105-1: EE4052

計算機程式設計

Computer Programming

Unit 15: 線性關係

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臺大電機系

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大綱

什麼是 abline

什麼是 線性回歸模型

U10：加入圖形元件 – 點 線 框

- points( )　　# 打點
- lines( )　　# 畫線
- abline( )　　# 畫 y = b x + a 的直線
- segments( )　　# 畫線段
- arrows( )　　# 畫箭頭
- box( )　　# 在原圖形最外圍加上框框
- lty　　# 直線的樣式
- lwd　　# 直線的寬度

U12：遺漏值處理

- 回歸模型預測值補差法：
- nhanes2[, 4]　　# 針對第4組數據
- sub <- which(is.na(nhanes2[, 4]) == TRUE)
- dataTR <- nhanes2[-sub,]
- dataTE <- nhanes2[sub,]
- dataTE
- lmout <- lm(chl ~ age, data = dataTR)
  - # 利用 dataTR 中 age 為引數，chl 為因變數，建構線性回歸模型
- dataTE[, 4] <- round(predict(lmout, dataTE))
  - # 用回歸模型預測值取代之
- dataTE

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三個資料庫

nhanes2, cars, iris

資料庫

- install.packages("mice") # 安裝 mice 軟體套件
- library(mice) # 載入 mice 軟體套件
- data(nhanes2)
- nrow(nhanes2) # nhanes2 資料集的橫列數
- ncol(nhanes2) # nhanes2 資料集的直行數
- summary(nhanes2) # nhanes2 資料集的概括資訊
- head(nhanes2)

```
> head( nhanes2 )
    age  bmi  hyp chl
1 20-39   NA <NA>  NA
2 40-59 22.7   no 187
3 20-39   NA   no 187
4 60-99   NA <NA>  NA
5 20-39 20.4   no 113
6 60-99   NA <NA> 184
```

```
> summary( nhanes2 )
    age          bmi          hyp          chl
 20-39:12   Min.   :20.40   no  :13   Min.   :113.0
 40-59: 7   1st Qu.:22.65   yes : 4   1st Qu.:185.0
 60-99: 6   Median :26.75   NA's: 8   Median :187.0
            Mean   :26.56             Mean   :191.4
            3rd Qu.:28.93             3rd Qu.:212.0
            Max.   :35.30             Max.   :284.0
            NA's   :9                 NA's   :10
```

遺漏值處理：回歸模型預測值補差法

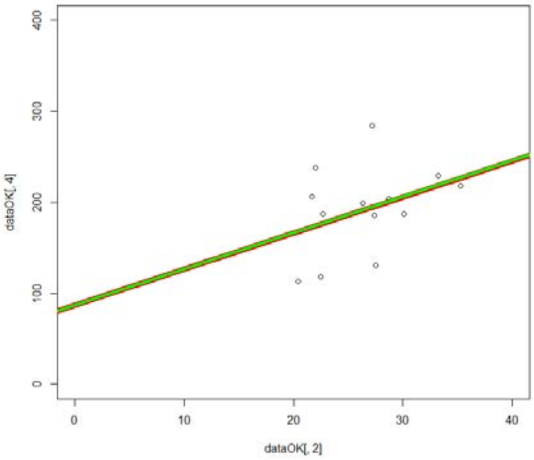
- 回歸模型預測值補差法：
- data0 <- nhanes2 # 針對第2, 4組數據
- subNA <- which(is.na(nhanes2[, 4]) == TRUE | is.na(nhanes2[, 2]) == TRUE)
- dataOK <- nhanes2[-subNA,]
- dataNA <- nhanes2[subNA,]
- dataOK
- dataNA
- lmchlbmi <- lm(chl ~ bmi, data = dataOK)
  - # 利用 dataOK 中 bmi 為引數，chl 為因變數，建構線性回歸模型

```
> lmout

Call:
lm(formula = chl ~ bmi, data = dataOK)

Coefficients:
(Intercept)          bmi
     87.130        3.963
```

chl = 3.963 * bmi + 87.130

遺漏值處理：回歸模型預測值補差法

- abline( ) # 畫 y = b x + a 的直線
- plot(dataOK[, 2], dataOK[, 4], xlim = c(0, 40), ylim = c(0, 400))
- abline(a = 87.130, b = 3.963, col = "red", lwd = 8)
- abline(lmchlbmi, col = "green", lwd = 4)

