$$(7.6) \Rightarrow \frac{d^2x}{dt^2} + 3\frac{dy}{dt} + 3y = 0$$

$$\text{||} \Rightarrow \frac{d^2x}{dt^2} + 3y = te^{-t}$$

$\mathcal{L}\{\}$ $x(0)=0,\quad x'(0)=2,\quad y(0)=0$

$$\Rightarrow s^2\mathcal{L}\{x\} - s\cancel{x(0)}^{\,0} - \cancel{x'(0)}^{\,2} + 3s\mathcal{L}\{y\} - \cancel{y(0)}^{\,0} + 3\mathcal{L}\{y\} = 0$$

$$s^2\mathcal{L}\{x\} - s\cancel{x(0)}^{\,0} - \cancel{x'(0)}^{\,2} + 3\mathcal{L}\{y\} = \mathcal{L}\{te^{-t}\}$$

$$\Rightarrow s^2\mathcal{L}\{x\} + (3s+3)\mathcal{L}\{y\} = 2$$

$$\ominus \quad s^2\mathcal{L}\{x\} + 3\mathcal{L}\{y\} = \frac{1}{(s+1)^2} \quad \times(s+1)$$

$$s^2(s+1)\mathcal{L}\{x\} + (3s+3)\mathcal{L}\{y\} = \frac{1}{s+1}$$

$$\Rightarrow \left(s^2(s+1) - s^2\right)\mathcal{L}\{x\} = \frac{1}{s+1} - 2$$

$$s^2(s)\,\mathcal{L}\{x\} = \frac{-2s-1}{s+1}$$

$$\Rightarrow \mathcal{L}\{x\} = \frac{-(2s+1)}{s^3(s+1)}$$

$$= \frac{1}{s} + \frac{1}{s^2} + \frac{1}{2}\frac{2}{s^3} - \frac{1}{s+1}$$

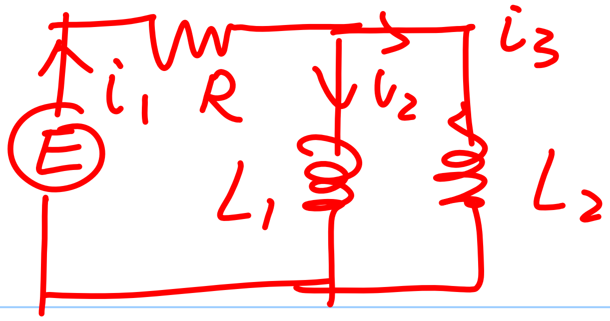
$$\Rightarrow \boxed{x(t) = 1 + t + \frac{1}{2}t^2 - e^{-t}} \quad \#$$

$$\Rightarrow \boxed{y(t)} = \frac{1}{3}te^{-t} - \frac{1}{3}\frac{d^2x}{dt^2}$$

$$= \frac{1}{3}te^{-t} - \frac{1}{3}\left(1 - e^{-t}\right)$$

$$\boxed{= \frac{1}{3}te^{-t} + \frac{1}{3}e^{-t} - \frac{1}{3}} \quad \#$$

(7.6) 15



(a) $i_1 = i_2 + i_3$

$$\begin{cases} E = R i_1 + L_1 i_2' \\ E = R i_1 + L_2 i_3' \end{cases}$$

$$\boxed{\begin{aligned} L_1 \frac{di_2}{dt} + R(i_2+i_3) &= E \\ L_2 \frac{di_3}{dt} + R(i_2+i_3) &= E \end{aligned}}$$ (a)

$$0.01\, i_2' + 5 i_2 + 5 i_3 = 100$$

(b) $$0.0125\, i_3' + 5 i_2 + 5 i_3 = 100$$

$\mathcal{L}\{\}$ $$\Rightarrow \begin{cases} (0.01 s + 5)\mathcal{L}\{i_2\} + 5\mathcal{L}\{i_3\} = \frac{100}{s} & \times 100 \\ (0.0125 s + 5)\mathcal{L}\{i_3\} + 5\mathcal{L}\{i_2\} = \frac{100}{s} & \times \frac{10000}{125} \end{cases}$$

$$\Rightarrow \begin{cases} (s+500)\mathcal{L}\{i_2\} + 500\mathcal{L}\{i_3\} = \frac{10000}{s} & \times 400 \\ (s+400)\mathcal{L}\{i_3\} + 400\mathcal{L}\{i_2\} = \frac{8000}{s} & \times (s+500) \end{cases}$$

$$\Rightarrow \left(400 \times 500 - (s+400)(s+500)\right)\mathcal{L}\{i_3\} = \frac{4000000}{s} - \frac{8000(s+500)}{s}$$

$$\Rightarrow -(s^2 + 900 s)\mathcal{L}\{i_3\} = -\frac{1}{s}(8000 s)$$

$$\Rightarrow \mathcal{L}\{i_3\} = \frac{8000}{s^2 + 900 s} = \frac{8000}{s(s+900)}$$

$$= \frac{80}{9}\frac{1}{s} - \frac{80}{9}\frac{1}{s+900}$$

$$\Rightarrow \boxed{i_3(t) = \frac{80}{9} - \frac{80}{9} e^{-900t}}$$

$$\Rightarrow 5i_2 = 100 - 0.0125\, i_3' - 5i_3$$

$$\boxed{i_2} = 20 - 0.0025\, i_3' - i_3$$

$$= 20 - 0.0025\left(8000\, e^{-900t}\right) - \frac{80}{9} - \frac{80}{9}\left(e^{-900t}\right)$$

$$= 20 - 20e^{-900t} - \frac{80}{9} - \frac{80}{9}e^{-900t}$$

$$\boxed{= \frac{100}{9} - \frac{100}{9}e^{-900t}}$$

(c) $\boxed{i_1} = i_2 + i_3$

$$= \frac{100}{9} - \frac{100}{9}e^{-900t} + \frac{80}{9} - \frac{80}{9}e^{-900t}$$

$$\boxed{= 20 - 20e^{-900t}}$$