Formal Methods

Midterm

Instructor: Farn Wang

Class hours: 9:10-12:00 Tuesday　　　　Course Nr.921 U7600

Room: BL 103　　　　Spring 2007

Student name:　　　　Student ID:

1. Please prove the following theorem with Natural Deduction. (10/10)

   $(P \wedge Q) \vee \neg R, P \Rightarrow R \models Q \vee \neg R$

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1. (P∧Q)∨¬R , premise
   2. P∧Q, assumption
   3. Q, ∧e 1
   4. ¬R, assumption
5. Q∨¬R, ∨1, 2-3, 4
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2. Please prove the following theorem with the resolution principle. Specifically, we need you to draw the resolution tree. (10/20)

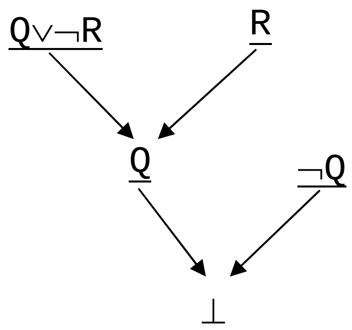
   $(P\wedge Q)\vee\neg R,\ P\Rightarrow R \vDash Q\vee \neg R$

$((P\wedge Q)\vee\neg R) \wedge(P\Rightarrow R) \wedge\neg(Q\vee \neg R)$; proof by refutation

$\equiv((P\wedge Q)\vee\neg R) \wedge(P\Rightarrow R) \wedge\neg(Q\vee \neg R)$; conversion to CNF

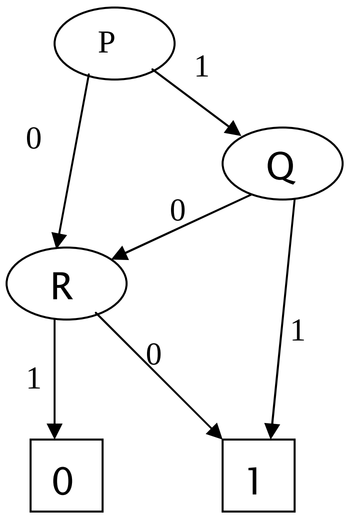
$\equiv(P\vee\neg R)\wedge(Q\vee\neg R)\wedge(\neg P\vee R) \wedge\neg Q\wedge R$

