電工學　HW 5 Solution

P5.16

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt} = \sqrt{\int_0^1 [3\exp(-t)]^2 dt} = \sqrt{\int_0^1 [9\exp(-2t)]dt}$$

$$= \sqrt{[-4.5\exp(-2t)]_{t=0}^{t=1}} = \sqrt{4.5[1-\exp(-2)]} = 1.973\text{ V}$$

P5.22*

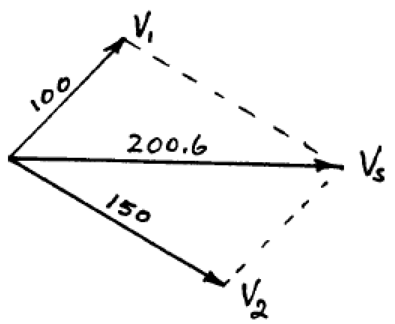
$$\omega = 2\pi f = 400\pi$$

$$v_1(t) = 10\cos(400\pi t + 30^\circ)$$
$$v_2(t) = 5\cos(400\pi t + 150^\circ)$$
$$v_3(t) = 10\cos(400\pi t + 90^\circ)$$

$v_1(t)$ lags $v_2(t)$ by 120°
$v_1(t)$ lags $v_3(t)$ by 60°
$v_2(t)$ leads $v_3(t)$ by 60°

P5.25

$$v_1(t) = 100\cos(\omega t + 45^\circ)$$
$$v_2(t) = 150\sin(\omega t + 60^\circ) = 150\cos(\omega t - 30^\circ)$$
$$\mathbf{V}_1 = 100\angle 45^\circ = 70.71 + j70.71$$
$$\mathbf{V}_2 = 150\angle -30^\circ = 129.9 - j75$$
$$\mathbf{V}_s = \mathbf{V}_1 + \mathbf{V}_2 = 200.6 - j4.29 = 200.6\angle -1.23^\circ$$
$$v_s(t) = 200.6\cos(\omega t - 1.23^\circ)$$



$\mathbf{V}_2$ lags $\mathbf{V}_1$ by 75°
$\mathbf{V}_s$ lags $\mathbf{V}_1$ by 46.23°
$\mathbf{V}_s$ leads $\mathbf{V}_2$ by 28.77°

P5.26 $V_m = 3\text{ V} \qquad T = 0.5\text{ s}$

$$f = \frac{1}{T} = 2\text{ Hz} \qquad \omega = 2\pi f = 4\pi\text{ rad/s}$$

$$\theta = -360^\circ \frac{t_{max}}{T} = -45^\circ$$

$$v(t) = 3\cos(4\pi t - 45^\circ)\text{ V}$$

$$\mathbf{V} = 3\angle -45^\circ\text{ V}$$

$$V_{rms} = \frac{3}{\sqrt{2}} = 2.121\text{ V}$$

P5.27

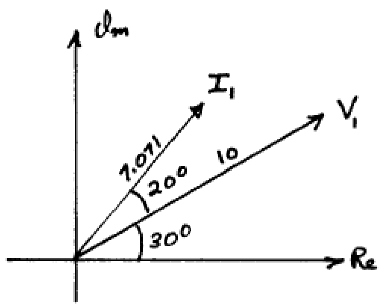
$$v_1(t) = 10\cos(\omega t + 30^\circ)$$

$$I_{1m} = \sqrt{2} \times I_{1rms} = 7.071$$

$$\mathbf{V}_1 = 10\angle 30^\circ$$

$$\mathbf{I}_1 = 7.071\angle 50^\circ$$

$$i_1(t) = 7.071\cos(\omega t + 50^\circ)$$



P5.35 (a) Notice that the voltage is a sine rather than a cosine.

$$\mathbf{V} = 100\angle -60^\circ \quad \mathbf{I} = 1\angle 30^\circ \quad Z = \frac{\mathbf{V}}{\mathbf{I}} = 100\angle -90^\circ = -j100$$

Because Z is pure imaginary and negative, the element is a capacitance.

$$\omega = 200 \qquad C = \frac{1}{|Z|\omega} = 50\ \mu\text{F}$$

(b) $\mathbf{V} = 500\angle 50^\circ \quad \mathbf{I} = 2\angle 50^\circ \quad Z = \frac{\mathbf{V}}{\mathbf{I}} = 250\angle 0^\circ = 250 + j0$

Because Z is pure real, the element is a resistance of 250 Ω.

