Fall 2007

線性系統
Linear Systems

Chapter 01
Information & Introduction

Feng-Li Lian

NTU-EE

Sep07 – Jan08

Materials used in these lecture notes are adopted from
"Linear System Theory & Design," 3rd. Ed., by C.-T. Chen (1999)

Syllabus

- Lecture Information:
  - Time: 9:10am-12noon, Tuesays
  - Room: EE-MD231
  - Webpage: http://cc.ee.ntu.edu.tw/~fengli/Teaching/LinearSystems/
- Instructor:
  - 連豊力(Feng-Li Lian),
  - Office: EE-MD717
  - E-mail: fengli@ntu.edu.tw, Phone: 02-3366-3606
- Teaching Assistant:
  - To be determined
- Grading:
  - Homework (30%) due in class, no late homework is accepted
  - Midterm exam (20+20%) on 10/23 and 11/27
  - Final exam (30%) on 1/15
- Textbook:
  - Linear System Theory & Design, 3rd. Ed., by C.-T. Chen (1999)

Homework and Exam

- Homework Rules:
  - No late homework will be graded!!!
  - 請註明：作業次別，姓名，學號，系級，日期
  - Problems
    - End-of-Chapter Problems
    - Special Problems

- Exam Rules:
  - 1-hour (midterm) & 2-hour (final) exam
  - Closed books and notes
  - No calculators are required
  - Coverage
    - Lecture notes: 1/3
    - End of Chapter Problems: 1/3
    - Special Problems: 1/3

What is Linear Systems about? – 1

- In Algebra:

$$a\,x \quad = \quad b$$

- In Linear Algebra:

$$\mathbf{A}\,\mathbf{x} \quad = \quad \mathbf{B}$$

What is Linear Systems about? – 2

- In Differential Equation:

$$\frac{dx(t)}{dt} = \dot{x}(t) = a\,x(t)$$

$$x(t_0) = x_0$$

$$\Rightarrow \quad x(t) =$$

What is Linear Systems about? – 3

- In Differential Equations:

$$\dot{x}(t) = a\,x(t) + b\,u(t)$$

$$x(t_0) = x_0$$

$$\Rightarrow \quad x(t) =$$

What is Linear Systems about? – 4

- In Linear Systems:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\,\mathbf{x}(t) + \mathbf{B}\,\mathbf{u}(t)$$

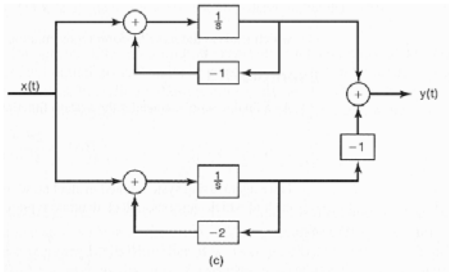
$$\mathbf{x}(t_0) = \mathbf{x}_0$$

$$\Rightarrow \quad \mathbf{x}(t) =$$

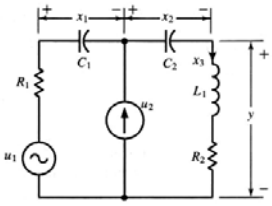
Key Ingredients in Linear Systems

- Linear Algebra:
- Differential Equations
- Signals and Systems

Engineering, Technology, System, Graph, Mathematics



$$2\,\frac{dx(t)}{dt} + 3\,x(t) = 5\,u(t)$$



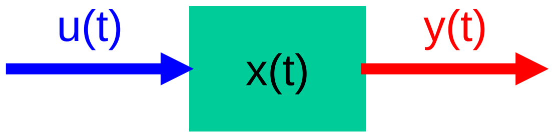
Analysis & Design Philosophy of Engineering Systems

- Mathematics, Statistics:
  - Differential Equations, Linear Algebra, Probability, Stochastic Processes
- Physics, Chemistry:
  - Electronics, Electrical Circuits, Electromagnetics, Dynamics, Thermodynamics, Heat Transfer
- Linear v.s. Nonlinear

- Stability
- Controllability
- Observability
- Performance
- Robustness

- Hardware vs Software
- Optimization vs Control
  - Optimal Control, Robust Control, Adaptive Control, Nonlinear Control

