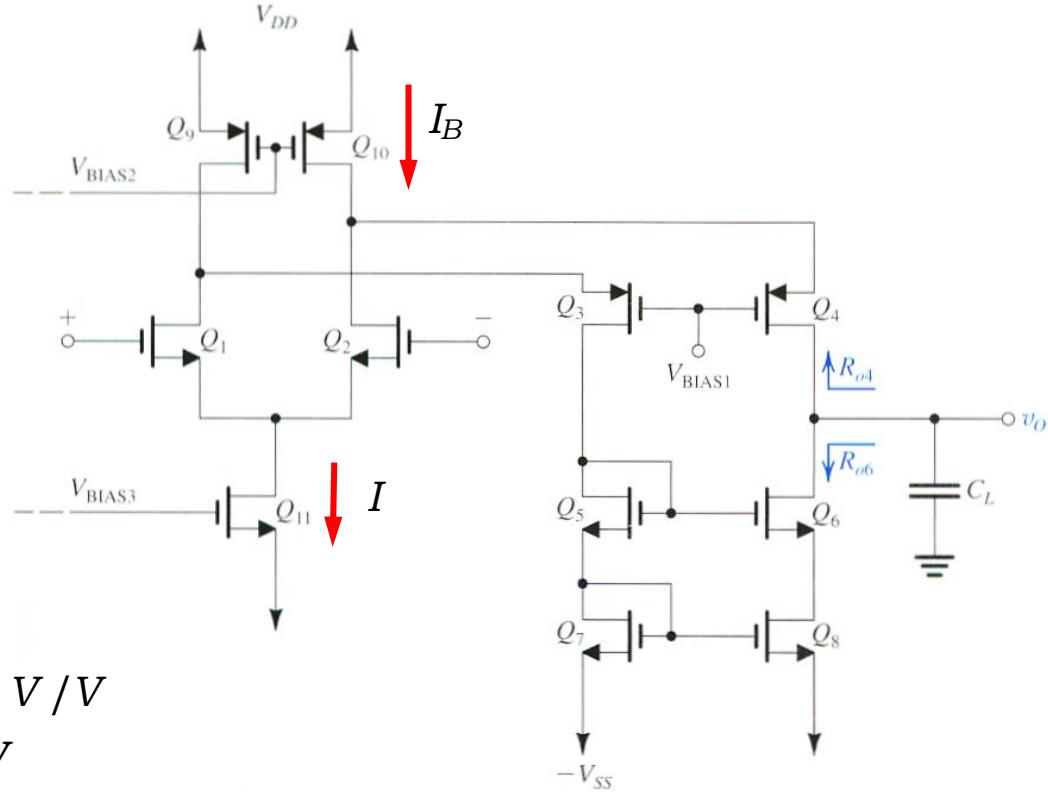
$$r_o = \frac{|V_A|}{I_D} = \frac{20}{I_D}$$

$$\left(\frac{W}{L}\right)_i = \frac{2I_{Di}}{k'V_{OV}^2}$$

Note: for all transistors

$$g_m r_o = 160 \text{ V/V}$$

$$V_{GS} = 1.0 \text{ V}$$



	$Q_1$	$Q_2$	$Q_3$	$Q_4$	$Q_5$	$Q_6$	$Q_7$	$Q_8$	$Q_9$	$Q_{10}$	$Q_{11}$
$I_D (\mu\text{A})$	120	120	30	30	30	30	30	30	150	150	240
$g_m (\text{mA/V})$	0.96	0.96	0.24	0.24	0.24	0.24	0.24	0.24	1.2	1.2	1.92
$r_o (\text{k}\Omega)$	167	167	667	667	667	667	667	667	133	133	83
$W/L$	38.4	38.4	24	24	9.6	9.6	9.6	9.6	120	120	76.8

- Find the allowable range of  $V_{ICM}$  and of the output voltage swing.

$$-1.25V \leq v_o \leq 2V$$

$$-1.25V \leq V_{ICM} \leq 3V$$

$$-V_{SS} + V_{OV11} + V_{OV1} + V_{tn} \leq V_{ICM} \leq V_{DD} - |V_{OV9}| + V_{tn}$$

$$\boxed{v_{o\max} = V_{DD} - |V_{OV10}| - |V_{OV4}|}$$

$$\boxed{v_{o\min} = -V_{SS} + V_{OV7} + V_{OV5} + V_{tn}}$$

- Determine the values of  $A_v$ ,  $f_t$ ,  $f_p$ , and  $SR$ .