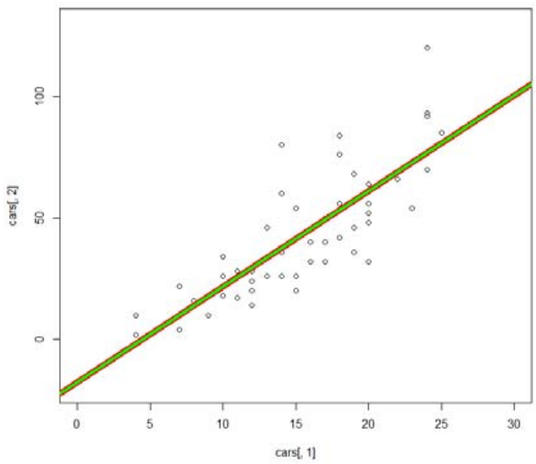


chl = 3.963 * bmi + 87.130

```
> lmchlbmi

Call:
lm(formula = chl ~ bmi, data = dataOK)

Coefficients:
(Intercept)          bmi
     87.130        3.963
```

另一個資料：cars

- cars
- plot(cars[, 1], cars[, 2], xlim = c(0, 30), ylim = c(-20, 130))
- lmcars <- lm(dist ~ speed, data = cars)
- abline(a = -17.579, b = 3.932, col = "red", lwd = 8)
- abline(lmcars, col = "green", lwd = 4)

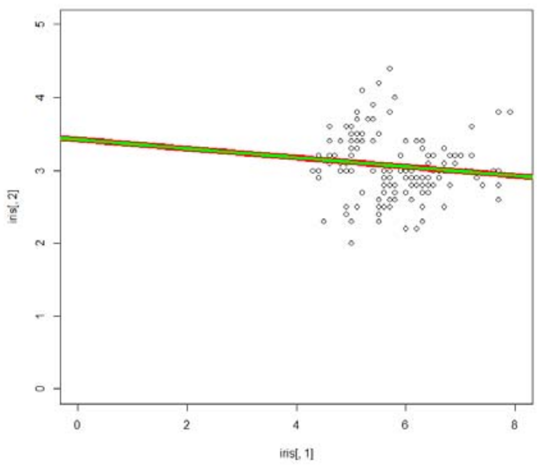



```
chl = 3.932 * speed – 17.579
```

```
> lmcars

Call:
lm(formula = dist ~ speed, data = cars)

Coefficients:
(Intercept)        speed
    -17.579        3.932
```


另一個資料：iris

- iris
- plot(iris[, 1], iris[, 2], xlim = c(0, 8), ylim = c(0, 5))
- lmiris1 <- lm(Sepal.Width ~ Sepal.Length, data = iris)
- abline(a = 3.41895, b = -0.06188 , col = "red", lwd = 8)
- abline(lmiris1, col = "green", lwd = 4)

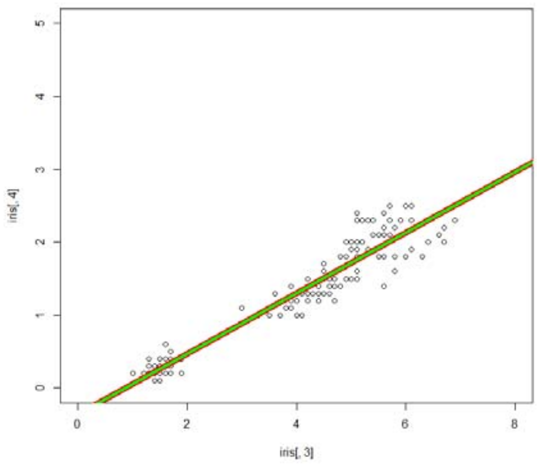



```
Sepal.Width = -0.06188 * Sepal.Length + 3.41895
```

```
> lmiris1

Call:
lm(formula = Sepal.Width ~ Sepal.Length,
data = iris)

Coefficients:
 (Intercept)  Sepal.Length
     3.41895      -0.06188
```

