Fall 2019

# 微分方程
# Differential Equations

## Unit 04.2
## Reduction of Order


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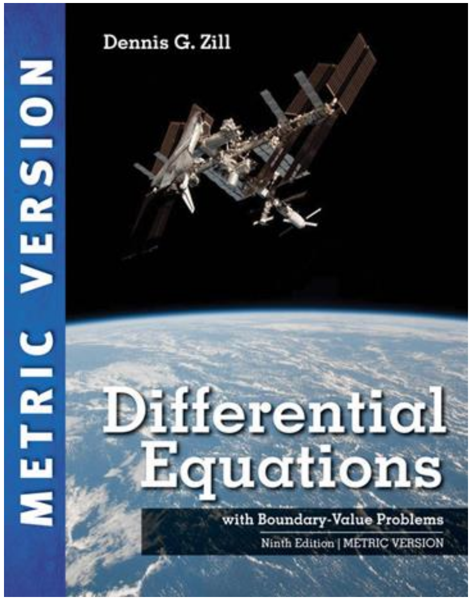
$$y_2(x) = y_1(x) \int \frac{e^{-\int P(x)dx}}{y_1^2(x)}\, dx$$




Figures and images used in these lecture notes are adopted from
**Differential Equations with Boundary-Value Problems**, 9th Ed., D.G. Zill, 2018 (Metric Version)

# Outline

- Given

$$a_2(x)\, y'' + a_1(x)\, y'(x) + a_0(x)\, y = 0$$

- IF $y_1(x)$ is a

- how to find

$$\Rightarrow\ y'' +$$

- assume $y_1(x)$ is a

$y_2(x) =$

# 4.2: Example 1

$y'' - y = 0 \qquad y_1(x) = e^x$, on $(-\infty, \infty)$

$y(x) = u(x)\, y_1(x)$

$y' =$

$y'' =$

# 4.2: Example 1

# 4.2: General Case

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$$x^2\, y'' - 3\, x\, y' + 4\, y = 0$$

$$y_1(x) = x^2$$

$$y_2 =$$

$$y_2' =$$

$$y_2'' =$$

# 4.2: Example 2

# Summary - 4.2: Reduction of Order

- Given $$y'' + P(x)\,y'(x) + Q(x)\,y = 0$$

- IF $y_1(x)$ is a solution

- ASSUME $y_2(x) = u(x)\,y_1(x)$

$$\rightarrow w(x) = u'(x)$$

$$\Rightarrow w' + \left(\frac{2y_1'}{y_1} + P\right) w = 0$$

$$\Rightarrow y_2(x) = y_1(x) \int \frac{e^{-\int P(x)dx}}{y_1^2(x)}\, dx$$