(7.5) 5

$$y'' + y = \delta(t-\tfrac{1}{2}\pi) + \delta(t-\tfrac{3}{2}\pi)$$

$$y(0)=0,\quad y'(0)=0$$

$$\mathcal{L}\{y''+y\} = \mathcal{L}\{\delta(t-\tfrac{1}{2}\pi)+\delta(t-\tfrac{3}{2}\pi)\}$$

$$\Rightarrow s^2\mathcal{L}\{y\} - sy(0) - y'(0) + \mathcal{L}\{y\} = e^{-\frac{\pi}{2}s} + e^{-\frac{3}{2}\pi s}$$

$$(s^2+1)\mathcal{L}\{y\} - 0 = e^{-\frac{\pi}{2}s} + e^{-\frac{3}{2}\pi s}$$

$$\Rightarrow \mathcal{L}\{y\} = \frac{1}{s^2+1}\left(e^{-\frac{\pi}{2}s} + e^{-\frac{3}{2}\pi s}\right)$$

$$= \underline{\frac{1}{s^2+1}e^{-\frac{\pi}{2}s}} + \frac{1}{s^2+1}e^{-\frac{3}{2}\pi s}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}e^{-\frac{\pi}{2}s}\right\} = \sin(t-\tfrac{\pi}{2})\,u(t-\tfrac{\pi}{2})$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}e^{-\frac{3}{2}\pi s}\right\} = \sin(t-\tfrac{3}{2}\pi)\,u(t-\tfrac{3}{2}\pi)$$

$$\Rightarrow y(t) = \boxed{\sin(t-\tfrac{\pi}{2})\,u(t-\tfrac{\pi}{2}) + \sin(t-\tfrac{3}{2}\pi)\,u(t-\tfrac{3}{2}\pi)}$$

$$\text{or} = \boxed{-\cos(t)\,u(t-\tfrac{\pi}{2}) + \cos(t)\,u(t-\tfrac{3}{2}\pi)}$$

(7.5) 12

$$y'' - 7y' + 6y = e^{t} + \delta(t-2) + \delta(t-4)$$

$$y(0)=0,\ y'(0)=0$$

$$\mathcal{L}\{y'' - 7y' + 6y\} = \mathcal{L}\{e^{t} + \delta(t-2) + \delta(t-4)\}$$

$$\Rightarrow s^2\mathcal{L}\{y\} - \cancelto{0}{s y(0)} - \cancelto{0}{y'(0)} - 7\left(s\mathcal{L}\{y\} - \cancelto{0}{y(0)}\right) + 6\mathcal{L}\{y\}$$

$$= \frac{1}{s-1} + e^{-2s} + e^{-4s}$$

$$\Rightarrow \underbrace{\left(s^2 - 7s + 6\right)}_{=(s-1)(s-6)} \mathcal{L}\{y\} = \frac{1}{s-1} + e^{-2s} + e^{-4s}$$

$$\mathcal{L}\{y\} = \frac{1}{(s-1)^2(s-6)} + \frac{e^{-2s} + e^{-4s}}{(s-1)(s-6)}$$

$$= -\frac{1}{25}\frac{1}{s-1} - \frac{1}{5}\frac{1}{(s-1)^2} + \frac{1}{25}\frac{1}{s-6}$$

$$+ \left[-\frac{1}{5}\frac{1}{s-1} + \frac{1}{5}\frac{1}{s-6}\right]\left(e^{-2s} + e^{-4s}\right)$$

$$\Rightarrow \boxed{\begin{aligned} y(t) &= -\frac{1}{25}e^{t} - \frac{1}{5}te^{t} + \frac{1}{25}e^{6t} \\ &+ \left[-\frac{1}{5}e^{t-2} + \frac{1}{5}e^{6(t-2)}\right]u(t-2) \\ &+ \left[-\frac{1}{5}e^{t-4} + \frac{1}{5}e^{6(t-4)}\right]u(t-4) \end{aligned}}$$

#