(c) Notice that the current is a sine rather than a cosine.

$$\mathbf{V} = 100\angle 30^\circ \quad \mathbf{I} = 1\angle -60^\circ \quad Z = \frac{\mathbf{V}}{\mathbf{I}} = 100\angle 90^\circ = j100$$

Because Z is pure imaginary and positive, the element is an inductance.

$$\omega = 400 \qquad L = \frac{|Z|}{\omega} = 0.25\text{ H}$$

5.38 (a) From the plot, we see that $T = 4$ ms, so we have $f = 1/T = 250$ Hz and $\omega = 500\pi$. Also, we see that the current lags the voltage by 1 ms or 90°, so we have an inductance. Finally, $\omega L = V_m / I_m = 5\ \Omega$, from which we find that $L = 3.18$ mH.

(b) From the plot, we see that $T = 8$ ms, so we have $f = 1/T = 125$ Hz and $\omega = 250\pi$. Also, we see that the current leads the voltage by 2 ms or

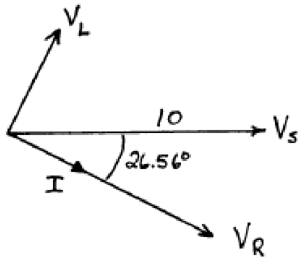
90°, so we have a capacitance. Finally, $1/\omega C = V_m / I_m = 2500\ \Omega$, from which we find that $C = 0.5093\ \mu$F.

P5.42

$$\mathbf{I} = \frac{\mathbf{V}_s}{R + j\omega L} = \frac{10\angle 0^\circ}{100 + j50} = 89.44\angle -26.56^\circ \text{ mA}$$

$$\mathbf{V}_R = R\mathbf{I} = 8.944\angle -26.56^\circ \text{ V}$$

$$\mathbf{V}_L = j\omega L\mathbf{I} = 4.472\angle 63.44^\circ \text{ V}$$



$\mathbf{I}$ lags $\mathbf{V}_s$ by 26.56°

P5.47

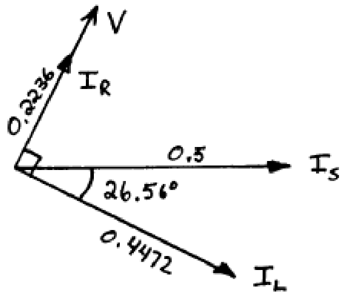
$$\mathbf{I}_s = 0.5\angle 0^\circ$$

$$\mathbf{V} = \mathbf{I}_s \frac{1}{1/200 + 1/j100} = 44.72\angle 63.44^\circ$$

$$\mathbf{I}_R = \mathbf{V}/R = 0.2236\angle 63.44^\circ$$

$$\mathbf{I}_L = \mathbf{V}/j\omega L = 0.4472\angle -26.56^\circ$$

$\mathbf{V}$ leads $\mathbf{I}_s$ by 63.44°



P5.50

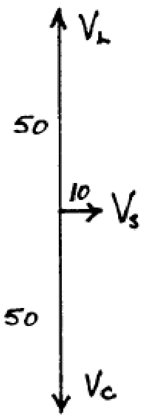
$$\mathbf{V}_s = 10\angle 0^\circ$$

$$\mathbf{I} = \frac{\mathbf{V}_s}{j\omega L + R - j/\omega C} = \frac{10}{j500 + 100 - j500} = 0.1\angle 0^\circ \text{ A}$$

$$\mathbf{V}_L = j500 \times \mathbf{I} = 50\angle 90^\circ$$

$$\mathbf{V}_R = 100\mathbf{I} = 10\angle 0^\circ$$

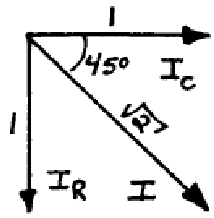
$$\mathbf{V}_C = -j500\mathbf{I} = 50\angle -90^\circ$$



The peak value of $v_L(t)$ is five times larger than the source voltage! This is possible because the impedance of the capacitor cancels the impedance of the inductance.

