Fall 2019

微分方程
Differential Equations

Unit 04.3
Homogeneous Linear Equations with Constant Coefficients

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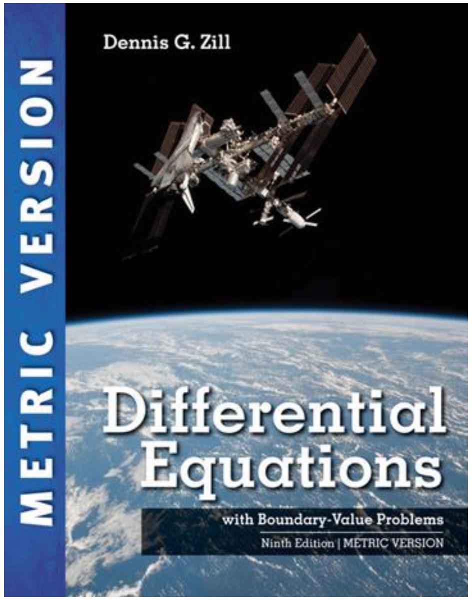
$$a\,y''(x) + b\,y'(x) + c\,y(x) = 0$$



Figures and images used in these lecture notes are adopted from
Differential Equations with Boundary-Value Problems, 9th Ed., D.G. Zill, 2018 (Metric Version)

Outline

- Auxiliary Equation:

- Consider a 2nd-order DE:

$$a\, y''(x) + b\, y'(x) + c\, y(x) = 0$$

$a, b, c \in \mathbb{R}$

$a \neq 0$

- Try a solution $y(x) =$

$$y'(x) =$$

$$y''(x) =$$

- Case 1: Two distinct real roots, m_1, m_2

$$\left\{ \qquad , \qquad \right\}:$$

$$y(x) =$$

- Case 2: Repeated real roots, $m_1 = m_2$

$\left\{ \quad , \quad \right\}$:

$y_1(x) =$

$y_2(x) =$

$y(x) =$

- Case 3: Conjugate complex roots, m_1, m_2

$m_1 =$

$m_2 =$

$y(x) =$

Euler's formula

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$e^{i\beta} + e^{-i\beta} = 2\cos\beta$$

$$e^{i\beta} - e^{-i\beta} = 2i\sin\beta$$

(a) $2\,y'' - 5\,y' - 3\,y = 0$

(b) $y'' - 10\,y' + 25\,y = 0$

(c) $y'' + 4\,y' + 7\,y = 0$

$$y'' + k^2 y = 0 \qquad\qquad y'' - k^2 y = 0$$

$$a_n\, y^{(n)} + a_{n-1}\, y^{(n-1)} + \cdots + a_1\, y' + a_0\, y = 0$$

$\Rightarrow$

- Case 1: All distinct real roots

  $y(x) =$

- Case 2: k repeated real roots, $k \leq n$

$$y(x) =$$

- Case 3: Complex roots in conjugate pair

$$y(x) =$$

- Consider a 2nd-order DE:

$$a\, y''(x) + b\, y'(x) + c\, y(x) = 0$$

$a, b, c \in \mathbb{R}$

$a \neq 0$

- Try a solution

$$y(x) = e^{mx}$$
$$y'(x) = m\, e^{mx}$$
$$y''(x) = m^2\, e^{mx}$$

$$\Rightarrow \left(a\, m^2 + b\, m + c\right) e^{mx} = 0$$

- Case 1: Two distinct real roots, m_1, m_2

$$y(x) = c_1\, e^{m_1 x} + c_2\, e^{m_2 x}$$

- Case 2: Repeated real roots, $m_1 = m_2$

$$y(x) = c_1\, e^{m_1 x} + c_2\, x\, e^{m_1 x}$$

- Case 3: Conjugate complex roots, $m_{1,2} = a \pm ib$

$$y(x) = e^{ax}\left(c_1 \cos bx + c_2 \sin bx\right)$$