

Fall 2019

微分方程 Differential Equations

Unit 04.2 Reduction of Order

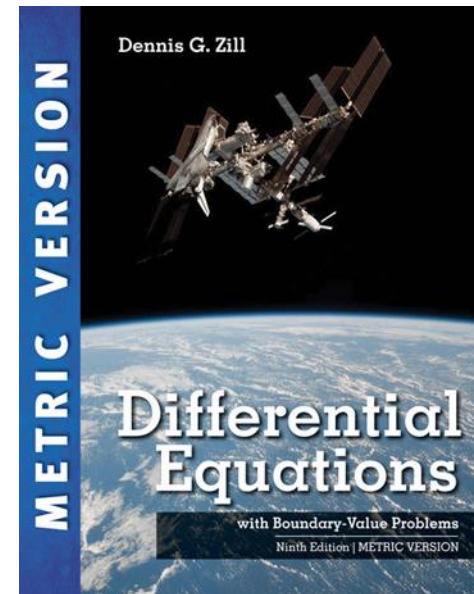
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Sep19 – Jan20

$$y_2(x) = y_1(x) \int \frac{e^{-\int P(x)dx}}{y_1^2(x)} dx$$

Figures and images used in these lecture notes are adopted from
Differential Equations with Boundary-Value Problems, 9th Ed., D.G. Zill, 2018 (Metric Version)



- 4.1: Linear Differential Equations: Basic Theory
 - 4.1.1: Initial-Value and Boundary-Value Problems
 - 4.1.2: Homogeneous Equations
 - 4.1.3: Nonhomogeneous Equations
- 4.2: Reduction of Order
- 4.3: Homogeneous Linear Eqns w/ Constant Coefficients
- 4.4: Undetermined Coefficients – Superposition Approach
- 4.5: Undetermined Coefficients – Annihilator Approach
- 4.6: Variation of Parameters
- 4.7: Cauchy-Euler Equations
- 4.8: Solving Systems of Linear Equations by Elimination
- 4.9: Nonlinear Differential Equations

- Given

$$a_2(x) y'' + a_1(x) y'(x) + a_0(x) y = 0$$

- IF $y_1(x)$ is a

- how to find

$$\Rightarrow y'' +$$

- assume $y_1(x)$ is a
 $y_2(x) =$

4.2: Example 1

$$y'' - y = 0 \quad y_1(x) = e^x, \text{ on } (-\infty, \infty)$$

$$y(x) = u(x) y_1(x)$$

$$y' =$$

$$y'' =$$

4.2: Example 2

$$x^2 y'' - 3 x y' + 4 y = 0$$

$$y_1(x) = x^2$$

$$y_2 =$$

$$y_2' =$$

$$y_2'' =$$

- Given $y'' + P(x) y'(x) + Q(x) y = 0$
- IF $y_1(x)$ is a solution
- ASSUME $y_2(x) = u(x) y_1(x)$

$$\rightarrow w(x) = u'(x)$$

$$\Rightarrow w' + \left(\frac{2y_1'}{y_1} + P \right) w = 0$$

$$\Rightarrow y_2(x) = y_1(x) \int \frac{e^{-\int P(x) dx}}{y_1^2(x)} dx$$