➔ $\{ P\vee\neg R,\ Q\vee\neg R,\ \neg P\vee R,\ \neg Q,\ R \}$; set clauses

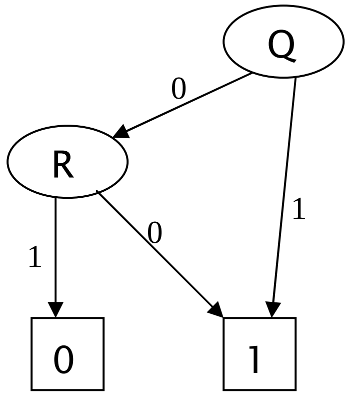


3. Please draw the BDDs for the following formulas. Assume that the variable ordering is P→Q→R. (10/30)

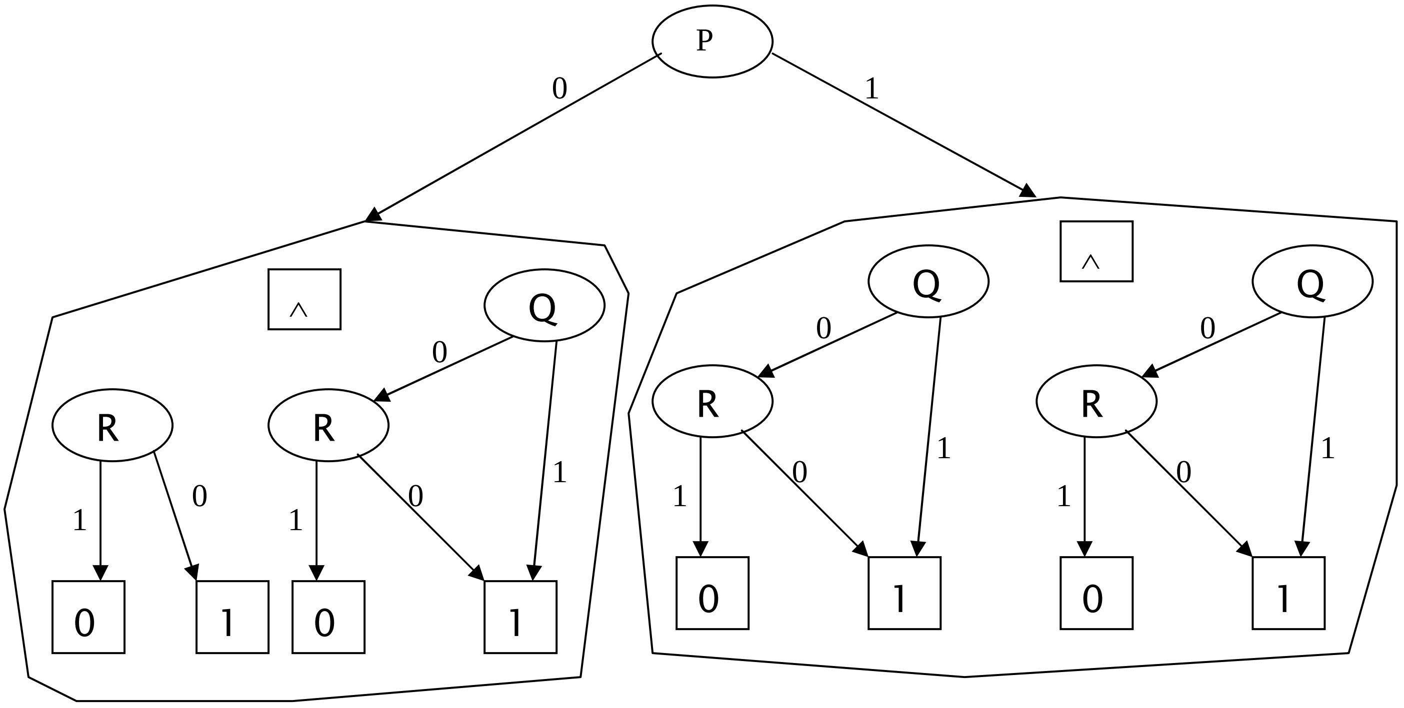
(a) $(P \wedge Q) \vee \neg R$

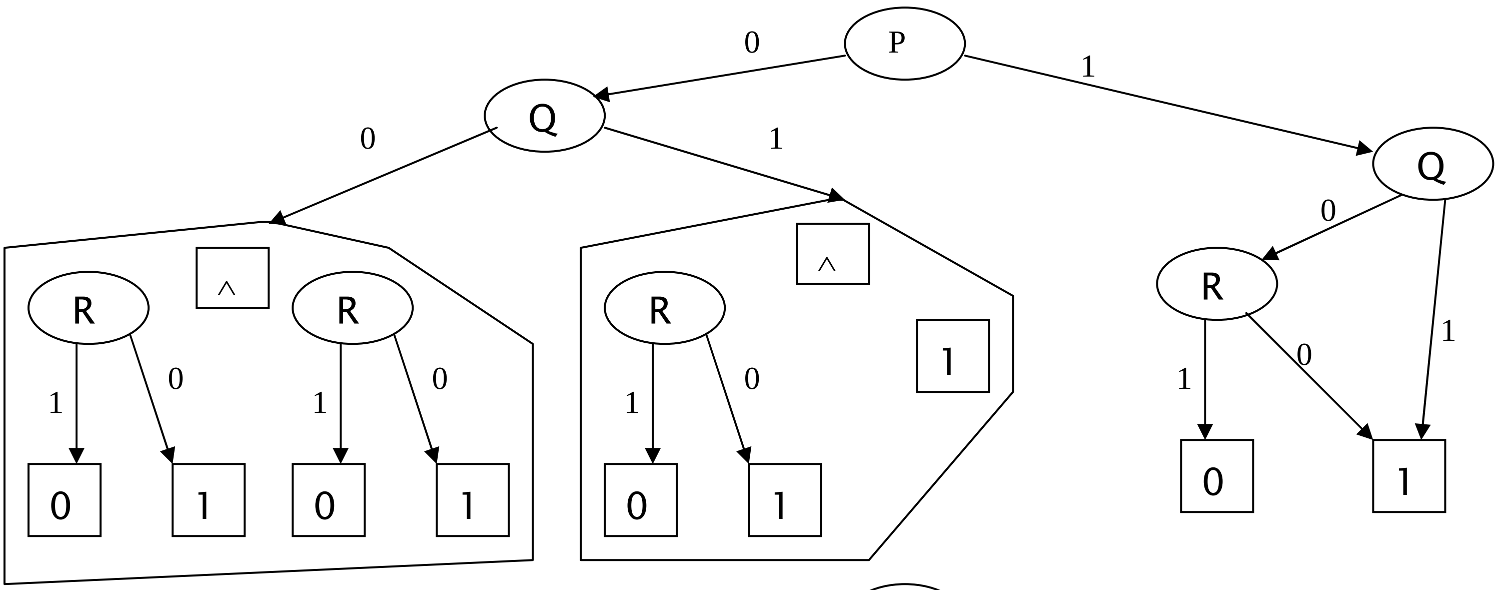


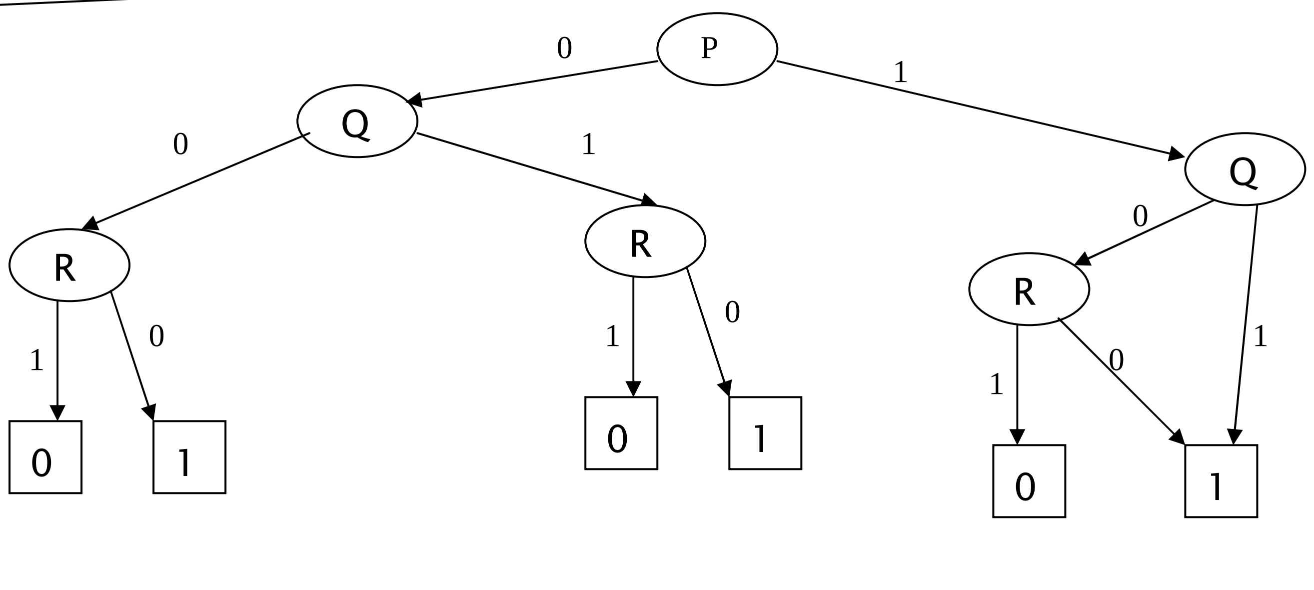
(b) $(P \Rightarrow R) \Rightarrow (Q \vee \neg R)$

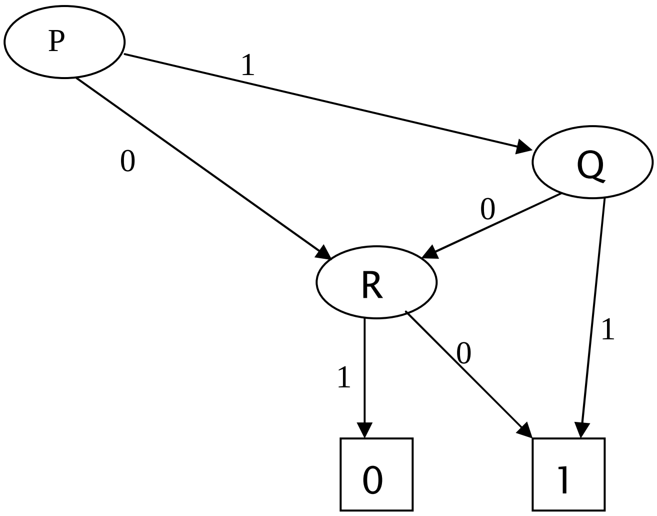


4. Given the BDDs in questions 3(a) and 3(b), please show us the steps to construct the BDD of their conjunction. In each step, please draw the result BDD of the step. (10/40)








P
1
0
Q
0
R
1
0
1
0
1

5. Please show us the Herbrand universe subsets H_0, H_1, and H_2 of the following formula. (10/50)

$$(P(x) \vee Q(f(x))) \wedge (R(x,y) \Rightarrow \neg Q(x)) \wedge (R(g(x,y),z) \Rightarrow \neg P(x))$$

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H0={a}, a is an introduced constant since there is no
constant.