另一個資料：iris

- iris
- plot(iris[, 3], iris[, 4], xlim = c(0, 8), ylim = c(0, 5))
- lmiris2 <- lm(Petal.Width ~ Petal.Length, data = iris)
- abline(a = -0.3631, b = 0.4158, col = "red", lwd = 8)
- abline(lmiris2, col = "green", lwd = 4)




```
Petal.Width = 0.4158 * Petal.Length - 0.3631
```

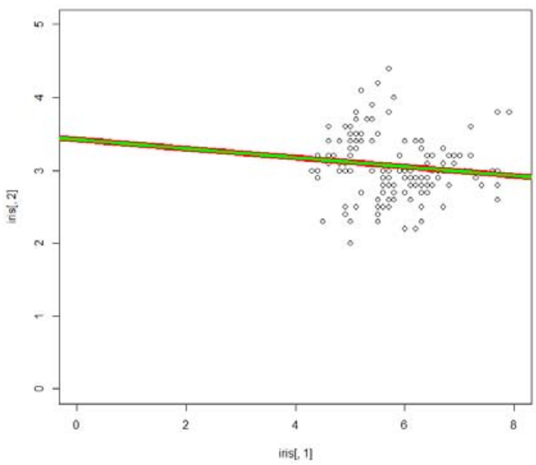
```
> lmiris2

Call:
lm(formula = Petal.Width ~ Petal.Length,
data = iris)

Coefficients:
 (Intercept)  Petal.Length
     -0.3631        0.4158
```


另一個資料：iris

- iris

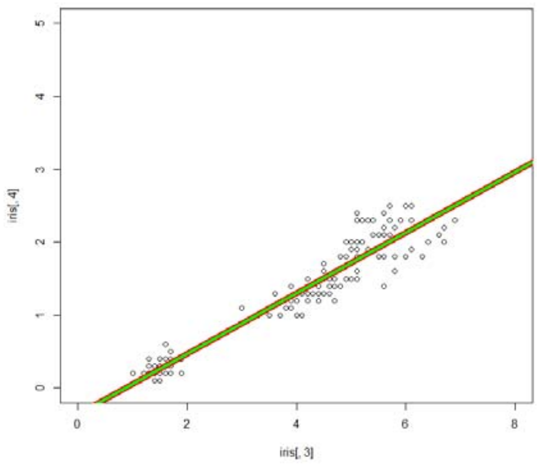
```
> lmiris1

Call:
lm(formula = Sepal.Width ~ Sepal.Length,
data = iris)

Coefficients:
 (Intercept)  Sepal.Length
     3.41895      -0.06188
```

```
Sepal.Width = -0.06188 * Sepal.Length + 3.41895
```



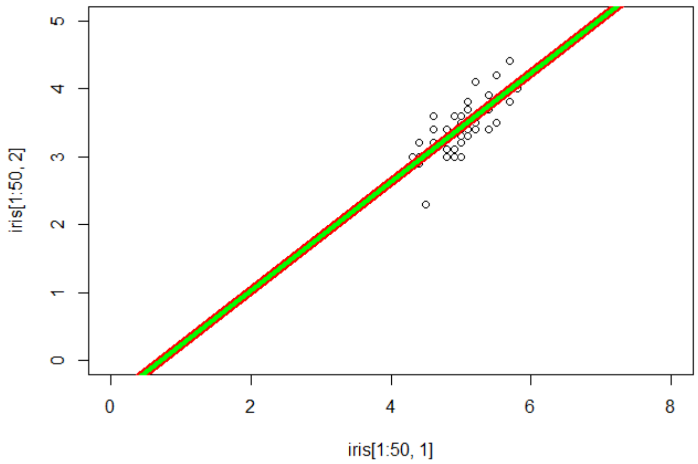

```
> lmiris2

Call:
lm(formula = Petal.Width ~ Petal.Length,
data = iris)

Coefficients:
 (Intercept)  Petal.Length
     -0.3631        0.4158
```

```
Petal.Width = 0.4158 * Petal.Length - 0.3631
```



另一個資料：iris, 依照種類

- iris
- plot(iris[1:50, 1], iris[1:50, 2], xlim = c(0, 8), ylim = c(0, 5))
- lmiris11 <- lm(Sepal.Width ~ Sepal.Length, data = iris[1:50,])
- abline(a = -0.5694, b = 0.7985, col = "red", lwd = 8)
- abline(lmiris11, col = "green", lwd = 4)

