HW1 參考解答

(2.2)-7

1. B07901111 林禹龍

2.2.7 $\frac{dy}{dx} = e^{3x+2y} \Rightarrow \frac{dy}{dx} = e^{3x} \cdot e^{2y} \Rightarrow \frac{1}{e^{2y}} dy = e^{3x} dx$

$\Rightarrow \int e^{-2y} dy = \int e^{3x} dx \Rightarrow \frac{1}{-2} e^{-2y} + C_1 = \frac{1}{3} e^{3x} + C_2$

$\Rightarrow e^{-2y} = -\frac{2}{3} e^{3x} + C_3 \quad .(C_3 = (-2) \times (C_2 - C_1))$

$\Rightarrow 3e^{-2y} = -2e^{3x} + C$ # $(C = -3C_3)$

2. B08901136 劉承瀚

2.2

+10

7. $\frac{dy}{dx} = e^{3x+2y}$

$\Rightarrow e^{-2y} dy = e^{3x} dx$

$\Rightarrow \int e^{-2y} dy = \int e^{3x} dx$

$\Rightarrow -\frac{1}{2} e^{-2y} + C_1 = \frac{1}{3} e^{3x} + C_2$

$\Rightarrow e^{-2y} = -\frac{2}{3} e^{3x} + C \quad (C = 2(C_1 - C_2))$

$\Rightarrow -2y = \ln\left(-\frac{2}{3} e^{3x} + C\right)$

$\Rightarrow y = \frac{-1}{2} \ln\left(-\frac{2}{3} e^{3x} + C\right)$ #

3. B06901094 劉又齊

2.2

+10

7. $\frac{dy}{dx} = e^{3x+2y} \Rightarrow e^{-2y} dy = e^{3x} dx \Rightarrow -\frac{1}{2}\int e^{-2y} d(-2y) = \frac{1}{3}\int e^{3x} d(3x)$

$\Rightarrow 3e^{-2y} + 2e^{3x} + C = 0$

(2.2)-25

1. B07901111 林禹龍

2.2.25

$$x^2\frac{dy}{dx} = y - xy \Rightarrow \frac{dy}{dx} = \frac{y}{x^2} - \frac{y}{x} \Rightarrow \frac{1}{y}dy = \left(\frac{1}{x^2} - \frac{1}{x}\right)dx \quad [x \neq 0,\ y \neq 0]$$

$$\Rightarrow \int \frac{1}{y}\,dy = \int \frac{1}{x^2}\,dx - \int \frac{1}{x}\,dx$$

+10

$$\Rightarrow \ln|y| + C_1 = \left(-\frac{1}{x} + C_2\right) - \left(\ln|x| + C_3\right)$$

$$\Rightarrow \ln|y| = \left(-\frac{1}{x} - \ln|x|\right) + C_4 \quad (C_4 = C_2 - C_3 - C_1)$$

$$\Rightarrow |y| = e^{-\frac{1}{x} - \ln|x| + C_4} = e^{-\frac{1}{x}} \times e^{-\ln|x|} \times e^{C_4} = \frac{C_5}{|x|}e^{-\frac{1}{x}} \quad (C_5 = e^{C_4})$$

$$\Rightarrow |y| \times |x| = C_5 e^{-\frac{1}{x}} \Rightarrow |xy| = C_5 e^{-\frac{1}{x}} \Rightarrow xy = \pm C_5 e^{-\frac{1}{x}} = Ce^{-\frac{1}{x}} \ (C = \pm C_5)$$

$$\Rightarrow y = \frac{Ce^{-\frac{1}{x}}}{x}$$

$y(-1) = -1$ 代入 $\Rightarrow -1 = \frac{Ce^{-\frac{1}{-1}}}{-1} \Rightarrow C \times e = 1 \Rightarrow C = \frac{1}{e} = e^{-1}$

$$y = \frac{e^{-1-\frac{1}{x}}}{x} \quad ※$$

2. B08901136 劉承瀚

25、 $x^2\frac{dy}{dx} = y - xy,\ y(-1) = -1$ ~~+8~~ +10

$$\Rightarrow \frac{dy}{dx} = y \cdot \frac{(1-x)}{x^2} \quad (\text{while } x \neq 0)$$

