

Fall 2019

# 微分方程 Differential Equations

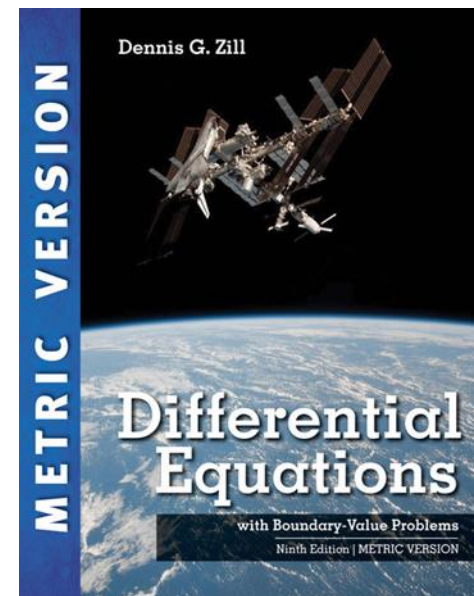
## Unit 04.3 Homogeneous Linear Equations with Constant Coefficients

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Sep19 – Jan20

$$a y''(x) + b y'(x) + c y(x) = 0$$



- 4.1: Linear Differential Equations: Basic Theory
  - 4.1.1: Initial-Value and Boundary-Value Problems
  - 4.1.2: Homogeneous Equations
  - 4.1.3: Nonhomogeneous Equations
- 4.2: Reduction of Order
- 4.3: Homogeneous Linear Eqns w/ Constant Coefficients
- 4.4: Undetermined Coefficients – Superposition Approach
- 4.5: Undetermined Coefficients – Annihilator Approach
- 4.6: Variation of Parameters
- 4.7: Cauchy-Euler Equations
- 4.8: Solving Systems of Linear Equations by Elimination
- 4.9: Nonlinear Differential Equations

- Auxiliary Equation:
- Consider a 2nd-order DE:

$$a y''(x) + b y'(x) + c y(x) = 0$$

$$a, b, c \in \mathbb{R}$$

$$a \neq 0$$

- Try a solution  $y(x) =$   
 $y'(x) =$   
 $y''(x) =$

- Case 1: Two distinct real roots,  $m_1, m_2$

$$\left\{ \quad , \quad \right\} :$$

$$y(x) =$$

- Case 2: Repeated real roots,  $m_1 = m_2$

$$\left\{ \quad , \quad \right\} :$$

$$y_1(x) =$$

$$y_2(x) =$$

$$y(x) =$$

- Case 3: Conjugate complex roots,  $m_1, m_2$

$$m_1 =$$

$$m_2 =$$

$$y(x) =$$

Euler's formula

$$e^{i\theta} = \cos \theta + i \sin \theta$$

$$e^{i\beta} + e^{-i\beta} = 2 \cos \beta$$

$$e^{i\beta} - e^{-i\beta} = 2i \sin \beta$$

(a)  $2y'' - 5y' - 3y = 0$

(b)  $y'' - 10y' + 25y = 0$

$$(c) \ y'' + 4y' + 7y = 0$$



## 4.3: Two Special Equations

$$y'' + k^2 y = 0$$

$$y'' - k^2 y = 0$$

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_1 y' + a_0 y = 0$$

⇒

- Case 1: All distinct real roots

$$y(x) =$$

- Case 2:  $k$  repeated real roots,  $k \leq n$

$$y(x) =$$

- Case 3: Complex roots in conjugate pair

$$y(x) =$$

- Consider a 2nd-order DE:

$$a y''(x) + b y'(x) + c y(x) = 0$$

$$a, b, c \in \mathbb{R}$$

$$a \neq 0$$

- Try a solution

$$y(x) = e^{mx}$$

$$y'(x) = m e^{mx} \quad \Rightarrow \quad (a m^2 + b m + c) e^{mx} = 0$$

$$y''(x) = m^2 e^{mx}$$

- Case 1: Two distinct real roots,  $m_1, m_2$

$$y(x) = c_1 e^{m_1 x} + c_2 e^{m_2 x}$$

- Case 2: Repeated real roots,  $m_1 = m_2$

$$y(x) = c_1 e^{m_1 x} + c_2 x e^{m_1 x}$$

- Case 3: Conjugate complex roots,  $m_{1,2} = a \pm ib$

$$y(x) = e^{ax} (c_1 \cos bx + c_2 \sin bx)$$