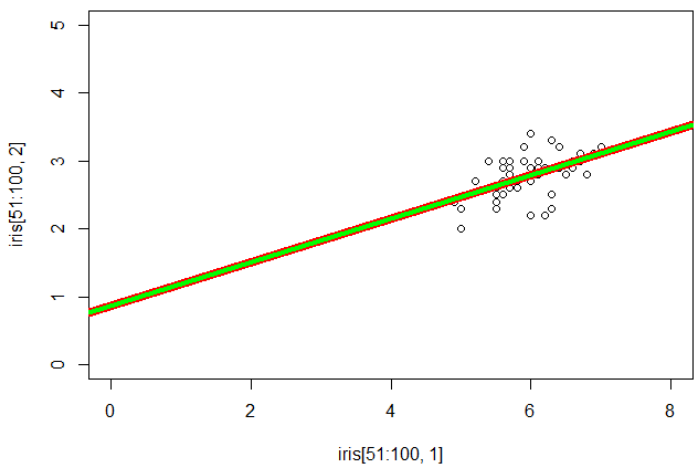



```
Sepal.Width = 0.7985 * Sepal.Length - 0.5694
```

```
> lmiris11

Call:
lm(formula = Sepal.Width ~ Sepal.Length,
data = iris[1:50, ] )

Coefficients:
 (Intercept)  Sepal.Length
     -0.5694        0.7985
```


另一個資料：iris, 依照種類

- iris
- plot(iris[51:100, 1], iris[51:100, 2], xlim = c(0, 8), ylim = c(0, 5))
- lmiris12 <- lm(Sepal.Width ~ Sepal.Length, data = iris[51:100,])
- abline(a = 0.8721, b = 0.3197, col = "red", lwd = 8)
- abline(lmiris12, col = "green", lwd = 4)

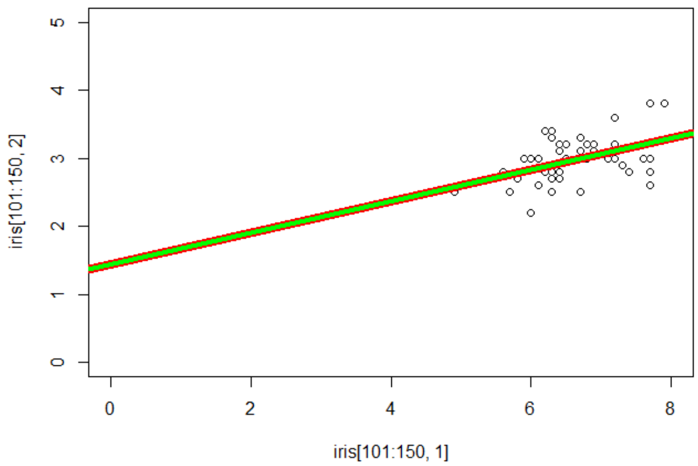



```
Sepal.Width = 0.3197 * Sepal.Length + 0.8721
```

```
> lmiris12

Call:
lm(formula = Sepal.Width ~ Sepal.Length,
data = iris[51:100, ] )

Coefficients:
 (Intercept)  Sepal.Length
      0.8721        0.3197
```

另一個資料：iris, 依照種類

- iris
- plot(iris[101:150, 1], iris[101:150, 2], xlim = c(0,8), ylim = c(0,5))
- lmiris13 <- lm(Sepal.Width ~ Sepal.Length, data = iris[101:150,])
- abline(a = 1.4463, b = 0.2319, col = "red", lwd = 8)
- abline(lmiris13, col = "green", lwd = 4)




```
Sepal.Width = 0.3197 * Sepal.Length + 1.4463
```

```
> lmiris13

Call:
lm(formula = Sepal.Width ~ Sepal.Length,
data = iris[101:150, ] )

Coefficients:
 (Intercept)  Sepal.Length
      1.4463        0.2319
```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另一個資料：iris, 依照種類

- lmiris11 <- lm(Sepal.Width ~ Sepal.Length, data = iris[1:50,])
- lmiris12 <- lm(Sepal.Width ~ Sepal.Length, data = iris[51:100,])
- lmiris13 <- lm(Sepal.Width ~ Sepal.Length, data = iris[101:150,])
- lmiris21 <- lm(Petal.Width ~ Petal.Length, data = iris[1:50,])
- lmiris22 <- lm(Petal.Width ~ Petal.Length, data = iris[51:100,])
- lmiris23 <- lm(Petal.Width ~ Petal.Length, data = iris[101:150,])

