Homework 1, Linear Systems, Fall 2007

- When turn in your homework, please write down: 作業次別, 姓名, 學號, 系級, 日期
- Assigned: 9/26/07, Due on 10/9/06

For the following problems, please find the associated state-space model. That is, find the matrices, **A**, **B**, **C**, **D**, in the equations:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t).$$

10 points/each

1 (A RLRC circuit)
Consider Figure 1. Let $x_1 = i_L, x_2 = v_C, u = v_s$, and $y = v_C$.

2 (A transistor circuit)
Consider Figure 2. Let $x_1 = i_b, x_2 = v_{out}, u = e_s$, and $y = v_{out}$.

3 (A parallel electrical circuit)
Consider Figure 3. Let $x_1 = i_{L1}, x_2 = v_{C1}, x_3 = i_{L2}, x_4 = v_{C2}, u = v_s$, and $y = v_{C2}$.

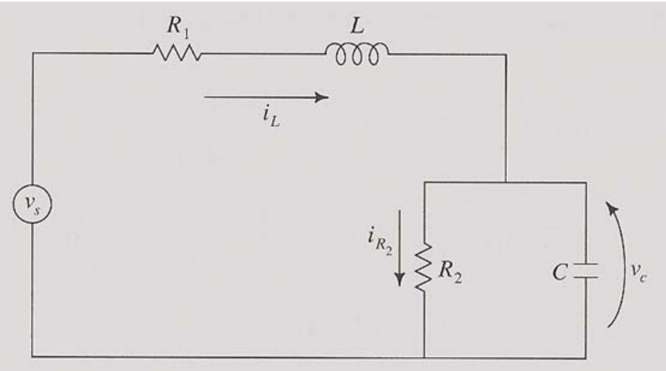

Fig. 1

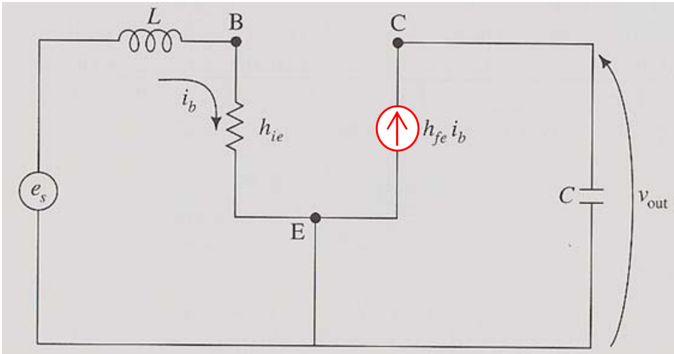

Fig. 2

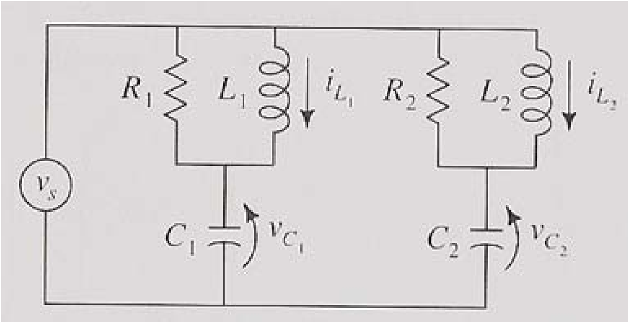

Fig. 3

4 (A 2-tank system)
Consider Figure 4. Let $x_1 = h_1, x_2 = h_2, u = u$, and $y = Q_1$, where h_i is the liquid level, Q_i is the flow, A_i is the cross-sectional area, R_i is a resistance constant of Tank i. In this case, the mass-continuity equation is applied, that is,

$$A_1 \frac{dh_1}{dt} = -\frac{h_1 - h_2}{R_1} + u(t)$$

$$A_2 \frac{dh_2}{dt} = \frac{h_1 - h_2}{R_1} - \frac{h_2}{R_2},$$

and Q_1 can be characterized as: $Q_1 = \frac{h_1 - h_2}{R_1}$.

5 (A 3-tank system)
Consider Figure 5. Let $x_i = h_i, u_i = u_i, i = 1, 2, 3$, and $y = h_2$, and $R_1 = R_2 = 1/2$ and $A_1 = A_3 = (2/3)A_2$ and $A_2 = 1$.