$$\Rightarrow \int \frac{dy}{y} = \int \frac{(1-x)}{x^2}\,dx$$

$$\Rightarrow \ln|y| + C_1 = -\frac{1}{x} - \ln|x| + C_2.$$

$$\Rightarrow |y| = e^{-\frac{1}{x} - \ln|x| + C_3} \quad (C_3 = C_2 - C_1)$$

$$\Rightarrow y = e^{-\frac{1}{x}} \cdot \frac{1}{x} \cdot C \quad (C = \pm e^{C_3})$$

$$\xRightarrow{y(-1)=-1} -1 = e \cdot (-1) \cdot C \Rightarrow C = \frac{1}{e}$$

$$\therefore y = e^{-\frac{1}{x} - 1} \cdot \frac{1}{x}$$

3. B06901094 劉又齊

25.

+10

$$x^2\frac{dy}{dx} = y - xy \Rightarrow \frac{dy}{dx} = y \cdot \frac{1-x}{x^2} \Rightarrow \frac{1}{y}dy = \left(x^{-2} - \frac{1}{x}\right)dx \Rightarrow \int \frac{dy}{y} = \int x^{-2} - \frac{1}{x}\,dx$$

$$\Rightarrow \ln|y| = -x^{-1} - \ln|x| + C \quad y(-1) = -1 \Rightarrow \ln|-1| = -\frac{1}{-1} - \ln|-1| + C \Rightarrow C = -1$$

$$\Rightarrow \ln|y| = -x^{-1} - \ln|x| - 1 \Rightarrow y = e^{-x^{-1} - \ln|x| - 1} = \frac{1}{x}e^{-\frac{1}{x} - 1}$$

(2.2)-32

1. B07901016 朱哲廣

#32

$(2y-2)\frac{dy}{dx} = 3x^2+4x+2, \quad y(1)=-2$

$2\int (y-1)\,dy = \int 3x^2+4x+2\ dx$

$y^2-2y = x^3+2x^2+2x+C \quad \because 8 = 5+C \quad \therefore C=3$

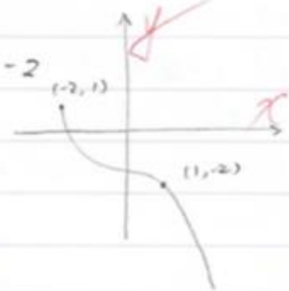
$\therefore (y-1)^2 = x^3+2x^2+2x+4, \quad y = 1 \pm \sqrt{x^3+2x^2+2x+4}$，取"$-$"以符 $y(1)=-2$

$\therefore y = 1-\sqrt{x^3+2x^2+2x+4}$ #

Good!

Interval: $x^3+2x^2+2x+4 = (x^2+2)(x+2) > 0 \Rightarrow x > -2$

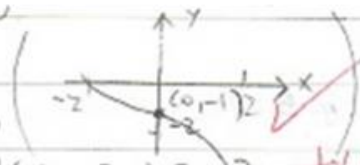
$\therefore$ Interval is $(-2, \infty)$ #



+10

2. B07901103 陳孟宏

2-2#32



$(2y-2)dy = (3x^2+4x+2)dx \Rightarrow y^2-2y = x^3+2x^2+2x+C, \because y(1)=-2 \Rightarrow C=3$

$\Rightarrow (y-1)^2 = x^3+2x^2+2x+4 \Rightarrow y = 1-\sqrt{x^3+2x^2+2x+4} \ (\because y(1)<0)$ #

& $x^3+2x^2+2x+4 = (x+2)(x^2+2) > 0 \Rightarrow x > -2$, $\therefore$ interval: $(-2,\infty)$ #

+10

(2.2)-38

1. B07901108 馬健凱

2.2.38 Show that an implicit solution of $2x\sin^2 y\,dx-(x^2+10)\cos y\,dy=0$ is given by $\ln(x^2+10)+\csc y=C$. Find the constant solutions, if any, that were lost in the solution of the differential equation. +10

<sol> Differentiate $\ln(x^2+10)+\csc y=C$ and we get $\frac{2x}{x^2+10}-\csc y\cot y\frac{dy}{dx}=0$

$\longrightarrow 2x\sin^2 y\,dx-(x^2+10)\cos y\,dy=0$

Writing the differential equation in the form $\frac{dy}{dx}=\frac{2x\sin^2 y}{(x^2+10)\cos y}$, we see that singular solutions occur when $\sin^2 y=0$, or $y=k\pi$, where k is an integer.