另一個資料：iris, 依照種類

- layout(matrix(1:6, nrow = 2, byrow = T))
- plot(iris[1:50, 1], iris[1:50, 2], xlim = c(0, 8), ylim = c(0, 5))
- abline(lmiris11, col = "green", lwd = 4)
- plot(iris[51:100, 1], iris[51:100, 2], xlim = c(0, 8), ylim = c(0, 5))
- abline(lmiris12, col = "green", lwd = 4)
- plot(iris[101:150, 1], iris[101:150, 2], xlim = c(0,8), ylim = c(0,5))
- abline(lmiris13, col = "green", lwd = 4)
- plot(iris[1:50, 3], iris[1:50, 4], xlim = c(0, 8), ylim = c(0, 5))
- abline(lmiris21, col = "green", lwd = 4)
- plot(iris[51:100, 3], iris[51:100, 4], xlim = c(0, 8), ylim = c(0, 5))
- abline(lmiris22, col = "green", lwd = 4)
- plot(iris[101:150, 3], iris[101:150, 4], xlim = c(0,8), ylim = c(0,5))
- abline(lmiris23, col = "green", lwd = 4)

另一個資料：iris, 依照種類

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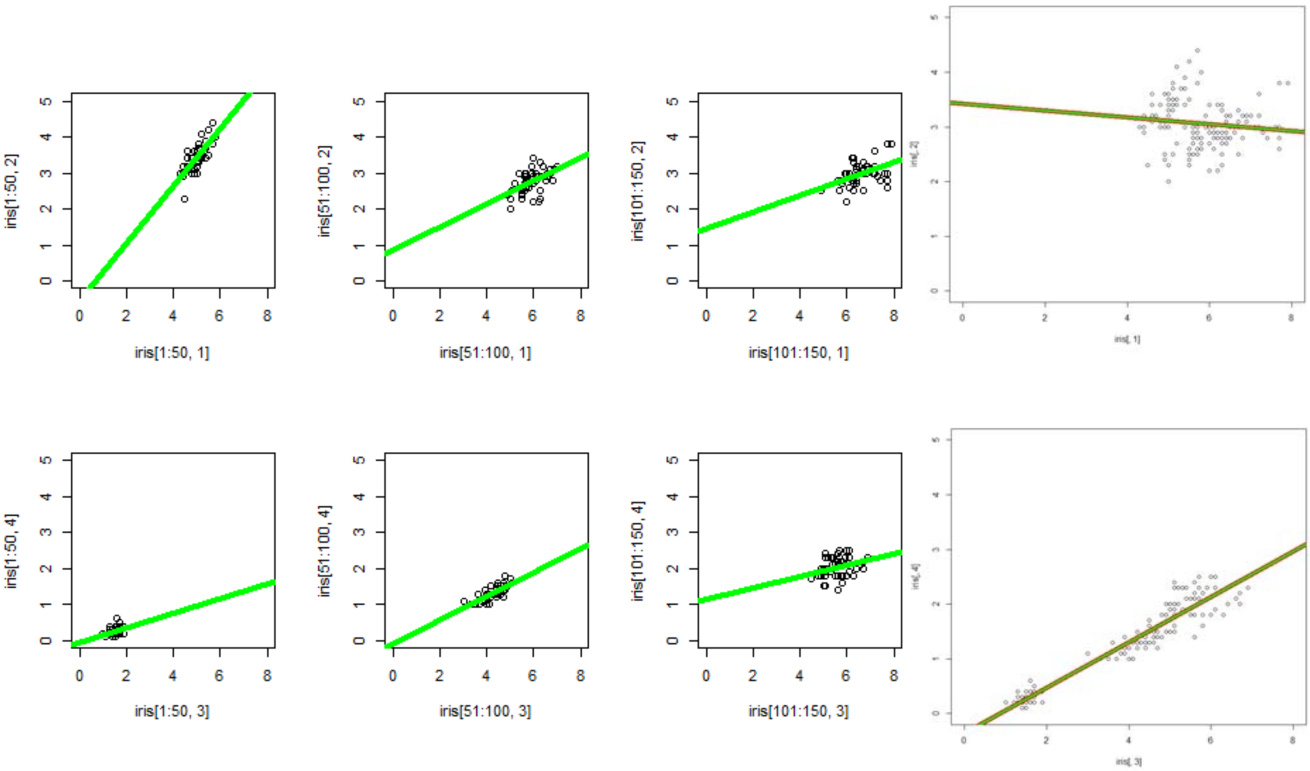

