(7.1) 5

$$f(t) = \begin{cases} \sin(t) & 0 \le t < \pi \\ 0 & t \ge \pi \end{cases}$$

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)\, e^{-st}\, dt$$

$$= \boxed{\int_0^\pi \underbrace{\sin(t)}_{u}\, \underbrace{e^{-st}}_{v'}\, dt} \quad = uv - \int u'v$$

$$= \sin t \left(\frac{1}{-s}\right) e^{-st} \Big|_0^\pi - \int_0^\pi \cos t \left(\frac{1}{-s}\right) e^{-st} dt$$

$$= \frac{-1}{s} \sin t\, e^{-st} \Big|_0^\pi + \frac{1}{s}\left[\int_0^\pi \underbrace{\cos t}_{u}\, \underbrace{e^{-st}}_{v'}\, dt\right] \quad = uv - \int u'v$$

$$= \bigcirc \quad + \frac{1}{s}\left[\cos t \left(\frac{1}{-s}\right) e^{-st} \Big|_0^\pi - \int_0^\pi (-\sin t)\left(\frac{1}{-s}\right) e^{-st} dt\right]$$

$$= \bigcirc \quad + \frac{1}{s}\left[\frac{-1}{s} \cos t\, e^{-st} \Big|_0^\pi - \frac{1}{s} \int_0^\pi \sin t\, e^{-st} dt\right]$$

$$= \left[\frac{-1}{s} \sin t - \frac{1}{s^2} \cos t\right] e^{-st} \Big|_0^\pi - \frac{1}{s^2} \boxed{\int_0^\pi \sin t\, e^{-st} dt}$$

$\frac{s^2+1}{s^2}$ $\|$

$$\left(1 + \frac{1}{s^2}\right) \boxed{\int_0^\pi \sin t\, e^{-st} dt} = \left[\frac{-\sin t}{s} + \frac{-\cos t}{s^2}\right] e^{-st} \Big|_0^\pi$$

$$\mathcal{L}\{f(t)\} = \int_0^\pi \sin t\, e^{-st} dt = \frac{s^2}{s^2+1}\left[\frac{-\sin t}{s} + \frac{-\cos t}{s^2}\right] e^{-st} \Big|_0^\pi$$

$$= \left[\frac{-s}{s^2+1} \sin t + \frac{-\cos t}{s^2+1}\right] e^{-st} \Big|_0^\pi$$

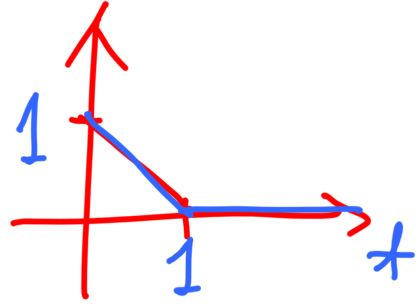
$$= \left[0 + \frac{1}{s^2+1}\right] e^{-s\pi} - \left[0 + \frac{-1}{s^2+1}\right] e^0$$

$$= \frac{1}{s^2+1}\left[e^{-s\pi} + 1\right]$$

$$\therefore \boxed{\mathcal{L}\{f(t)\} = \frac{1}{s^2+1}\left(1 + e^{-\pi s}\right), \quad s > 0}$$

(7.1)

9



$$\Rightarrow f(t) = \begin{cases} 1-t, & 0<t<1 \\ 0, & t \geq 1 \end{cases}$$

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)\, e^{-st}\, dt$$

$$= \int_0^1 (1-t)\, e^{-st} dt + \int_1^\infty (0)\, e^{-st}\, dt$$

$$= \int_0^1 (1-t)\, e^{-st}\, dt$$

$$= \int_0^1 e^{-st} - \int_0^1 \underset{u}{t}\, \underset{v'}{e^{-st}}\, dt$$

$$= \frac{1}{-s} e^{-st}\Big|_0^1 - \left[t\left(\frac{1}{-s} e^{-st}\right)\Big|_0^1 - \int_0^1 \frac{1}{-s} e^{-st}\, dt \right]$$

$$= \frac{-1}{s}(e^{-s}-1) - \left[\left(\frac{-1}{s} e^{-s} - 0\right) + \frac{-1}{s^2} e^{-st}\Big|_0^1 \right]$$

$$= \frac{1}{s}(1-e^{-s}) - \left[\frac{-e^{-s}}{s} + \frac{-1}{s^2}(e^{-s}-1) \right]$$

$$= \frac{1}{s} + \frac{1}{s^2}(e^{-s}-1)$$

$$\boxed{\mathcal{L}\{f(t)\} = \frac{1}{s^2} e^{-s} + \frac{1}{s} - \frac{1}{s^2}, \quad s>0}$$

