Fall 2019

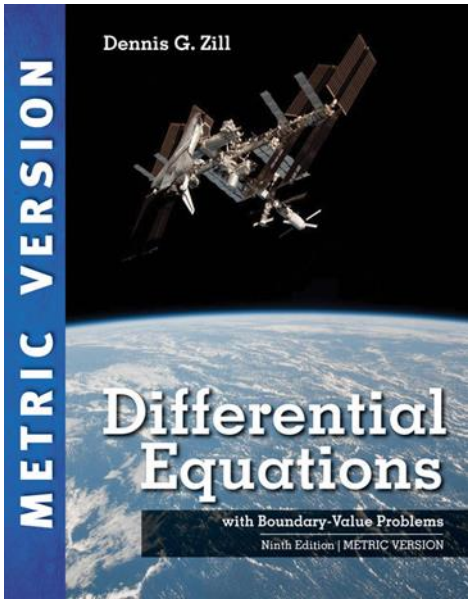
微分方程
Differential Equations

Unit 00.0
Introduction

Feng-Li Lian

NTU-EE

Sep19 – Jan20



Figures and images used in these lecture notes are adopted from
Differential Equations with Boundary-Value Problems, 9th Ed., D.G. Zill, 2018 (Metric Version)

Syllabus

- **Lecture Information:**
  - Time: Weds 10:20am-12:10pm
  - Room: MD-205
  - Office Hours: by e-mail appointment
  - Website: http://cc.ee.ntu.edu.tw/~fengli/Teaching/DifferentialEquations/
- **Instructor:**
  - 連豊力(Feng-Li Lian)
  - Office: MD-717
  - Email: fengli@ntu.edu.tw
  - Phone: 02-3366-3606
- **Textbook:**
  - Differential Equations with Boundary-Value Problems 9th Edition, 2018, Metric Version, Dennis G. Zill
- **Grading:**
  - Exams: Midterm, Final, 85%
  - Homeworks, Quizs etc., 15%

Lectures and Exams

- Midterm Exam
  - 1-1, 1-2, 1-3 (Introduction)
  - 2-1, 2-2, 2-3, 2-4, 2-5, 2-6 (First Order DE - Math)
  - 3-1, 3-2, 3-3 (First Order DE - Modeling)
  - 4-1, 4-2, 4-3, 4-4, 4-5 (Higher Order DE - Math)

- Final Exam
  - 4-6, 4-7 (Higher Order DE - Math)
  - 5-1 (Higher Order DE - Modeling)
  - 7-1, 7-2, 7-3, 7-4, 7-5, 7-6 (Transform – Laplace)
  - 11-1, 11-2, 11-3 (Transform – Fourier)
  - 12-1 (Partial DE)

Lectures and Exams

- **Introduction** — Chap 1: 1-1, 1-2, 1-3
- **First Order DE**
  - Math — Chap 2: 2-1, 2-2, 2-3, 2-4, 2-5, 2-6
  - Modeling — Chap 3: 3-1, 3-2, 3-3
  - Systematic — Chap 8 (Linear), Chap 10 (Nonlinear)
- **Higher Order DE**
  - Math — Chap 4: 4-1, 4-2, 4-3, 4-4, 4-5, 4-6, 4-7
  - Modeling — Chap 5: 5-1
  - Nonlinear — 4-10, 5-3
  - Polynomial — Chap 6
- **Partial DE**
  - Math — 12-1
  - Rectangular Coordinate — Chap 12
  - Polar, Cylindrical, Spherical Coordinates — Chap 13
- **Transforms**
  - Laplace Transforms — Chap 7: 7-1, 7-2, 7-3, 7-4, 7-5, 7-6
  - Fourier Series — Chap 11: 11-1, 11-2, 11-3
  - Fourier Transforms — Chap 14
- **Numerical**
  - ODE — Chap 9
  - PDE — Chap 15