P5.52

$$Z_{total} = j\omega L + \frac{1}{1/R + j\omega C}$$



$$= j100 + \frac{1}{0.01 + j0.01}$$

$$= j100 + 50 - j50$$

$$= 50 + j50$$

$$= 70.71\angle 45^\circ$$

$$\mathbf{I} = \frac{100\angle 0^\circ}{Z_{total}} = 1.414\angle -45^\circ$$

$$\mathbf{I}_R = \mathbf{I}\frac{Z_C}{R + Z_C} = (1.414\angle -45^\circ) \times \frac{-j100}{100 - j100}$$

$$= 1\angle -90^\circ$$

$$\mathbf{I}_C = \mathbf{I}\frac{R}{R + Z_C} = (1.414\angle -45^\circ) \times \frac{100}{100 - j100}$$

$$= 1\angle 0^\circ$$

P5.56 Writing KCL equations at nodes 1 and 2 we obtain

$$\frac{\mathbf{V}_1}{10} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{5 + j15} = 1\angle 0^\circ$$

$$\frac{\mathbf{V}_2}{-j10} + \frac{\mathbf{V}_2 - \mathbf{V}_1}{5 + j15} = 1\angle 30^\circ$$

Solving these equations, we obtain

$$\mathbf{V}_1 = 6.735\angle -38.54^\circ$$

$$\mathbf{V}_2 = 16.25\angle -55.52^\circ$$

P5.71 $\theta = \theta_v - \theta_i = 30^\circ - 60^\circ = -30^\circ$

power factor $= \cos(\theta) = 86.60\%$ leading

$P = V_{rms} I_{rms} \cos(\theta) = 12.99$ kW

$Q = V_{rms} I_{rms} \sin(\theta) = -7.5$ kVAR

apparent power $= V_{rms} I_{rms} = 1000 \times 15 = 15$ KVA

$$Z = \frac{\mathbf{V}}{\mathbf{I}} = \frac{1000\sqrt{2}\angle 30^\circ}{15\sqrt{2}\angle 60^\circ} = 66.67\angle -30^\circ$$

P5.76 $\mathbf{I} = \dfrac{240\sqrt{2}\angle 50^\circ - 220\sqrt{2}\angle 30^\circ}{1 + j2} = 52.03\angle 52.71^\circ$

$I_{rms} = 36.79 \quad A$

Delivered by Source *A*:

$P_A = 240 I_{rms} \cos(50 - 52.71) = 8.820 \quad \text{kW}$

$Q_A = 240 I_{rms} \sin(50 - 52.71) = -0.418 \quad \text{kVAR}$

Absorbed by Source *B*:

$P_B = 220 I_{rms} \cos(30 - 52.71) = 7.467 \quad \text{kW}$

$Q_A = 220 I_{rms} \sin(30 - 52.71) = -3.125 \quad \text{kVAR}$

Absorbed by resistor:

$P_R = I_{rms}^2 R = 1.353 \text{ kW}$

Absorbed by inductor:

$Q_L = I_{rms}^2 X = 2.707 \text{ kVAR}$

P5.82 Load *A*:

$P_A = 5 \text{ kW}$

$\theta_A = \cos^{-1}(0.9) = 25.84^\circ$

$Q_A = P_A \tan\theta_A = 2.421 \text{ kVAR}$

Load *B*:

$P_B = 10 \text{ kW}$

$\theta_B = \cos^{-1}(0.8) = -36.86^\circ$

(θ is negative for a leading power factor.)

$Q_B = P_B \tan\theta_B = -7.5 \text{ kVAR}$

Source:

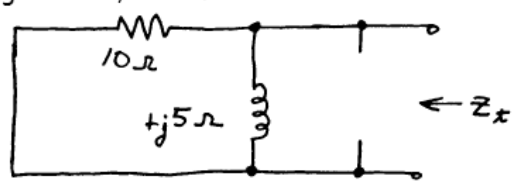
$P_s = P_A + P_B = 15 \text{ kW}$

$Q_s = Q_A + Q_B = -5.079 \text{ kVAR}$

$\text{Apparent power} = \sqrt{(P_s)^2 + (Q_s)^2} = 15.84 \text{ kVA}$

$\text{Power factor} = \dfrac{P_s}{\text{Apparent power}} = 0.9472 = 94.72\% \text{ leading}$

P5.90 Zeroing sources, we have:



Thus, the Thévenin impedance is

$$Z_t = \frac{1}{1/10 + 1/j5} = 4.472\angle 63.43^\circ = 2 + j4$$

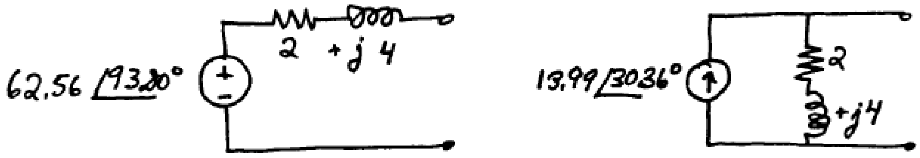
Writing a current equation for the node at the upper end of the current source under open circuit conditions, we have

$$\frac{\mathbf{V}_{oc} - 100\angle 45^\circ}{10} + \frac{\mathbf{V}_{oc}}{j5} = 5$$

$$\mathbf{V}_t = \mathbf{V}_{oc} = 62.56\angle 93.80^\circ$$

$$\mathbf{I}_n = \mathbf{V}_t / Z_t = 13.99\angle 30.36^\circ$$

Thus, the Thévenin and Norton equivalent circuits are:



For the maximum power transfer, the load impedance is

$$Z_{load} = 2 - j4$$

$$\mathbf{I}_{load} = \frac{\mathbf{V}_t}{Z_t + Z_{load}} = \frac{62.56\angle 93.80^\circ}{2 + j4 + 2 - j4} = 15.64\angle 93.80^\circ$$

$$P_{load} = R_{load}\left(I_{rms-load}\right)^2 = 244.6 \text{ W}$$

In the case for which the load must be pure resistance, the load for maximum power transfer is

$$Z_{load} = |Z_t| = 4.472$$

$$I_{load} = \frac{\mathbf{V}_t}{Z_t + Z_{load}} = \frac{62.56\angle 93.80^\circ}{2 + j4 + 4.472} = 8.223\angle -62.08^\circ$$

$$P_{load} = R_{load}\left(I_{rms-load}\right)^2 = 151.2 \text{ W}$$

P5.91 At the lower left-hand node under open-circuit conditions, KCL yields $\mathbf{I}_x = 0.5\mathbf{I}_x$ from which we have $\mathbf{I}_x = 0$. Then, the voltages across the 5-Ω resistance and the $j5$-Ω inductance are zero, and KVL yields

$$\mathbf{V}_t = \mathbf{V}_{ba-oc} = 3\angle 30^\circ$$

With short circuit conditions, we have

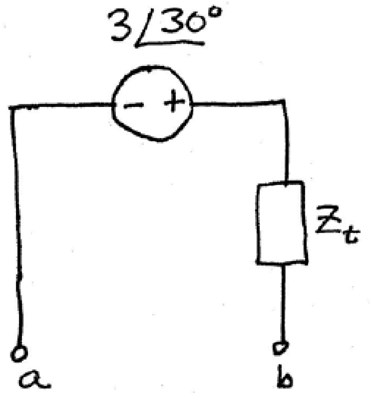
$$\mathbf{I}_x = -\frac{3\angle 30^\circ}{5 + j5} = 0.4243\angle 165^\circ$$

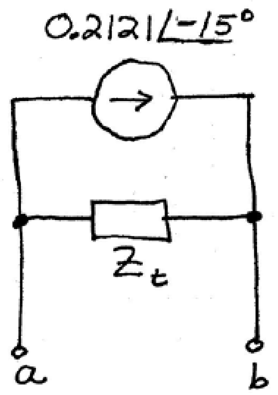
$$\mathbf{I}_n = \mathbf{I}_{ba-sc} = 0.5\mathbf{I}_x - \mathbf{I}_x = 0.2121\angle -15^\circ$$

The Thévenin impedance is given by

$$Z_t = \frac{\mathbf{V}_t}{\mathbf{I}_t} = \frac{3\angle 30^\circ}{0.2121\angle -15^\circ} = 14.14\angle 45^\circ$$

Finally, the equivalent circuits are:





P5.92 Under open-circuit conditions, we have

$$\mathbf{V}_t = \mathbf{V}_{ab-oc} = (4 + j3)2\angle 0^\circ = 10\angle 36.87^\circ \text{ V}$$

With the source zeroed, we look back into the terminals and see

$$Z_t = -j3 + j3 + 4 = 4\,\Omega$$

Next, the Norton current is

$$\mathbf{I}_n = \frac{\mathbf{V}_t}{Z_t} = 2.5\angle 36.87^\circ$$