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(7.1)
32

$f(t) = \cos 5t + \sin 2t$

$$\mathcal{L}\{\cos 5t\} = \int_0^\infty \underbrace{(\cos 5t)}_{u}\underbrace{e^{-st}\,dt}_{v'} = H(s)$$

$$= \cos 5t\left(\frac{-1}{s}e^{-st}\right)\Big|_0^\infty - \int_0^\infty (-5\sin 5t)\left(\frac{1}{-s}\right)e^{-st}\,dt$$

$$= \left[0-\left(\frac{-1}{s}\right)\right] - \frac{5}{s}\int_0^\infty \underbrace{\sin 5t}_{u}\,\underbrace{e^{-st}\,dt}_{v'}$$

$$= \frac{1}{s} - \frac{5}{s}\left[(\sin 5t)\left(\frac{1}{-s}\right)e^{-st}\Big|_0^\infty - \int_0^\infty 5\cos 5t\left(\frac{1}{-s}\right)e^{-st}\,dt\right]$$

$$= \frac{1}{s} - \frac{5}{s}\left[[0-0] + \frac{5}{s}\underbrace{\int_0^\infty \cos 5t\, e^{-st}\,dt}_{=H(s)}\right]$$

$$H(s) = \frac{1}{s} - \frac{5}{s}\frac{5}{s}H(s) = \frac{1}{s} - \frac{25}{s^2}H(s)$$

$$\left(1+\frac{25}{s^2}\right)H(s) = \frac{1}{s} \Rightarrow H(s) = \frac{s}{s^2+25}$$

同理 $\mathcal{L}\{\sin 2t\} = \int_0^\infty (\sin 2t)e^{-st}\,dt = G(s)$

$$= \cdots\cdots = \frac{2}{s}\left[\frac{1}{s} - \frac{2}{s}G(s)\right]$$

$$\Rightarrow \left(1+\frac{4}{s^2}\right)G(s) = \frac{2}{s^2}$$

$$G(s) = \frac{2}{s^2+4}$$

$\therefore$
$$\boxed{\mathcal{L}\{\cos 5t + \sin 2t\} = \frac{s}{s^2+25} + \frac{2}{s^2+4}}$$

(7,1) 41

$$\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t}\, dt \quad , \alpha > 0$$

$$\Gamma(\alpha+1) = \int_0^\infty \overset{u}{t^\alpha}\, \overset{v'}{e^{-t}}\, dt$$

$$= t^\alpha \left(\frac{1}{-1}\right) e^{-t} \Big|_0^\infty - \int_0^\infty \alpha t^{\alpha-1} \left(\frac{1}{-1}\right) e^{-t}\, dt$$

$$= -t^\alpha e^{-t} \Big|_0^\infty + \alpha \boxed{\int_0^\infty t^{\alpha-1} e^{-t}\, dt} = \Gamma(\alpha)$$

$$= [0 - 0] + \alpha\, \Gamma(\alpha)$$

$$\boxed{= \alpha\, \Gamma(\alpha)}$$

42

$$\mathcal{L}\{t^\alpha\} = \int_0^\infty t^\alpha e^{-st}\, dt$$

$$u \triangleq st \Rightarrow du = s\, dt$$

$$= \int_0^\infty \left(\frac{u}{s}\right)^\alpha e^{-u} \frac{1}{s}\, du$$

$$= \int_0^\infty \frac{1}{s^{\alpha+1}} u^\alpha e^{-u}\, du$$

$$= \frac{1}{s^{\alpha+1}} \boxed{\int_0^\infty u^\alpha e^{-u}\, du} = \Gamma(\alpha+1)$$

$$\boxed{\mathcal{L}\{t^\alpha\} = \frac{1}{s^{\alpha+1}}\, \Gamma(\alpha+1)}$$