2. B06901094 劉又齊

38.

$$\frac{2x}{x^2+10}dx=\frac{\cos y}{\sin^2 y}dy \Rightarrow \int\frac{1}{x^2+10}d(x^2)=\int\frac{d(\sin y)}{\sin^2 y} \Rightarrow \ln|x^2+10|=-\csc y+C$$

又 $x^2+10>0$ $\therefore \ln(x^2+10)+\csc y=c$ +10

sg. sol.: $\sin^2 y=0 \Rightarrow y=n\pi,\ n\in\mathbb{Z}$

(2.3)-9

1. B07901108 馬健凱

2.3.9 $x\frac{dy}{dx} - y = x^2 \sin x$ +10

<sol> $\frac{dy}{dx} - \left(\frac{1}{x}\right)y = x\sin x \quad (x \neq 0)$

Integrating Factor $= e^{\int -\frac{1}{x}dx} = e^{-\ln x} = \frac{1}{x}$

$\times\frac{1}{x}$

$\frac{1}{x}\frac{dy}{dx} - \left(\frac{1}{x^2}\right)y = \sin x \longrightarrow \frac{d}{dx}\left(\frac{y}{x}\right) = \sin x \longrightarrow \frac{y}{x} = \int \sin x\, dx = -\cos x + C$

$\rightarrow y = Cx - x\cos x$ for $0 < x < \infty$, and there is no transient term.

2. B07901144 周信頤

2.3 #9 $x\frac{dy}{dx} - y = x^2\sin x$ +10

$\frac{dy}{dx} - \frac{y}{x} = x\sin x$

$I.F = e^{-\int\frac{1}{x}dx} = \frac{1}{x}$

$\Rightarrow \frac{d}{dx}\left(\frac{1}{x}y\right) = \sin x$

$y = Cx - x\cos x \quad (0 < x < \infty)$, there is no transient term.

(2.3)-22

1. B07901108 馬健凱

2.3.22 $\frac{dP}{dt} + 2tP = P + 4t - 2$ +10

<sol> $\frac{dP}{dt} + (2t-1)P = 4t-2$

$\text{I.F.} = e^{\int (2t-1)dt} = e^{t^2-t} \rightarrow e^{t^2-t}\frac{dP}{dt} + e^{t^2-t}(2t-1)P = e^{t^2-t}(4t-2)$.

$\frac{d}{dt}(e^{t^2-t}P) = e^{t^2-t}(4t-2) \rightarrow e^{t^2-t}P = \int e^{t^2-t}(4t-2)dt = 2e^{t^2-t} + C$

$P = 2 + Ce^{-t^2+t}$ for $-\infty < t < \infty$, and the transient term is Ce^{-t^2+t}

2. B07901144 周信頤

2.3 # 22 $\frac{dP}{dt} + 2tP = P + 4t - 2$ +10

$\frac{dP}{dt} + (2t-1)P = 4t-2$

$\text{I.F}: e^{\int (2t-1)dt} = e^{t^2-t}$

$\Rightarrow \frac{d}{dt}(e^{t^2-t}P) = (4t-2)e^{t^2-t}$

$P = Ce^{t-t^2} + 2$ $(-\infty < t < \infty)$, the transient term is Ce^{t-t^2} #

(2.3)-27

1. B07901108 馬健凱

2.3.27 $xy' + y = e^x$, $y(1) = 2$ $+10$

<sol> $y' + \frac{1}{x}y = \frac{1}{x}e^x \; (x \neq 0)$

$\text{I.F.} = e^{\int \frac{1}{x}dx} = x \rightarrow xy' + y = e^x,\ \frac{d}{dx}(xy) = e^x \longrightarrow xy = e^x + c$