H1 = {a, f(a), g(a,a)}

H2 = { a, f(a), g(a,a),
       f(f(a)), f(g(a,a)),
       g(a,f(a)), g(f(a),a), g(f(a), f(a))}
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6. Please show us a Herbrand interpretation that satisfies the following formula. (10/60)

$$(P(x) \vee Q(f(x))) \wedge (R(x,y) \Rightarrow \neg Q(x)) \wedge (R(g(x,y),z) \Rightarrow \neg P(x))$$

$\{P(x) \mid x \in H\infty\}$

7. Please find the most general unifier between the two atoms in each of the following three items if it exists.   (10/70)

(a) P(x,f(a,h(y)),z), P(u,f(v,w),f(a))

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{ u/x, a/v, h(y)/w, a/z }
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(b) P(x,f(a,f(b,f(x)))), P(a, f(y,z))

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{ a/x, a/y, f(b,f(x))/z }
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(c) P(x,f(x,f(x,f(x)))), P(y,y)

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{ x/y, y/f(x,f(x,f(x))) }, There is no mgu since there is a cyclic substitution.
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8. Please use the resolution principle to prove the following theorem.
Specifically, please draw the resolution tree with unifiers. (10/80)

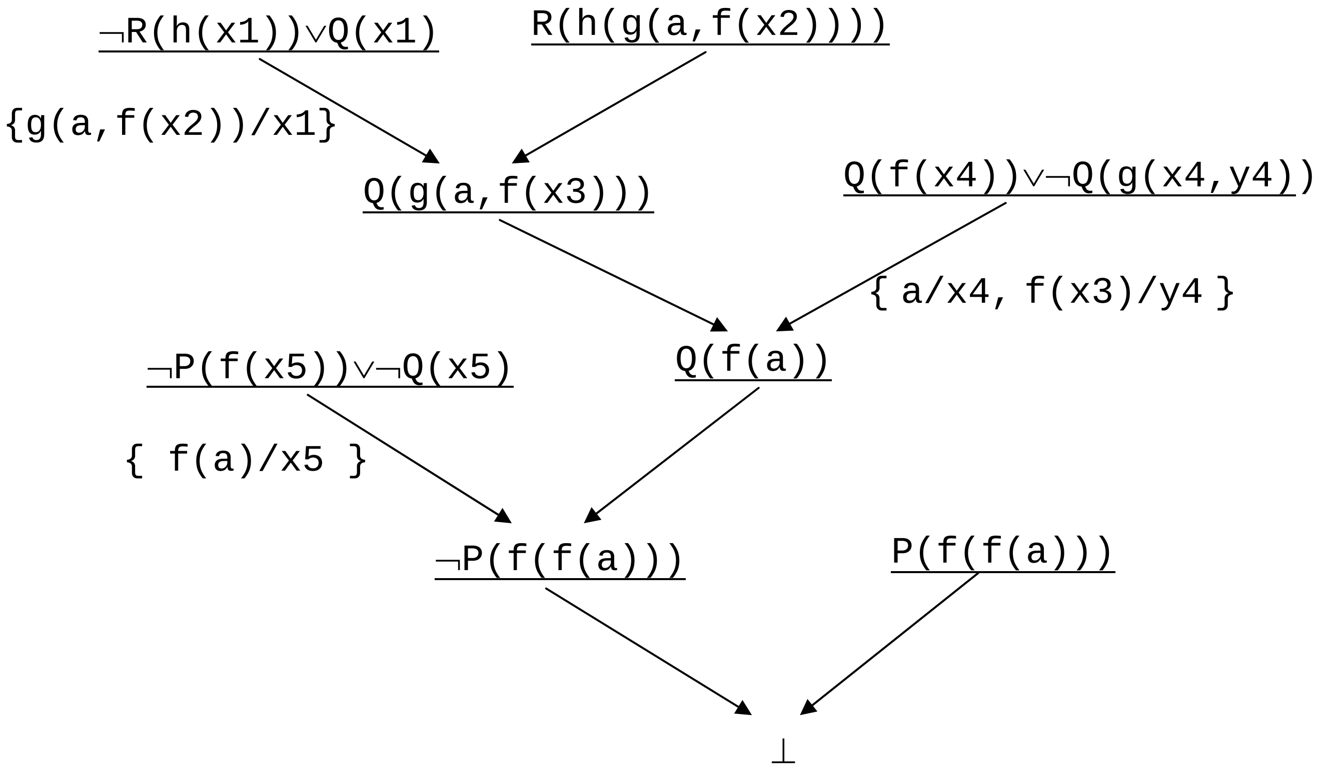
$Q(f(x)) \lor \neg Q(g(x,y))$,

$P(f(x)) \Rightarrow \neg Q(x)$,

$R(h(x)) \Rightarrow Q(x)$

$\vDash \quad \neg P(f(f(a))) \lor \neg R(h(g(a,f(x))))$

$Q(f(x)) \lor \neg Q(g(x,y)),\ \neg P(f(x)) \lor \neg Q(x),\ \neg R(h(x)) \lor Q(x),$
$\neg(\neg P(f(f(a))) \lor \neg R(h(g(a,f(x)))))$
$\equiv P(f(f(a))) \land R(h(g(a,f(x))))$



9. Please construct the clausal form of the following formula. (10/90)

$\exists x($ $Q(x)$

$\wedge \forall y((\neg Q(y)) \Rightarrow \exists z(P(y,z)))$

$\wedge \forall y \forall z \exists w(N(y,z) \Rightarrow (P(y,w) \wedge P(w,z)))$