$$R_{o4} = (g_{m4}r_{o4})(r_{o2} \| r_{o10}) = 160(167 \| 133) = 11.85M\Omega \quad R_{o6} = g_{m6}r_{o6}r_{o8} = 106.7M\Omega$$

$$R_o = R_{o4} \| R_{o6} = 10.7M\Omega$$

$$A_v = G_m R_o = 0.96 \times 10^{-3} \cdot 10.7 \times 10^6 = 10240V / V$$

$$f_t = \frac{G_m}{2\pi C_L} = \frac{0.96 \times 10^{-3}}{2\pi \cdot 5 \times 10^{-12}} = 30.6MHz$$

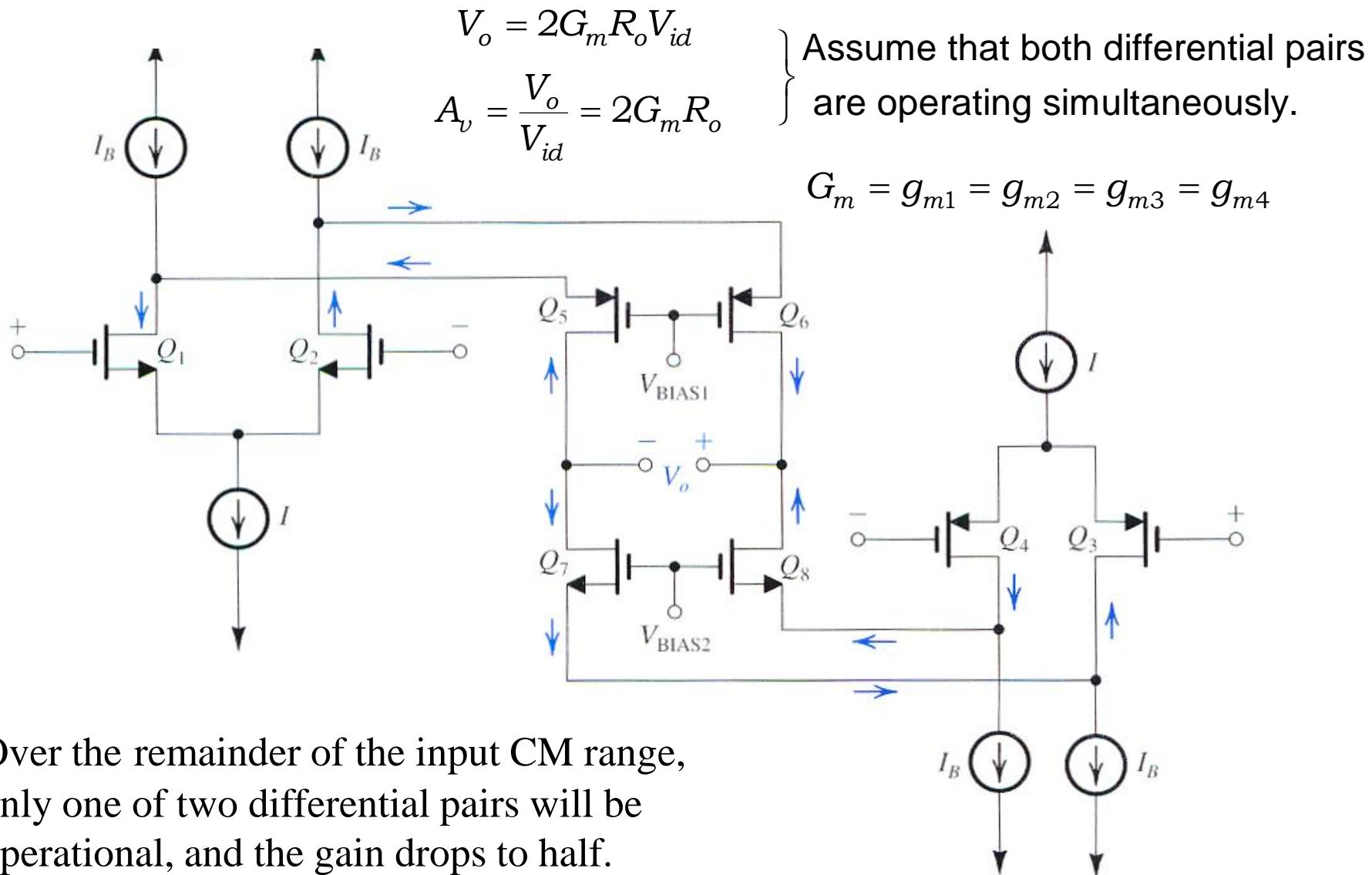
$$f_p = \frac{f_t}{A_v} \left( \text{or } = \frac{1}{2\pi C_L R_o} \right) = \frac{30.6MHz}{10240} = 3kHz$$

$$SR = \frac{I_B}{C_L} = \frac{150 \times 10^{-6}}{5 \times 10^{-12}} = 30V / \mu s$$