$\because y(1) = 2 \quad \therefore 2 = e + c,\quad c = 2 - e$

$y = \frac{1}{x}e^x + \frac{2-e}{x}$, for $0 < x < \infty$, $I = (0, \infty)$

2. B07901002 林柏沅

27. $y' + \frac{1}{x}y = \frac{1}{x}e^x$ $+10$

$e^{\int \frac{1}{x}} = x$

$\frac{d}{dx}[xy] = e^x$

$xy = C + e^x$

$y = \frac{C}{x} + \frac{e^x}{x}$ for $0 < x < \infty$

If $y(1) = 2$, $C = 2 - e$

$y = \frac{2 - e + e^x}{x}$. The solution is defined on

$I = (0, \infty)$ for what

3. B07901016 朱哲廣

#27

$x\frac{dy}{dx} + y = e^x$. $y(1) = 2$. $+10$

$\frac{dy}{dx} + \frac{1}{x}y = \frac{1}{x}e^x \qquad x \neq 0$

$e^{\int \frac{1}{x}dx} = e^{\ln x} = x$.

$\frac{d}{dx}[xy] = e^x$. $xy = e^x + c$. $y = \frac{1}{x}e^x + \frac{1}{x}c \qquad x \neq 0$

$x = 1$. $y = e + c = 2 \quad \therefore c = 2 - e$

$\therefore y = \frac{1}{x}e^x + \frac{1}{x}(2 - e)$. $x \in (0, \infty)$ #

(2.3)-37

1. B07901108 馬健凱

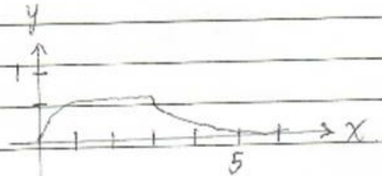
2.3.37 $\frac{dy}{dx}+2y=f(x)$, $y(0)=0$, where $f(x)=\begin{cases}1, & 0\le x\le 3\\ 0, & x>3\end{cases}$ +10

<sol> I.F. $=e^{2x} \rightarrow \frac{d}{dx}(e^{2x}y)=e^{2x}f(x) \rightarrow ye^{2x}=\begin{cases}\frac{1}{2}e^{2x}+C_1, & 0\le x\le 3\\ C_2, & x>3\end{cases}$

$\because y(0)=0,\ y=\frac{1}{2}+C_1 \rightarrow C_1=-\frac{1}{2}$.

For continuity, $y(3^-)=y(3^+)=\frac{1}{2}-\frac{1}{2}e^{-6}=C_2e^{-6}$ $\therefore C_2=\frac{1}{2}\frac{1-e^{-6}}{e^{-6}}=\frac{1}{2}e^{6}-\frac{1}{2}$

$y=\begin{cases}\frac{1}{2}-\frac{1}{2}e^{-2x}, & 0\le x\le 3\\ (\frac{1}{2}e^{6}-\frac{1}{2})e^{-2x}, & x>3\end{cases}$



2. B07901002 林柏沅

37. $\frac{d}{dx}[e^{2x}y]=e^{2x}f(x)$ +10

$e^{2x}y=\begin{cases}\frac{1}{2}e^{2x}+C_1, & 0\le x\le 3\\ C_2, & x>3\end{cases}$

If $y(0)=0$ then $C_1=-\frac{1}{2}$ and for continuity.

$y(3)=y(\lim_{x\to 3^+}x)$

$\frac{1}{2}e^{6}-\frac{1}{2}=C_2$

$\therefore y=\begin{cases}\frac{1}{2}-\frac{1}{2e^{2x}}, & 0\le x\le 3\\ \frac{1}{2}(e^{6}-1)e^{-2x}, & x>3\end{cases}$

3. B07901016 朱哲廣

4.37 +10

$\frac{dy}{dx}+2y=f(x)$. $y(0)=0$. $f(x)=\begin{cases}1, & 0\le x\le 3\\ 0, & x>3\end{cases}$

for $0\le x\le 3$

$\frac{dy}{dx}+2y=1$. $e^{\int 2dx}=e^{2x}$

$\frac{d}{dx}[e^{2x}y]=e^{2x} \rightarrow e^{2x}y=\int e^{2x}dx+C_1=\frac{1}{2}e^{2x}+C_1$