$)$

➔ $\exists x(Q(x) \wedge \forall y(Q(y) \vee \exists z(P(y,z)))$
$\wedge \forall y \forall z \exists w((\neg N(y,z)) \vee (P(y,w) \wedge P(w,z)))$
$)$

➔ $Q(a) \wedge \forall y(Q(y) \vee P(y,z(y)))$
$\wedge \forall y \forall z((\neg N(y,z)) \vee (P(y,w(y,z)) \wedge P(w(y,z),z)))$

➔ $Q(a) \wedge \forall y(Q(y) \vee P(y,z(y)))$
$\wedge \forall y \forall z((\neg N(y,z)) \vee (P(y,w(y,z)) \wedge P(w(y,z),z)))$

➔ $Q(a) \wedge \forall y1(Q(y1) \vee P(y,z(y1)))$
$\wedge \forall y2 \forall z2((\neg N(y2,z2)) \vee (P(y2,w(y2,z2)) \wedge P(w(y2,z2),z2)))$

➔ $\forall y1 \forall y2 \forall z2 (Q(a) \wedge (Q(y1) \vee P(y,z(y1)))$
$\wedge ((\neg N(y2,z2)) \vee (P(y2,w(y2,z2)) \wedge P(w(y2,z2),z2))))$

➔ $Q(a) \wedge (Q(y1) \vee P(y,z(y1)))$
$\wedge ((\neg N(y2,z2)) \vee (P(y2,w(y2,z2)) \wedge P(w(y2,z2),z2)))$

➔ $Q(a) \wedge (Q(y1) \vee P(y,z(y1)))$
$\wedge (\neg N(y2,z2) \vee P(y2,w(y2,z2)))$
$\wedge (\neg N(y2,z2) \vee P(w(y2,z2),z2))$

➔ $Q(a)$,
$Q(y1) \vee P(y,z(y1))$,
$\neg N(y2,z2) \vee P(y2,w(y2,z2))$,
$\neg N(y2,z2) \vee P(w(y2,z2),z2)$

10. Try to prove the following theorem with the resolution principle and *strategy set-of-support*. Specifically, we need you to draw the resolution tree with unifiers if you do it. Explain to me what and why the result is. (10/100)

$P(x) \vee Q(g(x)) \vee \neg Q(h(x)),\ P(x) \Rightarrow R(x),\ \neg R(a),\ \neg Q(g(a)),\ Q(h(a))$

$\models R(b)$

It is not possible to prove the theorem with the set-of-support strategy since the theorem is true due to the inconsistency of the premises. Thus the theorem is true regardless of the conclusion of the theorem. With the set-of-support strategy, every internal node in the resolution tree must be a decendant of the conclusion. This is not true since “contradiction” is not deducible from the resolution sequence from the conclusion.