(7.3)

10

$$\mathcal{L}\left\{e^{3t}\left(9-4t+10\sin\frac{t}{2}\right)\right\}$$

$$=\mathcal{L}\left\{9e^{3t}-4te^{3t}+10e^{3t}\sin\frac{t}{2}\right\}$$

$$=\mathcal{L}\left\{9e^{3t}\right\}-\mathcal{L}\left\{4te^{3t}\right\}+\mathcal{L}\left\{10e^{3t}\sin\frac{t}{2}\right\}$$

$$=9\mathcal{L}\left\{e^{3t}\right\}-4\mathcal{L}\left\{te^{3t}\right\}+10\mathcal{L}\left\{e^{3t}\sin\frac{t}{2}\right\}$$

$$\mathcal{L}\left\{e^{3t}\right\}=\frac{1}{s-3}$$

$$\mathcal{L}\left\{te^{3t}\right\}=\left.\frac{1}{s^2}\right|_{s\to s-3}=\frac{1}{(s-3)^2}$$

$$\mathcal{L}\left\{e^{3t}\sin\frac{t}{2}\right\}=\left.\frac{\frac{1}{2}}{s^2+\left(\frac{1}{2}\right)^2}\right|_{s\to s-3}=\frac{\frac{1}{2}}{(s-3)^2+\frac{1}{4}}$$

$$=\boxed{\frac{9}{s-3}-\frac{4}{(s-3)^2}+\frac{5}{(s-3)^2+\frac{1}{4}}}$$

OR $\Rightarrow \mathcal{L}\left\{9-4t-10\sin\frac{t}{2}\right\}$

$$=\frac{9}{s}-\frac{4}{s^2}-10\cdot\frac{\frac{1}{2}}{s^2+\left(\frac{1}{2}\right)^2}$$

$$=\frac{9}{s}-\frac{4}{s^2}-\frac{5}{s^2+\frac{1}{4}}$$

$\Rightarrow \mathcal{L}\left\{e^{3t}\left(9-4t+10\sin\frac{t}{2}\right)\right\}$ $s\to(s-3)$

$$=\boxed{\frac{9}{s-3}-\frac{4}{(s-3)^2}-\frac{5}{(s-3)^2+\frac{1}{4}}}$$

(7.3)

16

$$\mathcal{L}^{-1}\left\{\frac{2s+5}{\underbrace{s^2+6s+34}_{(s+3)^2+25}}\right\}$$

$$= \mathcal{L}^{-1}\left\{\underbrace{\frac{2(s+3)}{(s+3)^2+5^2}}_{\Downarrow} - \underbrace{\frac{1}{5}\frac{5}{(s+3)^2+5^2}}_{\Downarrow}\right\}$$

$$= \boxed{\left(2e^{-3t}\cos 5t\right) - \frac{1}{5}e^{-3t}\sin 5t} \,//$$

$$\mathcal{L}\{\cos 5t\} = \frac{s}{s^2+5^2}$$

$$\mathcal{L}\{\sin 5t\} = \frac{5}{s^2+5^2}$$

$$\mathcal{L}\{e^{at}f(t)\} = F(s-a)$$

(7.3) $y'' - 6y' + 9y = t, \quad y(0)=0, \quad y'(0)=1$

25

$$\mathcal{L}\{y'' - 6y' + 9y\} = \mathcal{L}\{t\}$$

$$\Rightarrow \mathcal{L}\{y''\} - 6\mathcal{L}\{y'\} + 9\mathcal{L}\{y\} = \mathcal{L}\{t\}$$

$$= s^2\mathcal{L}\{y\} - s\,y(0) - y'(0)$$

$$-6\left(s\mathcal{L}\{y\} - y(0)\right)$$

$$+9\left(\mathcal{L}\{y\}\right) \qquad = \frac{1}{s^2}$$

$$\Rightarrow \mathcal{L}\{y\}\left(s^2 - 6s + 9\right) = \frac{1}{s^2} + s\,\cancel{y(0)}^{\,0} + \cancel{y'(0)}^{\,1} + 6\,\cancel{y(0)}^{\,0}$$

$$= \frac{1}{s^2} + 1 = \frac{s^2+1}{s^2}$$

$$\Rightarrow \mathcal{L}\{y\} = \frac{s^2+1}{s^2(s^2-6s+9)} = \frac{s^2+1}{s^2(s-3)^2}$$

$$= \frac{2}{27}\frac{1}{s} + \frac{1}{9}\frac{1}{s^2} - \frac{2}{27}\frac{1}{s-3} + \frac{10}{9}\frac{1}{(s-3)^2}$$

$$\Rightarrow \boxed{y(t) = \frac{2}{27}\,1 + \frac{1}{9}t - \frac{2}{27}e^{3t} + \frac{10}{9}te^{3t}}$$

