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$$y''' + 2y'' - 13y' + 10y = xe^{-x}$$

$$\Rightarrow L(y) = g(x)$$

$$\frac{d}{dx} \triangleq D$$

$$\Rightarrow D^3 y + 2D^2 y - 13Dy + 10y = xe^{-x}$$

$$\Rightarrow (D^3 + 2D^2 - 13D + 10)\, y = xe^{-x}$$

$$\Rightarrow (D-1)(D-2)(D+5)\, y = xe^{-x}$$

$$\therefore \boxed{\begin{aligned} L &= D^3 + 2D^2 - 13D + 10 \\ &= (D-1)(D-2)(D+5) \end{aligned}}$$ #

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$$(D-2)(D+5) \Rightarrow y = e^{2x} + 3e^{-5x}$$

$$\Rightarrow (D-2)\underbrace{(D+5)\left(e^{2x} + 3e^{-5x}\right)}$$

$$= (D-2)\left[2e^{2x} + (-15)e^{-5x} + 5e^{2x} + 15e^{-5x}\right]$$

$$= (D-2)\left[7e^{2x}\right]$$

$$= 7\left[2e^{2x} - 2e^{2x}\right]$$

$$= 7 \cdot 0$$

$$= 0$$ # annihilated!

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$x + 3xe^{6x}$

$x \to D^2$

$\because D^2 x = D(Dx) = D(1) = 0$

$xe^{6x} \to (D-6)^2$

$\because (D-6)^2 xe^{6x}$

$= (D-6)\left[(D-6)xe^{6x}\right]$

$= (D-6)\left[e^{6x} + \cancel{x \cdot 6e^{6x}} - \cancel{6xe^{6x}}\right]$

$= (D-6)\left(e^{6x}\right)$

annihilated!

$= 6e^{6x} - 6e^{6x} = 0$ #

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D^2+5

$\Rightarrow \pm\sqrt{5}\,i$

$\Rightarrow \cos\sqrt{5}x \,,\ \sin\sqrt{5}x$

$\because (D^2+5)(\cos\sqrt{5}x)$

$= -(\sqrt{5})^2\cos\sqrt{5}x + 5\cos\sqrt{5}x = 0$ #

$(D^2+5)(\sin\sqrt{5}x)$

$= -(\sqrt{5})^2\sin\sqrt{5}x + 5\sin\sqrt{5}x = 0$ #

(4.5) $y'' - 2y' - 3y = \underbrace{4e^x - 9}$

45 for RHS $= 4e^x - 9$

$\Rightarrow D(D-1)(4e^x - 9)$

$= (D-1)(4e^x)$

$= 4e^x - 4e^x = 0$

$\Rightarrow D(D-1)\left[D^2 - 2D - 3\right] y = 0$

$\Rightarrow D(D-1)(D-3)(D+1)\, y = 0$

$\Rightarrow y = \underbrace{c_1 e^{3x} + c_2 e^{-x}}_{y_c} + \underbrace{c_3 e^x + c_4}_{y_p}$

$y_p = Ae^x + B$

$y_p' = Ae^x \Rightarrow$

$y_p'' = Ae^x$

$Ae^x - 2(Ae^x) - 3(Ae^x + B) = 4e^x - 9$

$-4Ae^x - 3B = 4e^x - 9$

$\Rightarrow A = -1, \ B = 3$

$\Rightarrow \boxed{y = c_1 e^{3x} + c_2 e^{-x} - e^x + 3}$ #

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$$y'' + y = 8\cos 2x - 4\sin x$$

$$y\left(\tfrac{\pi}{2}\right) = -1, \quad y'\left(\tfrac{\pi}{2}\right) = 0$$

AHE : $y'' + y = 0 \Rightarrow (D^2+1)\,y = 0$

$$y_c = c_1 \cos x + c_2 \sin x$$

RHS $= 8\cos 2x - 4\sin x$

$\Rightarrow (D^2+4)\,(D^2+1)$ → repeated!

$\because (D^2+4)\cos 2x = 0$

$\quad (D^2+1)\sin x = 0$

$\therefore y_p = A\underline{x}\cos x + B\underline{x}\sin x + C\cos 2x + E\sin 2x$

$$y_p' = A\cos x - Ax\sin x + B\sin x + Bx\cos x - 2C\sin 2x + 2E\cos 2x$$

$$y_p'' = -A\sin x - A\sin x - Ax\cos x + B\cos x + B\cos x - Bx\sin x - 4C\cos 2x - 4E\sin 2x$$

$$= (-2A)\sin x + (2B)\cos x \underline{- Ax\cos x} \underline{- Bx\sin x} - 4C\cos 2x - 4E\sin 2x$$

$$y_p'' + y_p = (-2A)\sin x + (2B)\cos x - 3C\cos 2x - 3E\sin 2x$$

$$= 8\cos 2x - 4\sin x$$

$$\Rightarrow \quad A = 2 \qquad C = -\frac{8}{3}$$
$$B = 0 \qquad E = 0$$

$$y = c_1 \cos x + c_2 \sin x + 2x\cos x - \frac{8}{3}\cos 2x$$

$$y' = -c_1 \sin x + c_2 \cos x + 2\cos x - 2x\sin x + \frac{16}{3}\sin 2x$$

$$y\left(\frac{\pi}{2}\right) = -1$$

$$-1 = 0 + C_2 + 0 - \frac{8}{3}(-1) \Rightarrow C_2 = -\frac{11}{3}$$

$$y'\left(\frac{\pi}{2}\right) = 0$$

$$0 = -C_1 + 0 + 0 - 2\cdot\frac{\pi}{2}\cdot 1 + 0$$

$$\Rightarrow C_1 = -\pi$$

$$\boxed{y = -\pi\cos x - \frac{11}{3}\sin x + 2x\cos x - \frac{8}{3}\cos 2x}$$

#