Automatica, Vol. 19, No. 6, pp. 737–740, 1983
Printed in Great Britain.

0005-1098/83 $3.00 + 0.00
Pergamon Press Ltd.

Brief Paper

Analysis of Rider and Single-track-vehicle System; Its Application to Computer-controlled Bicycles*

M. NAGAI†

Key Words—Man–machine system; feedback control; stability; computer-controlled vehicle; robots; bicycles.

Abstract—Single-track-vehicles are dynamically more unstable and more complicated than automobiles, so that a rider must play a more important role in a man–machine system. The purpose of this paper is to analyze the balancing and tracking stabilities of this rider-cycle system, in order to design an automatically balancing and tracking robot bicycle which is expected to be used in a laboratory for researching good handling performance. A dynamical model of it is presented as a feedback control system with two inputs, handlebar steering and rider upper body leaning. The experimental bicycle is controled by a robotical rider which is realized by a microcomputer. According to a lane change experiment, the cycle runs successfully, and the results of dynamical responses are verified by the theoretical analysis.

1. *Introduction*

SINGLE-TRACK-VEHICLES, i.e. motorcycles and bicycles, present unique problems of stability and control, requiring more or less continuous attention by a rider. Because of the dynamical instability and complicated nature of single-track-vehicles, rider handling behaviour plays a more important role than it does for automobiles. The rider operates the cycle with handlebar steering and his upper body leaning.

The closed-loop system of rider and cycle has long been of practical and theoretical interest. Van Lunteren and Stassen (1969) have studied the bicycle balancing problem with an electrohydraulic simulator, and identified human-transfer functions for handlebar steering and rider leaning, but they were not concerned with tracking problems. They used a low-order dynamical model of bicycle assuming massless wheels and massless front fork assembly, following the earlier research by Whipple (1899). Van Zytveld (1975) studied the automatic bicycle balancing problem with a high-order dynamical model of a bicycle. He pointed out that stability is induced over a certain range of speeds by original bicycle characteristics such as the front fork assembly, and also a modest increase of stability is due to rider leaning. But his work has been concerned with 'no-hands' operation and the balancing problem only. Weir and Zellner (1979) reviewed recent research, and studied the closed-loop system of rider and cycle in order to design the desirable handling properties, dealing with balancing problem only.

Although there are many studies of the rider and cycle system, I cannot find references concerned with both balancing and tracking problems of a bicycle controlled with two inputs, handlebar steering and rider leaning. The purpose of this paper is to realize a driving robot which may function in a manner similar to a real rider. In order to design this automatically balancing and tracking robot bicycle, the stability and controllability of the system is analyzed using automatic control theory.

2. *Model and analysis*

2.1. *Model of cycle.* It is necessary to make a complicated bicycle model including front fork assembly, tire and wheel properties, etc., so as to explain a sensitive balancing problem such as no-hands running. But the handlebar steering plays a significant role in the rider and bicycle system, in a case of tracking or lane changing. In this paper, a two-input and two-output bicycle model is used for analysis of the rider and bicycle system. As it is convenient to use a low-order dynamical model in order to find apparent relationships between inputs and outputs, a simple dynamical model is presented.

From the following assumptions:

(a) side slip of tires is neglected;
(b) masses of front fork assembly and wheels are neglected;
(c) all masses of bicycle and rider are respectively lumped on a vertical line including the center of gravity, and the roll motion of bicycle is analogous to an inverted pendulum shown in Fig. 1;
(d) all equations are linearized:

equations of motions are described as follows:

$$\dot{r} = B_1 V u_s, \quad (B_1 = q/L) \tag{1}$$

$$\dot{y} = -B_2 V u_s - Vr, \quad (B_2 = qL_r/L) \tag{2}$$

$$\ddot{\theta} + (w^2/g)\ddot{y} + f_u \ddot{u}_l = w^2\theta + f_0 w^2 u_l \tag{3}$$

