Quiz 1

1) $x^2 \frac{dy}{dx} = y - xy, \quad y(-1) = -1$

$\Rightarrow x^2 \frac{dy}{dx} = y(1-x)$

let $x \neq 0, y \neq 0$, $\frac{1}{y} y x^2$, $dx \to$ RHS

$\frac{1}{y} dy = \frac{1}{x^2}(1+x)\,dx$

Integrate both sides

$\int \frac{1}{y} dy = \int \frac{1-x}{x^2} dx$

$= \int \frac{1}{x^2} dx - \int \frac{1}{x} dx$

$\int \frac{1}{u} du = \ln|u| + c$

$\int u^n du = \frac{u^{n+1}}{n+1} + c \qquad n \neq -1$

$|y| + c_1 = \frac{x^{-2+1}}{-2+1} + c_2 - \ln|x| + c_3$

c_1, c_2, c_3 some constants

$\ln|y| = \frac{x^{-1}}{-1} - \ln|x| + c_4$

$c_4 = c_3 + c_2 - c_1$

$= -\frac{1}{x} - \ln|x| + c_4$

$= \ln|y| + \ln|x| = -\frac{1}{x} + c_4$

$\Rightarrow \ln|yx| = -\frac{1}{x} + c_4$

$$\Rightarrow |yx| = e^{-\frac{1}{x}+c_4}$$

$$= e^{-\frac{1}{x}} \cdot e^{c_4}$$

$$yx = (\pm e^{c_4})\, e^{-\frac{1}{x}}$$

$$= c$$

$$yx = c\, e^{-\frac{1}{x}}$$

$$y(-1) = -1$$

$$(-1)(-1) = c\, e^{-\frac{1}{-1}} = c\, e$$

$$1 = c\, e$$

$$\Rightarrow c = e^{-1}$$

$$\Rightarrow yx = e^{-1} e^{-\frac{1}{x}} = e^{-(1+\frac{1}{x})}$$

$$\text{or } y = \frac{1}{x} e^{-(1+\frac{1}{x})}$$

Quiz

$$u(x,y) = xy$$

(2) $\left(-xy\sin(x) + 2y\cos(x)\right)dx + \underbrace{(2x\cos(x))}_{\bar{N}}dy = 0$

$\bar{M}$

$\bar{M}_y = -x\sin x + 2\cos x$

$\bar{N}_x = 2\cos x - 2x\sin x$

$\Rightarrow \bar{M}_y \neq \bar{N}_x$

$M = \bar{M}u = -x^2y^2\sin x + 2xy^2\cos x$

$N = \bar{N}u = 2x^2y\cos x$

$M_y = -2x^2y\sin x + 4xy\cos x$

$N_x = 4xy\cos x - 2x^2y\sin x$

$\Rightarrow M_y = N_x$

$f_y \equiv 2x^2y\cos x$

$f = x^2y^2\cos x + h(x)$

$\Rightarrow f_x = 2xy^2\cos x - x^2y^2\sin x + h'(x)$

$= M$

$\therefore h'(x) = 0 \Rightarrow h(x) = C$ some constant

$\Rightarrow \boxed{f(x,y) = x^2y^2\cos x + C = 0}$

Quiz

13)

$$16\frac{d^4y}{dx^2} + 24\frac{d^2y}{dx^2} + 9y = 0$$ ← input

Let $y(x) = e^{mx} \Rightarrow y'(x) = me^{mx}$

become $\Rightarrow y''(x) = m^2e^{mx}$, $y''' = m^3e^{mx}$, $y^{(4)} = m^4e^{mx}$

$$16(m^4e^{mx}) + 24(m^2e^{mx}) + 9(e^{mx}) = 0$$

$$\Rightarrow e^{mx}\left[16m^4 + 24m^2 + 9\right] = 0$$

$\because e^{mx} \neq 0 \Rightarrow 16m^4 + 24m^2 + 9 = 0$

$$\Rightarrow 16\left(m^2 + \frac{3}{4}\right)^2 = 0$$

$$\Rightarrow m^2 = -\frac{3}{4}, -\frac{3}{4}$$

$$\Rightarrow m = \frac{\sqrt{3}}{2}i, \frac{\sqrt{3}}{2}i \quad -\frac{\sqrt{3}}{2}i, -\frac{\sqrt{3}}{2}i$$

$$\Rightarrow y(x) = c_1\cos\left(\frac{\sqrt{3}}{2}x\right) + c_2\sin\left(\frac{\sqrt{3}}{2}x\right)$$

$$+ c_3 x\cos\left(\frac{\sqrt{3}}{2}x\right) + c_4 x\sin\left(\frac{\sqrt{3}}{2}x\right)$$