Outline of Textbook & Lectures

- 1: Introduction to Differential Equations
- 2: First-Order Differential Equations
- 3: Modeling with First-Order DEs
- 4: Higher-Order Differential Equations
- 5: Modeling with Higher-Order DEs
- 6: Series Solutions of Linear Equations
- 7: The Laplace Transform
- 8: Systems of **Linear** First-Order DEs
- 9: **Numerical** Solutions of Ordinary DEs
- 10: Systems of **Nonlinear** First-Order DEs
- 11: Fourier Series
- 12: Boundary-Value Problems in Rectangular Coordinates
- 13: Boundary-Value Problems in Other Coordinate Systems
- 14: Integral Transforms
- 15: Numerical Solutions of Partial DEs

- 信號與系統

- 線性系統
- 數值方法
- 非線性系統

- 信號與系統

Key Ingredients and Purpose on First-Order DEs

$$(1+x)\,dy - y\,dx = 0 \qquad \frac{dy}{y} = \frac{dx}{1+x}$$

◆ Separable DE

$$\frac{dy}{dx} - 3y = 6$$

◆ Linear DE

$$2xy\,dx + (x^2 - 1)\,dy = 0$$

◆ Exact DE

$$(x^2 + y^2)\,dx + (x^2 - xy)\,dy = 0$$

◆ Substitution

Key Ingredients and Purpose on First-Order DEs

$$\frac{dP(t)}{dt} = k\,P(t), \qquad P(0) = P_0$$

- Linear DE

$$\frac{dP(t)}{dt} = k(a - P)(b - P) \qquad P(0) = P_0$$

- Nonlinear DE

$$\frac{dx}{dt} = -\,a\,x + b\,xy$$

$$\frac{dy}{dt} = +\,c\,y - e\,xy$$

- Systems of DEs

Key Ingredients and Purpose on Higher-Order DEs

$$y'' - 3y' + 2y = 0 \qquad \begin{cases} y(0) = 3 \\ y'(0) = 1 \end{cases}$$

◆ nth-Order IVP

$$y'' + 16y = 0 \qquad \begin{cases} y(0) = 0 \\ y(\frac{\pi}{2}) = 1 \end{cases}$$

◆ nth-Order BVP

$$y'' - 3y' + 2y = 4x^2$$

$$y'' - 3y' + 2y = 4x + 10\sin(x)$$

$$x^2y'' - 3xy' + 3y = 2x^4e^x$$

◆ Particular Methods

$$\begin{cases} x'' + 2x' - x + y'' - 3y = \sin t \\ x' + 4x + y' - 2y = e^{-t} \end{cases}$$

◆ Systems of DEs

$$(y)(y'') = (y')^2$$

◆ Nonlinear DEs

Key Ingredients and Purpose on Higher-Order DEs

$$L\frac{di(t)}{dt} + R\,i(t) + \frac{1}{C}\int i(t)dt = E(t)$$

$$L\frac{d^2q(t)}{dt^2} + R\frac{dq(t)}{dt} + \frac{1}{C}q(t) = E(t)$$

- Circuit

$$EI\frac{d^4y}{dx^4} = w_0$$

$$EI\frac{d^2y}{dx^2} = -Py$$

- Beam

$$m\frac{d^2x}{dt^2} + k_1x + k_2x^3 = 0$$

- Nonlinear Models

Key Ingredients and Purpose on Transform Methods

$$\frac{dy(t)}{dt} + 3\,y(t) = 13\,\sin(2t)$$

◆ Laplace Transform

$$(s+3)\,Y(s) = 6 + \frac{26}{s^2+4}$$

$$\frac{dy}{dx} + 3\,y = 0$$

$$y(x) = c_1 e^{-3x}$$

◆ Fourier Series

$$\frac{d^2y}{dx^2} + 3^2\,y = 0$$

$$y(x) = c_1\cos(3x) + c_2\sin(3x)$$