Solution of Homework 5, Linear Systems, Fall 2008

- (Minimal Realization)

3 (A parallel electrical circuit)

As disscused in HW 4, the system is neither controllable nor observable, Hence, it is NOT minimal.
If the input-output behavior is described by the last two states, that is,

$$\begin{bmatrix} \dot{i_{L_2}} \\ \dot{v_{C_2}} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L_2} \\ \frac{1}{C_2} & -\frac{1}{C_2R_2} \end{bmatrix} \begin{bmatrix} i_{L_2} \\ v_{C_2} \end{bmatrix} + \begin{bmatrix} \frac{1}{L_2} \\ \frac{1}{C_2R_2} \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} i_{L_2} \\ v_{C_2} \end{bmatrix},$$

the reduced-order system is both controllable and observable.
Hence, it is minimal.
It can be verified by check the rank of the controllability and observability matrices.

$$\mathcal{C} = \begin{bmatrix} \frac{1}{L_2} & -\frac{1}{L_2C_2R_2} \\ \frac{1}{C_2R_2} & \frac{1}{L_2C_2} - \frac{1}{C_2^2R_2^2} \end{bmatrix}, \quad \mathcal{O} = \begin{bmatrix} 0 & 1 \\ \frac{1}{C_2} & -\frac{1}{C_2R_2} \end{bmatrix}$$

Both matrices are full rank.

5 (A 3-tank system)

As disscused in HW 4, the system is neither controllable nor observable, Hence, it is minimal. Let us define

$$x_4 = x_1 + x_3$$

and then adding the first and third equations above we will get

$$\begin{aligned} \dot{x}_4 &= -3x_4 + 6x_2 \\ \dot{x}_2 &= 2x_4 - 4x_2 + u \\ \text{and} \quad y &= x_2. \end{aligned}$$

the reduced-order system is both controllable and observable.
Hence, it is NOT minimal.
It can be verified by check the rank of the controllability and observability matrices.

$$\mathcal{C} = \begin{bmatrix} 0 & 6 \\ 1 & -4 \end{bmatrix}, \quad \mathcal{O} = \begin{bmatrix} 0 & 1 \\ 2 & -4 \end{bmatrix}$$

Both matrices are full rank.