Formal Model and Verification

Exercise 1: How to make propositions

1. Which of these sentences are propositions ?   What are the truth values of those that are propositions ?

(1a) Boston is the capital of Massachusetts.

(1b) Miami is the capital of Florida.

(1c) 2+3=5

(1d) 5+7=10

(1e) x+2=11

(1f) Answer this question!

(1g) x+y=y+x for every pair of real numbers x and y.

2. What is the negation of each of these propositions ?

(2a) Today is Thursday.

(2b) There is no pollution in New Jersey.

(2c) 2+1=3

(2d) The summer in Maine is hot and sunny.

3. Let *p*, *q*, and *r* be the propositions.
    p : You have the flu.
    q : You miss the final examination.
    r : You pass the course.
Express each of these formulas as an English sentence.

(3a) $p \rightarrow q$

(3b) $\neg q \leftrightarrow r$

(3c) $q \rightarrow \neg r$

(3d) $p \lor q \lor r$

(3e) $(p \rightarrow \neg r) \lor (q \rightarrow \neg r)$
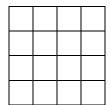
(3f) $(p \land q) \lor (\neg q \land r)$

4. Let p, q, and r be the propositions.
    p : You get an A on the final exam.
    q : You do every exercise in this course.
    r : You get an A in this class.
Write these statements using p, q, and r and logical connectives.

(4a) You get an A in this class, but you do not do every exercise in this course.

(4b) You get an A on the final, you do every exercise in this course, and you get an A in this class.

(4c) To get an A in this class, it is necessary for you to get an A on the final.

(4d) You gent an A on the final, but you don't do every exercise in this course; nevertheless, you get an A in this class.

(4e) Getting an A on the final and doing every exercise in this course is sufficient for getting an A in this class.

(4f) You will get an A in this class if and only if you either do every exercise in this course or you get an A on the final.

5. We have a 4×4 Sudoku game board.

The board is divided into 4 2×2 zones, the top-right, the top-left, the bottom-right, and the bottom-left zones.   Each cell contains an integer value between 1 and 4.   We have the following rules for a solution to the Sudoku game.

(5a) Any two numbers in the same column cannot be the same.
(5b) Any two numbers in the same row cannot be the same.
(5c) Any two numbers in the same zone can neither be the same.

For example, we have the following solution to the game.

1	2	3	4
3	4	1	2
2	1	4	3
4	3	2	1

Please use propositional logics to define solutions.   The only atomic propositions that you can use are of the following form.

$$s\langle i,j,v\rangle$$

Here i, j, and v are integers in interval [1,4].