U11：相關性

cor( ), correlation 相關係數

cor(x, y)

cor_matrix <- cor(data_all, use = "pairwise")

cor_iris <- cor(iris[, 1:4], use = "pairwise")

```
> cor_iris

             Sepal.Length Sepal.Width Petal.Length Petal.Width
Sepal.Length    1.0000000  -0.1175698    0.8717538   0.8179411
Sepal.Width    -0.1175698   1.0000000   -0.4284401  -0.3661259
Petal.Length    0.8717538  -0.4284401    1.0000000   0.9628654
Petal.Width     0.8179411  -0.3661259    0.9628654   1.0000000
```


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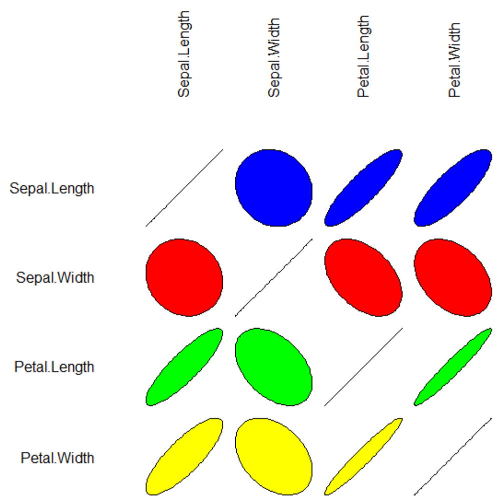
U11：相關性

plotcoor( ), 繪製相關圖

install.packages("ellipse")

library(ellipse)

plotcorr(cor_iris, col = c("blue", "red", "green", "yellow"))




```
> cor_iris

             Sepal.Length Sepal.Width Petal.Length Petal.Width
Sepal.Length    1.0000000  -0.1175698    0.8717538   0.8179411
Sepal.Width    -0.1175698   1.0000000   -0.4284401  -0.3661259
Petal.Length    0.8717538  -0.4284401    1.0000000   0.9628654
Petal.Width     0.8179411  -0.3661259    0.9628654   1.0000000
```

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lm: Linear Model

Least Squares Approximation

Least Squares Approximation

- 參考資料：http://www.ms.uky.edu/~ma138/Spring15/Curve_fitting.pdf

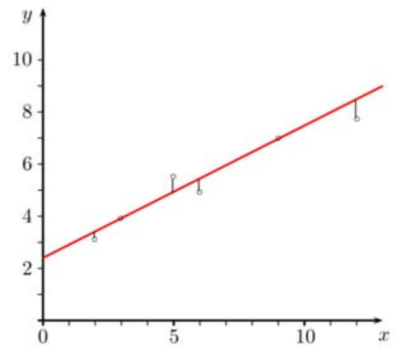

FIGURE 1: Fitting a straight line to data by the method of least squares

$$\boxed{y = ax + b}$$

$$\begin{cases} ax_1 + b = y_1 \\ ax_2 + b = y_2 \\ \quad\vdots \\ ax_n + b = y_n \end{cases} \rightsquigarrow \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

$$\delta_1 = (ax_1 + b) - y_1, \quad \delta_2 = (ax_2 + b) - y_2, \quad \ldots, \quad \delta_n = (ax_n + b) - y_n.$$

$$\sqrt{\delta_1^2 + \delta_2^2 + \ldots + \delta_n^2} \text{ is as small as possible.}$$

$$\widehat{a} = \frac{n\left(\sum_{i=1}^{n} x_i y_i\right) - \left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right)}{n\left(\sum_{i=1}^{n} x_i^2\right) - \left(\sum_{i=1}^{n} x_i\right)^2} \qquad \widehat{b} = \frac{1}{n}\left(\sum_{i=1}^{n} y_i - \widehat{a}\sum_{i=1}^{n} x_i\right),$$