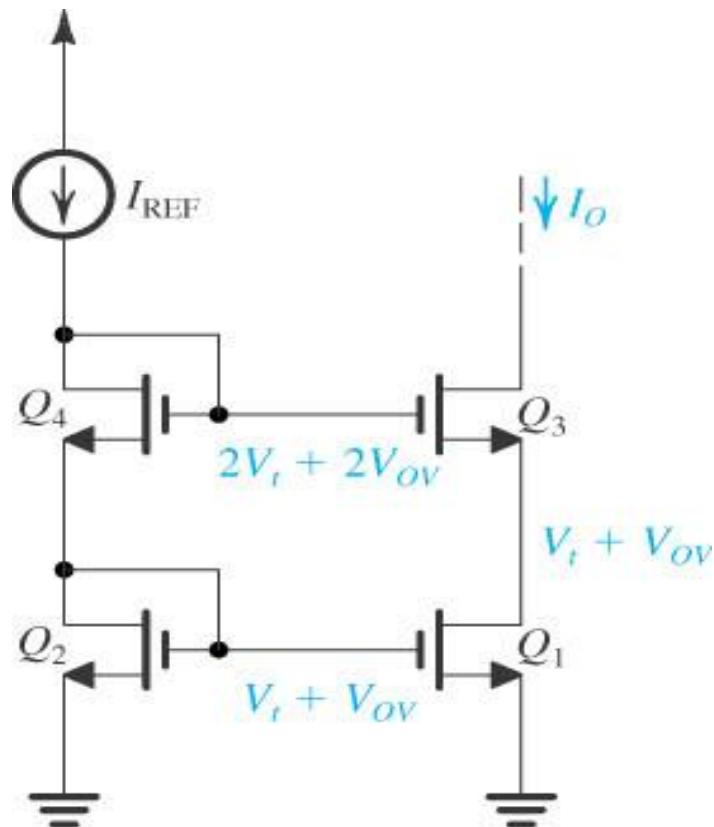
- What is the power dissipation of the op amp?

$$P_D = 5V \times 0.3mA = 1.5mW$$

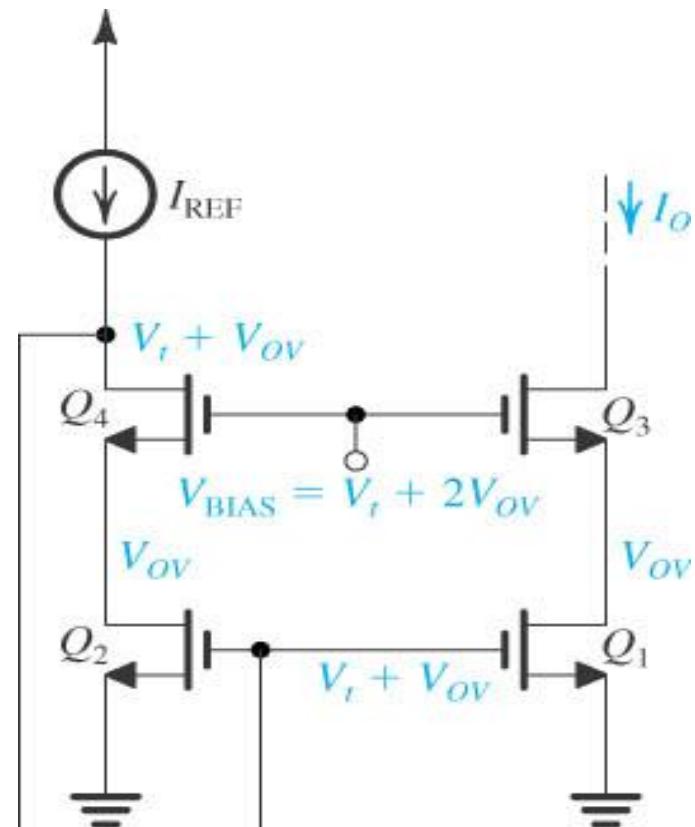
## 12.2.6 Increasing the Input Common-Mode Range: Rail-to-Rail Input Operation



## 12.2.7 Wide-Swing Current Mirror (Increasing the Output Voltage Range)



(a)



(b)

*For Q4 in saturation,  $V_{ds} > V_{gs} - V_t$*   
 $V_{ds} = V_t$ ,  $V_{gs} - V_t = V_{ov}$   
 $\rightarrow V_t > V_{ov}$