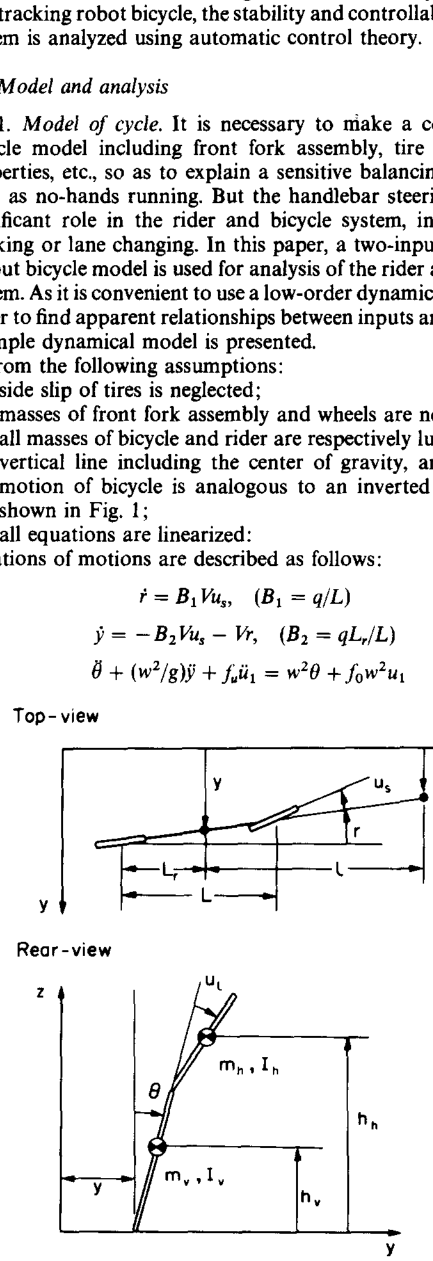

FIG. 1. Schematic model of single-track-vehicle.

*Received 12 January 1983; revised 19 May 1983. The original version of this paper was presented at the IFAC/IFIP/IFORS/IEA Conference on Analysis, Design, and Evaluation of Man–Machine Systems which was held in Baden-Baden, F.R.G. during September 1982. The published proceedings of this IFAC meeting may be ordered from Pergamon Press Ltd, Headington Hill Hall, Oxford OX3 0BW, U.K. This paper was recommended for publication in revised form by editor A. Sage.

† Department of Mechanical Engineering, Tokyo University of Agriculture and Technology, Koganei, Tokyo 184, Japan.

where

$$w^2 = (m_v h_v + m_h h_h)g/(I_v + I_h + m_v h_v^2 + m_h h_h^2)$$
$$f_u = (m_h h_h^2 + I_n)h/h_h(I_v + I_h + m_v h_v^2 + m_h h_h^2)$$
$$f_0 = m_h h/(m_v h_v + m_h h_h)$$

where r denotes a yaw angle, y a lateral position, θ a roll angle, u_1 a lean angle of rider, u_s a handlebar steer angle and V a running speed. Let h be a distance between the rider center of gravity and the hinge point, then hu_1 means a lateral position of the rider center of gravity from the extended home plane. The other parameters are those of cycle and rider (cf. Table 1). Let variables v_1 and v_2 be defined as

$$v_1 = y + (\theta + f_u u_1)g/w^2 \quad (4)$$

$$v_2 = \dot{v}_1 \quad (5)$$

then a fourth-order dynamical equation is described in state space form, deduced from (1)–(5)

$$\begin{bmatrix}\dot r\\ \dot y\\ \dot v_1\\ \dot v_2\end{bmatrix} = \begin{bmatrix}0 & 0 & 0 & 0\\ -V & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & -w^2 & w^2 & 0\end{bmatrix}\begin{bmatrix}r\\ y\\ v_1\\ v_2\end{bmatrix} + \begin{bmatrix}B_1 V & 0\\ -B_2 V & 0\\ 0 & 0\\ 0 & F\end{bmatrix}\begin{bmatrix}u_s\\ u_1\end{bmatrix} \quad (6)$$

$$(F = (f_0 - f_u)g).$$

For stabilizing both roll and directional motions, at least two variables must be perceived by rider. The directional motion can be visually perceived as by an automobile driver, while the roll motion can be vestibularly perceived. Therefore two output variables are defined here to be the lateral deviation of a previewed point y_f and the roll angle θ. Then the output equation of this system is described as

$$\begin{bmatrix}y_f\\ \theta\end{bmatrix} = \begin{bmatrix}-l & 1 & 0 & 0\\ 0 & -w^2/g & w^2/g & 0\end{bmatrix}\begin{bmatrix}r\\ y\\ v_1\\ v_2\end{bmatrix} + \begin{bmatrix}0 & 0\\ 0 & -f_u\end{bmatrix}\begin{bmatrix}u_s\\ u_1\end{bmatrix} \quad (7)$$

where y_f is the lateral deviation of the point at preview length l, that is, $y_f = y - lr$. Preview time $t_p = l/V$ is empirically about 1–3 s. Now the fourth-order dynamical system equation with two inputs and two outputs can be rewritten as

$$\dot{\mathbf{X}} = \mathbf{AX} + \mathbf{BU} \quad (6)$$

$$\mathbf{Y} = \mathbf{CX} + \mathbf{DU} \quad (7)$$

2.2. *Observability and controllability.* For the system equations (6) and (7), a condition for observability is that the matrix

$$\mathbf{U}_0 = (\mathbf{C}^t(\mathbf{CA})^t(\mathbf{CA}^2)^t(\mathbf{CA}^3)^t)^t \quad (8)$$