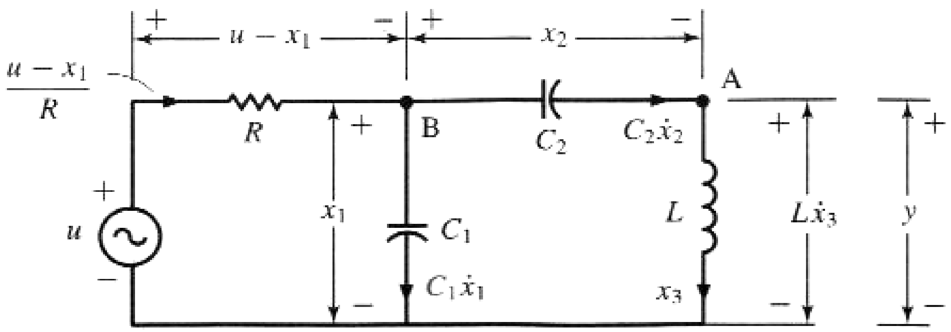
System, Input, Output, Parameter, Variable



- Input-Output Model

- State-Space Model

An RLC Electrical Circuit – 1



At node A: $C_2\dot{x}_2 = x_3$

At node B: $\dfrac{u - x_1}{R} = C_1\dot{x}_1 + C_2\dot{x}_2$

At RHS loop: $L\dot{x}_3 = x_1 - x_2$ $\qquad y = L\dot{x}_3$

$$\Rightarrow \hat{\mathbf{y}}(s) = \hat{\mathbf{G}}(s)\hat{\mathbf{u}}(s)$$

An RLC Electrical Circuit – 2

$$\Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1/RC_1 & 0 & -1/RC_1 \\ 0 & 0 & 1/C_2 \\ 1/L & -1/L & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} -1/RC_1 \\ 0 \\ 0 \end{bmatrix} u$$

- Let $\mathbf{x} := \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$$\Rightarrow \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$

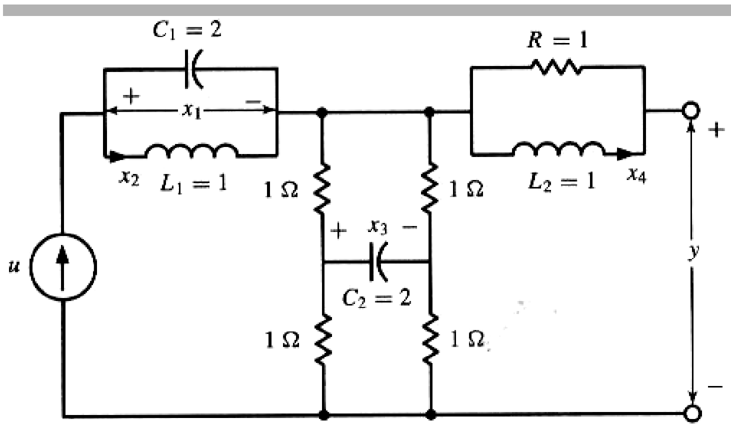
- $y = L\dot{x}_3 = x_1 - x_2 \quad = [1 \ -1 \ \ 0]\mathbf{x} + 0 \cdot u$

$$\Rightarrow y = \mathbf{C}\mathbf{x} + \mathbf{D}u$$

- In general, $\dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)\mathbf{u}(t)$

$$\mathbf{y}(t) = \mathbf{C}(t)\mathbf{x}(t) + \mathbf{D}(t)\mathbf{u}(t)$$

Another RLC Electrical Circuit



$$\mathbf{x} := \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

- $\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)\mathbf{u}(t) \\ \mathbf{y}(t) = \mathbf{C}(t)\mathbf{x}(t) + \mathbf{D}(t)\mathbf{u}(t) \end{cases}$

- Solution
- Stability
- Controllability
- Observability
- Minimal Form
- State Feedback
- State Estimation

Course Outline

- Mathematical Description of Systems (Ch 02)
- Linear Algebra (Ch 03)
- State-Space Solutions and Realizations (Ch 04)
- Stability (Ch 05)
- Controllability & Observability (Ch 06)
- Minimal Realization (Ch 07)
- State Feedback and State Estimation (Ch 08)