$\#$

(7.3) $\mathcal{L}^{-1}\left\{\frac{(1+e^{-2s})^2}{s+2}\right\}$

44

$$= \mathcal{L}^{-1}\left\{ \frac{1}{s+2} + \frac{2e^{-2s}}{s+2} + \frac{e^{-4s}}{s+2}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\} + \mathcal{L}^{-1}\left\{\frac{2e^{-2s}}{s+2}\right\} + \mathcal{L}^{-1}\left\{\frac{e^{-4s}}{s+2}\right\}$$

$$= e^{-2t} + 2e^{-2t}\Big|_{t\to t-2} + e^{-2t}\Big|_{t\to t-4}$$

$$= \boxed{e^{-2t} + 2e^{-2(t-2)}\mathcal{U}(t-2) + e^{-2(t-4)}\mathcal{U}(t-4)}$$

(7.3)

59

$$f(t) = \begin{cases} t, & 0 \le t < 2 \\ 0, & t \ge 2 \end{cases}$$

$$f(t) = t - t\,\mathcal{U}(t-2)$$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{t - t\,\mathcal{U}(t-2)\}$$

$$= \mathcal{L}\{t\} - \mathcal{L}\{t\,\mathcal{U}(t-2)\}$$

$$= \underline{\mathcal{L}\{t\}} - \mathcal{L}\{\underline{(t-2)\,\mathcal{U}(t-2)} + \underline{2\,\mathcal{U}(t-2)}\}$$

$$\boxed{= \frac{1}{s^2} - \frac{e^{-2s}}{s^2} - \frac{2e^{-2s}}{s}}$$ //

$$\mathcal{L}\{t\} = \frac{1}{s^2}$$

$$\mathcal{L}\{t\,\mathcal{U}(t)\} = \frac{1}{s^2}$$

$$\mathcal{L}\{(t-2)\,\mathcal{U}(t-2)\} = \frac{1}{s^2}\,e^{-2s}$$

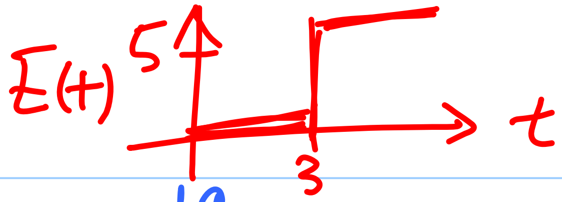
$$\mathcal{L}\{1\} = \frac{1}{s}$$

$$\mathcal{L}\{\mathcal{U}(t)\} = \frac{1}{s}$$

$$\mathcal{L}\{\mathcal{U}(t-2)\} = \frac{1}{s}\,e^{-2s}$$

(7.3) $q(0)=0$, $R=2.5\,\Omega$, $C=0.08\,f$,

73



$$R\frac{dq}{dt}+\frac{1}{C}q=E(t)$$

$$\Rightarrow \quad 2.5\frac{dq}{dt}+12.5\,q=5\,\mathcal{U}(t-3)$$

$$\Rightarrow \quad \frac{dq}{dt}+5\,q=2\,\mathcal{U}(t-3)$$

$$\mathcal{L}\Rightarrow \quad s\mathcal{L}\{q\}-\cancel{q(0)}^{\,0}+5\mathcal{L}\{q\}=\mathcal{L}\{2\,\mathcal{U}(t-3)\}$$

$$s\mathcal{L}\{q\}+5\mathcal{L}\{q\}=2\,\frac{1}{s}e^{-3s}$$

$$\Rightarrow \quad \mathcal{L}\{q\}(s+5)=\frac{2}{s}e^{-3s}$$

$$\Rightarrow \quad \mathcal{L}\{q\}=\frac{2}{s(s+5)}e^{-3s}$$

$$=\left(\frac{2}{5}\,\frac{1}{s}-\frac{2}{5}\,\frac{1}{s+5}\right)e^{-3s}$$

$$\mathcal{L}^{-1}\Rightarrow \quad \boxed{q(t)=\frac{2}{5}\,\mathcal{U}(t-3)-\frac{2}{5}\,e^{-5(t-3)}\,\mathcal{U}(t-3)}$$