Fall 2019

微分方程
Differential Equations

Unit 04.5
Undetermined Coefficients – Annihilator

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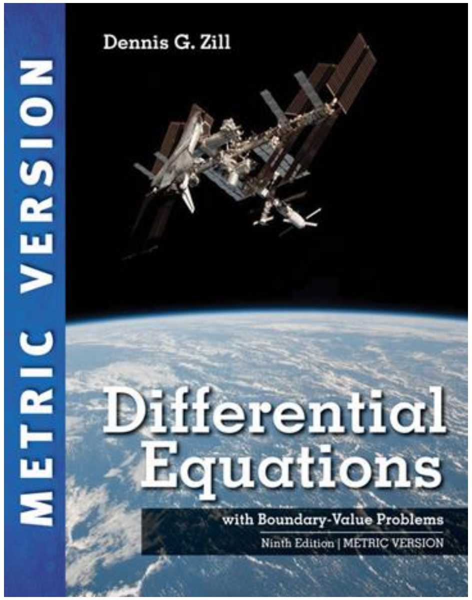
Sep19 – Jan20



Figures and images used in these lecture notes are adopted from
Differential Equations with Boundary-Value Problems, 9th Ed., D.G. Zill, 2018 (Metric Version)

Outline

$$y'' + 3y' + 2y = 4x^2$$

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$$y'' + 3y' + 2y = 4x^2$$

4.5: Undetermined Coefficients – Annihilator Approach

$$y'' + 3y' + 2y = e^{2x}$$

$$y'' + 3y' + 2y =$$

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⇒ purpose:

- Annihilator Operator: $D =$

$$L = a_n D^n + a_{n-1} D^{n-1} + \cdots + a_1 D + a_0$$

  - L : a linear differential operator with constant coefficients
  - $f(x)$: a sufficiently differentiable function

- IF

- THEN L is said to be an of function $f(x)$

4.5: Examples

(1) $1 \qquad x \qquad x^2 \qquad \cdots \qquad x^{n-1}$

(2) $e^{ax} \qquad xe^{ax} \qquad x^2e^{ax} \qquad \cdots \qquad x^{n-1}e^{ax}$

(3) $e^{ax}\cos bx \qquad xe^{ax}\cos bx \qquad \cdots \quad x^{n-1}e^{ax}\cos bx$

$e^{ax}\sin bx \qquad xe^{ax}\sin bx \qquad \cdots \quad x^{n-1}e^{ax}\sin bx$

4.5: Example 3

$$y'' + 3y' + 2y = 4x^2$$

$L =$

$P(D) =$

$$\begin{cases} L(f(x)) = g(x) \\ L(f(x)) = \end{cases}$$

- y_c

4.5: Example 5

$$y'' + y = x \cos x$$

- y_c

- y_p

4.5: Example 6

$$y'' - 2y' + y = 10e^{-2x}\cos(x)$$

$$y'' + 4\,y' - 2\,y = 2x^2 + 6 + 10\sin 3x - 3e^{-2x}$$

$$\Rightarrow y = y_c + y_p$$

4.4: $\Rightarrow \; y_p = (Ax^2+Bx+C) + (E\cos 3x+F\sin 3x) + (Ge^{-2x})$

4.5: $\Rightarrow \; P(D)\left(y'' + 4\,y' - 2\,y\right)$

$$= P(D)\left(2x^2 + 6 + 10\sin 3x - 3e^{-2x}\right) = 0$$

e.g., $\Rightarrow \; y^{(4)} - 9\,y^{(3)} + 7\,y'' - 6\,y' + 12\,y = 0$

$$\Rightarrow y = y_c + y_p$$