Solution of Homework 1, Linear Systems, Fall 2007

1 (A RLRC circuit)

$$\mathbf{A} = \begin{bmatrix} -\frac{R_1}{L} & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{CR_2} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}, \quad \mathbf{C} = [\ 0 \quad 1\ ], \quad \mathbf{D} = 0$$

2 (A transistor circuit)

$$\mathbf{A} = \begin{bmatrix} -\frac{h_{ie}}{L} & 0 \\ \frac{h_{fe}}{C} & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}, \quad \mathbf{C} = [\ 0 \quad 1\ ], \quad \mathbf{D} = 0$$

3 (A parallel electrical circuit)

$$\mathbf{A} = \begin{bmatrix} 0 & \frac{-1}{L_1} & 0 & 0 \\ \frac{1}{C_1} & \frac{-1}{C_1R_1} & 0 & 0 \\ 0 & 0 & 0 & \frac{-1}{L_2} \\ 0 & 0 & \frac{1}{C_2} & \frac{-1}{C_2R_2} \end{bmatrix}, \mathbf{B} = \begin{bmatrix} \frac{1}{L_1} \\ \frac{1}{C_1R_1} \\ \frac{1}{L_2} \\ \frac{1}{C_2R_2} \end{bmatrix}, \quad \begin{aligned} \mathbf{C} &= [\ 0 \quad 0 \quad 0 \quad 1\ ], \\ \mathbf{D} &= 0 \end{aligned}$$

4 (A 2-tank system)

$$\mathbf{A} = \begin{bmatrix} -\frac{1}{R_1A_1} & \frac{1}{R_1A_1} \\ \frac{1}{R_1A_2} & -\left(\frac{1}{R_1A_2} + \frac{1}{R_2A_2}\right) \end{bmatrix}, \mathbf{B} = \begin{bmatrix} \frac{1}{A_1} \\ 0 \end{bmatrix}, \quad \begin{aligned} \mathbf{C} &= \begin{bmatrix} \frac{1}{R_1} & -\frac{1}{R_1} \end{bmatrix}, \\ \mathbf{D} &= 0 \end{aligned}$$

5 (A 3-tank system)

$$\mathbf{A} = \begin{bmatrix} -3 & 3 & 0 \\ 2 & -4 & 2 \\ 0 & 3 & -3 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} \frac{3}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{3}{2} \end{bmatrix}, \quad \begin{aligned} \mathbf{C} &= [\ 0 \quad 1 \quad 0\ ], \\ \mathbf{D} &= [\ 0 \quad 0 \quad 0\ ], \end{aligned}$$

6 (A bicycle-rider system)

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -V & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -w^2 & w^2 & 0 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} B_1V & 0 \\ -B_2V & 0 \\ 0 & 0 \\ 0 & F \end{bmatrix}, \quad \begin{aligned} \mathbf{C} &= \begin{bmatrix} -l & 1 & 0 & 0 \\ 0 & -\frac{w^2}{g} & \frac{w^2}{g} & 0 \end{bmatrix}, \\ \mathbf{D} &= \begin{bmatrix} 0 & 0 \\ 0 & -f_u \end{bmatrix} \end{aligned}$$