$y=\frac{1}{2}+C_1e^{-2x}$

for $x>3$

$\frac{dy}{dx}+2y=0$

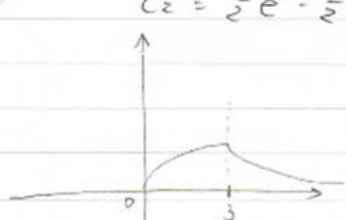
$\frac{d}{dx}[e^{2x}y]=0$. $e^{2x}y=C_2$ $y=e^{-2x}C_2$

$\Rightarrow y=\begin{cases}\frac{1}{2}+C_1e^{-2x}, & 0\le x\le 3\\ C_2e^{-2x}, & x>3\end{cases}$ $\because y(0)=0 \therefore C_1=-\frac{1}{2}$

$\because y$ is continuous $\therefore \frac{1}{2}-\frac{1}{2}e^{-6}=C_2e^{-6}$

$C_2=\frac{1}{2}e^{6}-\frac{1}{2}$

$\Rightarrow y=\begin{cases}\frac{1}{2}(1-e^{-2x}), & 0\le x\le 3\\ \frac{1}{2}(e^{6}-1)e^{-2x}, & x>3\end{cases}$ #



(2.4)-3

1. B06901094 劉又齊

3.
$M=5x+4y \quad \frac{\partial}{\partial y}M=4$
$N=4x-8y^3 \quad \frac{\partial}{\partial x}N=4$

$M_y=N_x \Rightarrow F(x,y)=\int M dx+g(y)=\frac{5}{2}x^2+4yx+g(y)$ +10
exact!
$F_y=N \Rightarrow 4x+g'(y)=4x-8y^3 \Rightarrow g(y)=\int -8y^3=-2y^4$

$\therefore \frac{5}{2}x^2+4xy-2y^4=C$

2. B07901069 劉奇聖

$\boxed{2.4}$

3、$(5x+4y)dx+(4x-8y^3)dy=0$ +10

(sol) $M=5x+4y$
$N=4x-8y^3$
$M_y=4$, $N_x=4$
$\because M_y=N_x \therefore$ it is exact
$f=\int(5x+4y)dx+g(y)$
$=\frac{5}{2}x^2+4xy+g(y)=C$
$\frac{\partial f}{\partial y}=4x+g'(y)=4x-8y^3$
$\Rightarrow g'(y)=-8y^3 \Rightarrow g(y)=-2y^4$
$\therefore \frac{5}{2}x^2+4xy-2y^4=C$ ✓

3. B07901111 林禹龍

4.3
$(5x+4y)dx+(4x-8y^3)dy=0$ +10

$M_y=\frac{d}{dy}(5x+4y)=4$
$N_x=\frac{d}{dx}(4x-8y^3)=4$
$\because M_y=N_x \quad \therefore$ This is an exact equation

$M=\frac{\partial f}{\partial x} \Rightarrow f(x,y)=\int M dx=\int(5x+4y)dx=\frac{5}{2}x^2+4xy+h(y)$

$\frac{\partial f}{\partial y}=4x+h'(y)=(4x-8y^3) \Rightarrow h'(y)=-8y^3$

$\int h'(y)dy=\int -8y^3 dy=-2y^4+C_1$ (C_1 is a constant)

$f(x,y)=\frac{5}{2}x^2+4xy-2y^4+C_1=0 \Rightarrow \frac{5}{2}x^2+4xy-2y^4=C \quad (C=-C_1)$

(2.4)-17

1. B06901094 劉又齊

17. $M = \tan x - \sin x \sin y$, $M_y = -\sin x \cos y$

$N = \cos x \cos y$, $N_x = -\sin x \cos y$

$M_y = N_x \Rightarrow$ exact!