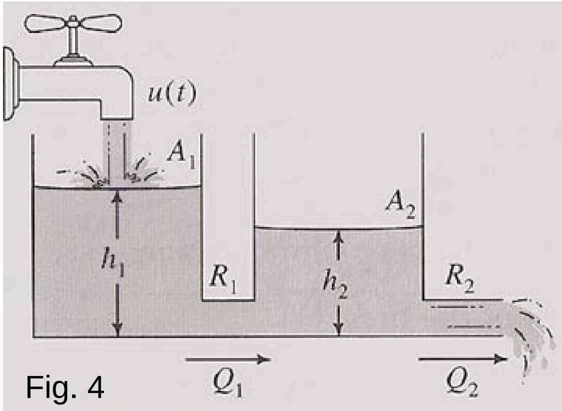

Fig. 4

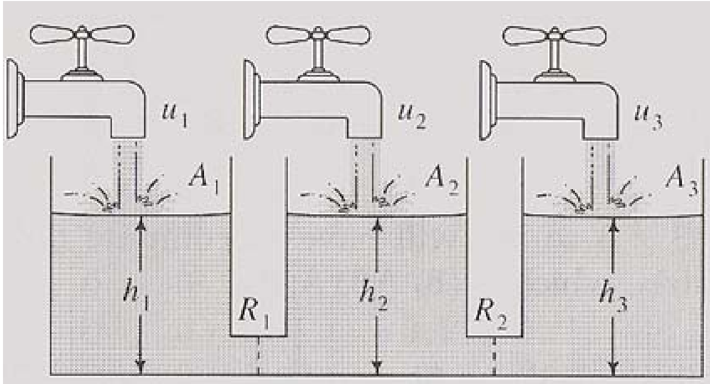

Fig. 5

6 (A bicycle-rider system)
Consider Figure 6. Let $x_1 = r, x_2 = y, x_3 = v_1, x_4 = v_2, u_1 = u_v, u_2 = u_h$, and $y_1 = y_f, y_2 = \theta$. Assume that the equations of motion are discribed as follows.

$$
\begin{aligned}
\dot{r} &= \tfrac{1}{L} V u_v \\
\dot{y} &= -\tfrac{L_r}{L} V u_v - V r \\
\ddot{\theta} &+ \tfrac{w^2}{g}\ddot{y} + f_u \ddot{u}_h = w^2\theta + f_0 w^2 u_h
\end{aligned}
$$

where

$$
\begin{aligned}
w^2 &= \tfrac{(m_v h_v + m_h h_h) g}{(I_v + I_h + m_v h_v^2 + m_h h_h^2)} \\
f_u &= \tfrac{(m_h h_h^2 + I_n) h}{[h_h (I_v + I_h + m_v h_v^2 + m_h h_h^2)]} \\
f_0 &= \tfrac{m_h h_h}{(m_v h_v + m_h h_h)} \\
V &= \text{ the forward speed} \\
v_1 &= y + (\theta + f_u u_h) g / w^2 \\
v_2 &= \dot{v}_1
\end{aligned}
$$

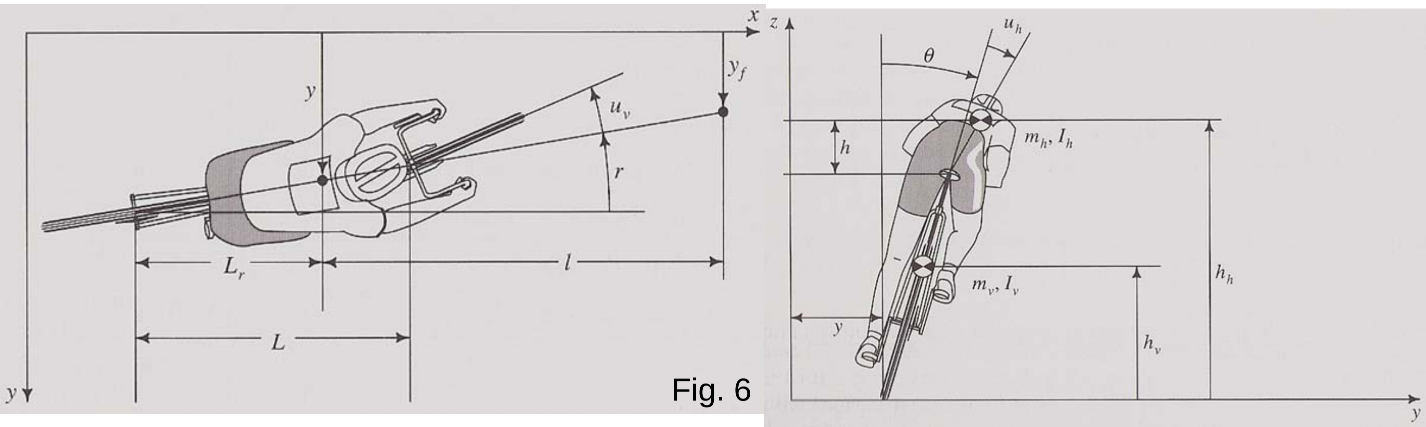

Fig. 6