TABLE 1. SPECIFICATIONS OF EXPERIMENTAL BICYCLE

Weight of cycle	$m_v g$	136.0 N
Weight of on-board mass	$m_h g$	9.11 N
Height of cycle c.g.	h_v	0.334 m
Height of on-board mass	h_h	0.465 m
Moment of inertia of cycle	I_v	2.25 kg m^2
Moment of inertia of mass	I_h	0.201 kg m^2
Cosine of caster angle	q	0.946
Wheel base	L	0.665 m
Longitudinal distance between rear wheel and c.g.	L_r	0.313 m

should have rank 4. This condition is satisfied when a forward speed is not zero, i.e. $V > 0$. In the same way, a condition for controllability is that the matrix

$$\mathbf{U}_c = (\mathbf{B}\,\mathbf{AB}\,\mathbf{A}^2\mathbf{B}\,\mathbf{A}^3\mathbf{B}) \quad (9)$$

should have rank 4. This condition is satisfied when the forward speed is not equal to $L_r w$, or when the control matrix element $B(4,2)$ in (6) is not equal to zero. The second condition for controllability is always satisfied, then the system is controllable not only by handlebar steering but also upper body leaning.

2.3. *Stability.* Because the system is controllable and observable, it is theoretically possible to obtain an optimal closed loop system which may be realized by a skillful rider. In this case, an optimal control technique is applicable to this system. But if it is assumed that the cycle is controlled by a non-experienced rider with the feedback of output variables

$$\begin{bmatrix}u_s\\ u_1\end{bmatrix} = -\begin{bmatrix}k_{sy} & k_{s\theta}\\ k_{ly} & k_{l\theta}\end{bmatrix}\begin{bmatrix}y_f\\ \theta\end{bmatrix}, \quad \mathbf{U} = -\mathbf{KY} \quad (10)$$

then it is necessary to study the feedback gains for realizing the stable system. The characteristic function of the closed-loop system, shown in Fig. 2 consisting of (6), (7) and (10), is reduced to

$$|s\mathbf{I}_4 - \mathbf{A} + \mathbf{BK}(\mathbf{I}_2 + \mathbf{DK})^{-1}\mathbf{C}| = 0. \quad (11)$$

The conditions for stability are found by the Hurwitz method, so that several instructive relations concerned with the feedback gains are obtained, in a few cases as follows.

Case A. When a directional control is done only by upper body leaning, and a lateral roll control is independently done only by handlebar steering, that is, $u_s = -k_{s\theta}\theta$, $u_1 = -k_{ly}y_f$, the stability conditions are reduced to

$$0 < k_{ly} < P_l \quad (12)$$

$$k_{s\theta} > F(k_{ly})/V^2 \quad (13)$$

where P_l is a constant, $F(k_{ly})$ a function of the gain k_{ly}, respectively depending on the system parameters. The first inequality (12) means that a rider must lean to the left side if he intends to change the lane to the left. The second inequality (13) means that the rider must steer the handlebar to the left if the cycle rolls to the left. If the running speed becomes lower, the gain $k_{s\theta}$, the ratio of the steer angle to the roll angle, must be increased so that the rider's task may become heavier.

Case B. When the cycle is controlled only by handlebar steering without upper body leaning, that is, $u_s = -k_{sy}y_f - k_{s\theta}\theta$, $u_1 = 0$, the stability conditions are reduced to

$$k_{sy} > 0 \quad (14)$$

$$k_{s\theta} > P_2 k_{sy} + P_3/V^2 \quad (15)$$

where P_2 and P_3 are constants depending on the system parameters. The first inequality (14) means that the rider must steer the handlebar to the right side at first if he intends to change the lane to the left. The fact is empirically known as a reverse handling action. The second inequality (15) provides nearly the same relation as the inequality (13), concerning the forward speed.

Case C. When no compensation gains are zero, the property of the closed-loop system will become more desirable than above. Although the conditions for stability in this case cannot be obtained in such a simple form as above, similar constraints about feedback gains can be obtained.

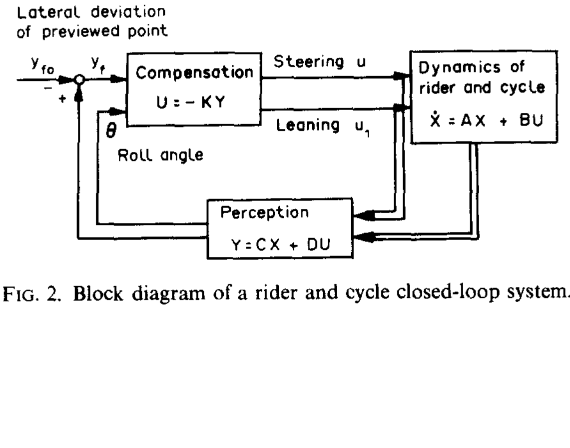

FIG. 2. Block diagram of a rider and cycle closed-loop system.