$F(x,y) = \int N dy + g(x) = \cos x \sin y + g(x)$

$F_x = M \Rightarrow -\sin x \sin y + g'(x) = \tan x - \sin x \sin y$

$\Rightarrow g(x) = \int \tan x \, dx = -\ln|\cos x|$

$\therefore \cos x \sin y - \ln|\cos x| = C$

+10

2. B07901069 劉奇聖

17、 $(\tan x - \sin x \sin y)dx + \cos x \cos y \, dy = 0$

+10

(sol) $M = \tan x - \sin x \sin y$

$N = \cos x \cos y$

$M_y = -\sin x \cos y$, $N_x = -\sin x \cos y$

$\because M_y = N_x$ $\therefore$ it is exact

$N = \frac{\partial f}{\partial y} \Rightarrow f = \int \cos x \cos y \, dy + g(x)$

$= \cos x \sin y + g(x) = c$

$\frac{\partial f}{\partial x} = -\sin x \sin y + g'(x) = \tan x - \sin x \sin y$

$\Rightarrow g'(x) = \tan x \Rightarrow g(x) = \ln|\sec x|$

$\therefore \cos x \sin y + \ln|\sec x| = c$

3. B07901111 林禹龍

2.4.17 $(\tan x - \sin x \sin y)dx + (\cos x \cos y)dy = 0$

+10

$M_y = -\sin x \cos y$

$N_x = -\sin x \cos y$

$\because M_y = N_x$ $\therefore$ This is an exact equation

$M = \frac{\partial f}{\partial x} \Rightarrow f(x,y) = \int M dx = \int \tan x - \sin x \sin y \, dx$

$= \ln|\sec x| + \sin y \cos x + h(y)$

$\frac{\partial f}{\partial y} = \cos y \cos x + h'(y) = \cos x \cos y \Rightarrow h'(y) = 0$

$\int h'(y) dy = \int 0 \, dy = C_1$ (C_1 is a constant)

$f(x,y) = \ln|\sec x| + \sin y \cos x + C_1 = 0 \Rightarrow \ln|\sec x| + \sin y \cos x = C$ $(C = -C_1)$

(2.4)-25

1. B06901094 劉又齊

25. $M=y^2\cos x-3x^2y-2x$, $M_y=2y\cos x-3x^2$

$N=2y\sin x-x^3+\ln y$, $N_x=2y\cos x-3x^2$ $\quad M_y=N_x\Rightarrow$ exact!

+10

$F(x,y)=\int M dx=y^2\sin x-x^3y-x^2+g(y)$ $\quad F_y=N\Rightarrow 2y\sin x-x^3+g'(y)=2y\sin x-x^3+\ln y$

$\Rightarrow g(y)=\int \ln y\, dy=y\ln y-y$

$\Rightarrow y^2\sin x-x^3y-x^2+y\ln y-y=C$

$C|_{(0,e)}=0$ $\quad\therefore y^2\sin x-x^3y-x^2+y\ln y-y=0$

2. B07901069 劉奇聖

25、 $M=y^2\cos x-3x^2y-2x$

$N=2y\sin x-x^3+\ln y$

+10

$\left.\begin{array}{l}M_y=2y\cos x-3x^2\\ N_x=2y\cos x-3x^2\end{array}\right\}\Rightarrow M_y=N_x$

$\frac{\partial f}{\partial x}=M\Rightarrow f=\int(y^2\cos x-3x^2y-2x)\,dx+g(y)$

$=y^2\sin x-x^3y-x^2+g(y)=c$

$\frac{\partial f}{\partial y}=2y\sin x-x^3+g'(y)=2y\sin x-x^3+\ln y$

$\Rightarrow g'(y)=\ln y\Rightarrow g(y)=y\ln y-y$

$\therefore y^2\sin x-x^3y-x^2+y\ln y-y=c$

$y(0)=e\Rightarrow 0-0-0+e-e=c\Rightarrow c=0$

$\therefore y^2\sin x-x^3y-x^2+y\ln y-y=0$

3. B07901111 林禹龍

2.4.25

$(y^2\cos x-3x^2y-2x)dx+(2y\sin x-x^3+\ln y)dy=0$

+10

$M_y=2y\cos x-3x^2$

$N_x=2y\cos x-3x^2$ $\quad\because M_y=N_x\quad\therefore$ This is an equation

$M=\frac{\partial f}{\partial x}\Rightarrow f(x,y)=\int M dx=\int(y^2\cos x-3x^2y-2x)dx$