$$\boxed{y = \widehat{a}\, x + \widehat{b}}$$

Least Squares Approximation

- 參考資料：http://www.ms.uky.edu/~ma138/Spring15/Curve_fitting.pdf

t (sec)	0.5	1.1	1.5	2.1	2.3
T (°C)	32.0	33.0	34.2	35.1	35.7

$\boxed{T = at + b,}$

$$\begin{cases} 0.5\,a+b &= 32.0\\ 1.1\,a+b &= 33.0\\ 1.5\,a+b &= 34.2\\ 2.1\,a+b &= 35.1\\ 2.3\,a+b &= 35.7 \end{cases} \rightsquigarrow \begin{bmatrix} 0.5 & 1\\ 1.1 & 1\\ 1.5 & 1\\ 2.1 & 1\\ 2.3 & 1 \end{bmatrix}\begin{bmatrix} a\\ b\end{bmatrix} = \begin{bmatrix} 32.0\\ 33.0\\ 34.2\\ 35.1\\ 35.7\end{bmatrix}.$$

$$A^TA = \begin{bmatrix} 0.5 & 1.1 & 1.5 & 2.1 & 2.3\\ 1 & 1 & 1 & 1 & 1\end{bmatrix}\begin{bmatrix} 0.5 & 1\\ 1.1 & 1\\ 1.5 & 1\\ 2.1 & 1\\ 2.3 & 1 \end{bmatrix} = \begin{bmatrix} 13.41 & 7.5\\ 7.5 & 5\end{bmatrix}$$

$$A^T\mathbf{b} = \begin{bmatrix} 0.5 & 1.1 & 1.5 & 2.1 & 2.3\\ 1 & 1 & 1 & 1 & 1\end{bmatrix}\begin{bmatrix} 32.0\\ 33.0\\ 34.2\\ 35.1\\ 35.7\end{bmatrix} = \begin{bmatrix} 259.42\\ 170\end{bmatrix}$$

$$\begin{bmatrix} 13.41 & 7.5\\ 7.5 & 5\end{bmatrix}\begin{bmatrix} a\\ b\end{bmatrix} = \begin{bmatrix} 259.42\\ 170\end{bmatrix}$$

$$\left[\begin{array}{cc|c} 13.41 & 7.5 & 259.42\\ 7.5 & 5 & 170\end{array}\right] \text{ is equivalent to } \left[\begin{array}{cc|c} 1 & 0 & 2.0463\\ 0 & 1 & 30.93\end{array}\right]$$

$\widehat{a} = 2.0463$ and $\widehat{b} = 30.93$. $\boxed{T(t) = 2.0463\,t + 30.93}$

year	1980	1985	1990	1995
population	227	237	249	262

$\boxed{P(t) = at + b.}$

$$\begin{bmatrix} 0 & 1\\ 5 & 1\\ 10 & 1\\ 15 & 1\end{bmatrix}\begin{bmatrix} a\\ b\end{bmatrix} = \begin{bmatrix} 227\\ 237\\ 249\\ 262\end{bmatrix}$$

$$A^TA = \begin{bmatrix} 0 & 5 & 10 & 15\\ 1 & 1 & 1 & 1\end{bmatrix}\begin{bmatrix} 0 & 1\\ 5 & 1\\ 10 & 1\\ 15 & 1\end{bmatrix} = \begin{bmatrix} 350 & 30\\ 30 & 4\end{bmatrix}$$

$$A^T\mathbf{b} = \begin{bmatrix} 0 & 5 & 10 & 15\\ 1 & 1 & 1 & 1\end{bmatrix}\begin{bmatrix} 227\\ 237\\ 249\\ 262\end{bmatrix} = \begin{bmatrix} 7605\\ 975\end{bmatrix}$$

$$\begin{bmatrix} 350 & 30\\ 30 & 4\end{bmatrix}\begin{bmatrix} a\\ b\end{bmatrix} = \begin{bmatrix} 7605\\ 975\end{bmatrix}$$

$$\left[\begin{array}{cc|c} 350 & 30 & 7605\\ 30 & 4 & 975\end{array}\right] \text{ is equivalent to } \left[\begin{array}{cc|c} 1 & 0 & 117/50\\ 0 & 1 & 1131/5\end{array}\right]$$

$\boxed{P(t) = 117/50 \cdot t + 1131/5.}$