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$$12y'' - 5y' - 2y = 0$$

$$y(x) = e^{mx}$$

$$y'(x) = m e^{mx}$$

$$y''(x) = m^2 e^{mx}$$

$$12(m^2 e^{mx}) - 5(m e^{mx}) - 2(e^{mx}) = 0$$

$$e^{mx}\left[12m^2 - 5m - 2\right] = 0$$

$\because e^{mx} \neq 0$, $\quad 12m^2 - 5m - 2 = 0$

$$\Rightarrow 12\left(m^2 - \tfrac{5}{12}m - \tfrac{1}{6}\right) = 0$$

$$\Rightarrow 12\left(m + \tfrac{1}{4}\right)\left(m - \tfrac{2}{3}\right) = 0$$

$$\Rightarrow m = -\tfrac{1}{4} \text{ or } \tfrac{2}{3}$$

$$\Rightarrow y(x) = c_1 e^{-\frac{1}{4}x} + c_2 e^{\frac{2}{3}x}$$

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$$16\frac{d^4y}{dx^2} + 24\frac{d^2y}{dx^2} + 9y = 0 \quad \leftarrow \text{input}$$

Let $y(x) = e^{mx} \Rightarrow y'(x) = m e^{mx}$

become $\Rightarrow y''(x) = m^2 e^{mx}$, $y''' = m^3 e^{mx}$, $y^{(4)} = m^4 e^{mx}$

$$16(m^4 e^{mx}) + 24(m^2 e^{mx}) + 9(e^{mx}) = 0$$

$$\Rightarrow e^{mx}\left[16m^4 + 24m^2 + 9\right] = 0$$

$\because e^{mx} \neq 0 \quad \Rightarrow 16m^4 + 24m^2 + 9 = 0$

$$\Rightarrow 16\left(m^2 + \frac{3}{4}\right)^2 = 0$$

$$\Rightarrow m^2 = -\frac{3}{4}, -\frac{3}{4}$$

$$\Rightarrow m = \frac{\sqrt{3}}{2}i, \frac{\sqrt{3}}{2}i, -\frac{\sqrt{3}}{2}i, -\frac{\sqrt{3}}{2}i$$

$$\Rightarrow y(x) = c_1\cos\left(\frac{\sqrt{3}}{2}x\right) + c_2\sin\left(\frac{\sqrt{3}}{2}x\right) + c_3 x\cos\left(\frac{\sqrt{3}}{2}x\right) + c_4 x\sin\left(\frac{\sqrt{3}}{2}x\right)$$

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$y'' + y = 0, \quad y'(0)=0, \quad y'(\frac{\pi}{2})=0$

$y(x) = e^{mx} \Rightarrow y' = me^{mx}, \quad y'' = m^2 e^{mx}$

$m^2 e^{mx} + e^{mx} = 0$

$e^{mx}[m^2+1] = 0$

$\because e^{mx} \neq 0 \Rightarrow m^2+1=0$

$m = \pm i$

$\Rightarrow y(x) = c_1 \cos(x) + c_2 \sin(x)$

$\Rightarrow y' = -c_1 \sin(x) + c_2 \cos(x)$

$\left.\begin{array}{l} y'(0) = 0 + c_2 = 0 \\ y'(\frac{\pi}{2}) = -c_1 + 0 = 0 \end{array}\right\} \Rightarrow \begin{array}{l} c_2 = 0 \\ c_1 = 0 \end{array}$

$\Rightarrow \underline{y(x) = 0}$ ✱

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$y'' - 3y = 0, \quad y(0) = 1, \quad y'(0) = 5$

$y(x) = e^{mx}, \quad y' = me^{mx}, \quad y'' = m^2 e^{mx}$

$e^{mx}[m^2 - 3] = 0$

$\Rightarrow m^2 - 3 = 0 \Rightarrow m = \pm\sqrt{3}$

By (10) $\Rightarrow y(x) = c_1 e^{\sqrt{3}x} + c_2 e^{-\sqrt{3}x}$

By (11) $\Rightarrow y(x) = c_1 \cosh(\sqrt{3}x) + c_2 \sinh(\sqrt{3}x)$

$y(0) = 1, \quad y'(0) = 5$

By (10)

$$\left.\begin{aligned} y(0) &= c_1 + c_2 = 1 \\ y'(0) &= c_1\sqrt{3} + c_2(-\sqrt{3}) = 5 \end{aligned}\right\}$$

$\Rightarrow c_1 = \frac{1}{2}\left(1 + \frac{5}{\sqrt{3}}\right), \quad c_2 = \frac{1}{2}\left(1 - \frac{5}{\sqrt{3}}\right)$

$\therefore \; y(x) = \frac{1}{2}\left(1 + \frac{5}{\sqrt{3}}\right) e^{\sqrt{3}x} + \frac{1}{2}\left(1 - \frac{5}{\sqrt{3}}\right) e^{-\sqrt{3}x}$ //

By (11)

$y(0) = c_1 + 0 = 1 \Rightarrow c_1 = 1$

$y'(0) = c_1\sqrt{3}\sinh(0) + c_2\sqrt{3}\cosh(0) = 0 + \sqrt{3}c_2 = 5$

$\Rightarrow c_2 = \frac{5}{\sqrt{3}}$

$y(x) = \cosh(\sqrt{3}x) + \frac{5}{\sqrt{3}}\sinh(\sqrt{3}x)$ //