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$$\mathcal{L}^{-1}\left\{\frac{2s-4}{(s^2+s)(s^2+1)}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{2s-4}{s(s+1)(s^2+1)}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{-4}{s} + \frac{3}{s+1} + \frac{s}{s^2+1} + \frac{3}{s^2+1}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{-4}{s}\right\} + \mathcal{L}^{-1}\left\{\frac{3}{s+1}\right\} + \mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\} + \mathcal{L}^{-1}\left\{\frac{3}{s^2+1}\right\}$$

$$= -4 + 3e^{-t} + \cos t + 3\sin t$$

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$$y' + 6y = e^{4t} \qquad y(0) = 2$$

$$\mathcal{L}\{y' + 6y\} = \mathcal{L}\{e^{4t}\}$$

$$\Rightarrow s\mathcal{L}\{y\} - y(0) + 6\mathcal{L}\{y\} = \frac{1}{s-4}$$

$$\Rightarrow \mathcal{L}\{y\}(s+6) - 2 = \frac{1}{s-4}$$

$$\mathcal{L}\{y\} = \left(\frac{1}{s-4} + 2\right)\frac{1}{s+6}$$

$$= \frac{1 + (2s-8)}{(s-4)(s+6)} = \frac{2s-7}{(s-4)(s+6)}$$

$$= \frac{1}{10}\frac{1}{s-4} + \frac{19}{10}\frac{1}{s+6}$$

$$\Rightarrow \boxed{y(t) = \frac{1}{10}e^{4t} + \frac{19}{10}e^{-6t}}$$

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$$y'' - 4y' = 6e^{3t} - 3e^{-t} \qquad y(0)=1,\ y'(0)=-1$$

$$\Rightarrow s^2\mathcal{L}\{y\} - s\,y(0) - y'(0) - 4\left(s\mathcal{L}\{y\} - y(0)\right) = 6\mathcal{L}\{e^{3t}\} - 3\mathcal{L}\{e^{-t}\}$$

$$\Rightarrow \mathcal{L}\{y\}\left[s^2-4s\right] + \left[-sy(0) - y'(0) + 4y(0)\right] = 6\frac{1}{s-3} - 3\frac{1}{s+1}$$

$$\Rightarrow \mathcal{L}\{y\} = \frac{1}{s^2-4s}\left[sy(0)+y'(0)-4y(0)\right] + \frac{1}{s^2-4s}\left[\frac{6}{s-3} - \frac{3}{s+1}\right]$$

zero input response

$$\mathcal{L}\{y_{zi}(t)\} = \frac{1}{s^2-4s}\left[s-1-4\right] = \frac{s-5}{s^2-4s}$$

$$y_{zi}(t) = \mathcal{L}^{-1}\left\{\frac{s-5}{s^2-4s}\right\} = \mathcal{L}^{-1}\left\{\frac{5}{4}\frac{1}{s} - \frac{1}{4}\frac{1}{s-4}\right\}$$

$$= \frac{5}{4} - \frac{1}{4}e^{4t}$$

zero-state response

$$\mathcal{L}\{y_{zs}(t)\} = \frac{1}{s^2-4s}\left[\frac{6}{s-3} - \frac{3}{s+1}\right]$$

$$y_{zs}(t) = \mathcal{L}^{-1}\left\{\frac{6}{(s^2-4s)(s-3)} - \frac{3}{(s^2-4s)(s+1)}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{27}{20}\frac{1}{s-4} - \frac{2}{s-3} + \frac{5}{4}\frac{1}{s} - \frac{3}{5}\frac{1}{s+1}\right\}$$

$$y_{zs}(t) = \frac{27}{20}e^{4t} - 2e^{3t} + \frac{5}{4} - \frac{3}{5}e^{-t}$$