$=y^2\sin x-x^3y-x^2+h(y)$

$\frac{\partial f}{\partial y}=2y\sin x-x^3+h'(y)=2y\sin x-x^3+\ln y$

$\Rightarrow h'(y)=\ln y$

$\int h'(y)dy=\int\ln y\,dy=y\ln y-\int y\frac{1}{y}dy=y\ln y-y+C_1$

$f(x,y)=y^2\sin x-x^3y-x^2+y\ln y-y+C_1=0\Rightarrow y^2\sin x-x^3y-x^2+y\ln y-y=C$

$(C=-C_1)$

(2.4)-29

1. B07901111 林禹龍

2.4.29

$+10$

$(-xy\sin x+2y\cos x)dx+(2x\cos x)dy=0$

$M_y=-x\sin x+2\cos x$

$\because M_y\neq N_x \therefore$ This is a nonexact equation

$N_x=2\cos x-2x\sin x$

$\times u(x,y)\Rightarrow \times(xy)\Rightarrow(-x^2y^2\sin x+2xy^2\cos x)dx+(2x^2y\cos x)dy=0$

$M'_y=-2x^2y\sin x+4xy\cos x$

$\because M'_y=N'_x \therefore$ New equation is exact

$N'_x=4xy\cos x-2x^2y\sin x$

$M'=\frac{\partial f}{\partial x}\Rightarrow f(x,y)=\int M'\,dx=\int(-x^2y^2\sin x+2xy^2\cos x)\,dx$

$=(x^2y^2\cos x-\int 2xy^2\cos x\,dx)+\int 2xy^2\cos x\,dx$

$=x^2y^2\cos x+h(y)$

$\frac{\partial f}{\partial y}=2x^2y\cos x+h'(y)=2x^2y\cos x\Rightarrow h'(y)=0$

$\int h'(y)\,dy=\int 0\,dy=C_1$

$f(x,y)=x^2y^2\cos x+C_1=0\Rightarrow x^2y^2\cos x=C\quad(C=-C_1)$

(2.4)-29

2. B07901114 王錦盛

29. $(\overset{M}{-xy\sin x + 2y\cos x})\,dx + \overset{N}{2x\cos x}\,dy = 0$ +10

$M_y = -x\sin x + 2\cos x$

$N_x = 2(\cos x + x(-\sin x))$

$= 2\cos x - 2x\sin x$

$M_y \neq N_x \Rightarrow$ not exact

by multiply $\mu(x,y) = xy \Rightarrow (\overset{M}{-x^2y^2\sin x + 2xy^2\cos x})\,dx + \overset{N}{2x^2y\cos x}\,dy = 0$

$M_y = -2x^2y\sin x + 4xy\cos x$

$N_x = 2y(2x\cos x - x^2\sin x)$

$= -2x^2y\sin x + 4xy\cos x$

$M_y = N_x \rightarrow$ 已變成 Exact

$M = \frac{df}{dx} = -x^2y^2\sin x + 2xy^2\cos x$

$f = -y^2\int x^2\sin x\,dx + 2y^2\int x\cos x\,dx + h(y)$

solve $\int x^2\sin x\,dx$

$u = x^2 \qquad dv = \sin x\,dx$

$du = 2x\,dx \qquad v = -\cos x$

$= -x^2\cos x + \int 2x\cos x\,dx$

solve $\int 2x\cos x\,dx$

$u = x \qquad dv = \cos x\,dx$

$du = 1\,dx \qquad v = \sin x$

$= 2[x\sin x - \int \sin x\,dx]$

$= 2x\sin x + 2\cos x$

solve $\int x\cos x\,dx$

$= x\sin x + \cos x$

$\int x^2\sin x\,dx = -x^2\cos x + 2x\sin x + 2\cos x$

$f = -y^2[-x^2\cos x + 2x\sin x + 2\cos x] + 2y^2[x\sin x + \cos x] + h(y)$

$= x^2y^2\cos x - \cancel{2xy^2\sin x} - \cancel{2y^2\cos x} + \cancel{2xy^2\sin x} + \cancel{2y^2\cos x} + h(y)$

$= x^2y^2\cos x + h(y)$

$f_y = 2x^2y\cos x + h'(y) = 2x^2y\cos x \Rightarrow h'(y) = 0 \Rightarrow h(y) = C_1$

Solution $\Rightarrow x^2y^2\cos x = C$ //