3. *Experiment*

The experimental bicycle was made to be operated automatically by a microcomputer. Figure 3 shows the schematic features of the experimental apparatus. The bicycle can run relatively on a moving belt, which is driven by an electric motor. Although the bicycle is mechanically constrained in the longitudinal direction, the other motions are all free from any constraints. There is an on-board mass which is permitted to move in the lateral direction. This mass is driven by a servomotor in order that it may play the same role as an upper body of rider. There is another motor for steering the handlebar. The control signals are transmitted from the microcomputer to these motors.

All state variables can be observed being derived from the previewed point deviation $y_f(t)$ and roll angle $\theta(t)$, as the cycle system is theoretically observable. But in this experiment, the lateral deviation of present point of cycle $y(t)$ and yaw angle $r(t)$ are directly measured instead of the previewed point deviation. The lateral deviation of the previewed point $y_f(t)$ is calculated from $y(t)$ and $r(t)$ in the microcomputer. Therefore it is necessary to measure the lateral positions of three different points of the main frame of cycle in order to get the lateral deviation $y(t)$, yaw angle $r(t)$ and roll angle $\theta(t)$.

To measure the attitude of the bicycle, an acoustic measuring method is used with 40 kHz acoustic wave. Pulsated acoustic waves are transmitted from the transmitters attached to the cycle, to the receivers attached to the frame of moving belt apparatus. The lateral displacement of each point of bicycle is calculated by counting the transmitted time between the transmitter and the receiver. With the method especially developed to suppress the measuring error in our laboratory, the absolute accuracy of lateral position error is less than 0.2 mm. The accuracies of roll angle and yaw angle are less than 0.05°.

Two input variables, hadlebar angle $u_s(t)$ and on-board mass position $hu_1(t)$, are also measured electrically by potentiometers. The position signal of the on-board mass is fed back to the control force of the servomotor for position control. The measured signals are all sampled every 10–40 ms and put into the microcomputer (16-bits).

The fundamental specifications of the experimental bicycle are shown in Table 1. The speed of the moving rubber belt is changeable up to $10\,\mathrm{km\,h^{-1}}$. The lane change experiment is mainly carried out at a constant speed to get the dynamical transient response.

4. *Results and discussions*

At first the experimental bicycle is made to run on a straight line on the moving belt. According to the model analysis, the dynamical characteristics of the closed-loop system depend on a running speed, and moreover the system has a tendency to become unstable when the bicycle slows down. To verify this tendency, the bicycle is made to slow down gradually from $10\,\mathrm{km\,h^{-1}}$. From the experiment, the bicycle system is stable enough to run smoothly at first. But at a certain speed about $1\text{–}2\,\mathrm{km\,h^{-1}}$, the bicycle system becomes oscilatory and then unstable.

The lane change experiment is carried out when the bicycle runs constantly at $5\,\mathrm{km\,h^{-1}}$. The transient responses of input and output variables are observed, when a step input signal is added to the course reference signal. Figures 4–7 show the experimental results of the transient responses of lateral position $y(t)$, roll angle $\theta(t)$, handlebar steer angle $u_s(t)$ and on-board mass position $hu_1(t)$. The results of theoretical calculations are also shown in these figures.

Figure 4 shows at first the results in the case B explained in Section 2.3 when the bicycle is controlled only by handlebar steering. The on-board mass is fixed to the bicycle body. Even in this case, the bicycle is able to run successfully in accordance with the theoretical considerations in the stability analysis.

Figures 5 and 6 show the results in the case C when the bicycle is controlled not only by handlebar steering but also by on-board mass moving. The on-board mass moves in proportion to the lateral deviation of the previewed point, and the feedback gain hk_{ly} is changed from zero to $0.1\,\mathrm{m\,m^{-1}}$ in Fig. 5, and to $0.2\,\mathrm{m\,m^{-1}}$ in Fig. 6. The other values in these figures are equal to those in Fig. 4. The time required for the lane change with on-board mass action is shorter than that without on-board mass action. In other words, an upper body leaning action has a function to shorten the time for lane change. On the other hand, the values of handlebar steer angle and roll angle responses become larger when the movement of on-board mass is larger. These results of experiments are qualitatively and quantitatively well consistent with those of calculations.

Figure 7 shows the results in the case A explained in Section 2.3 when two inputs are independently determined from two outputs. In this case, the bicycle is able to run also successfully, as analyzed previously. But the experimental response waves are more oscilatory than those in the other cases. It is mainly because there is mechanical backlash and a time delay between the control force and the resultant position of the on-board mass.

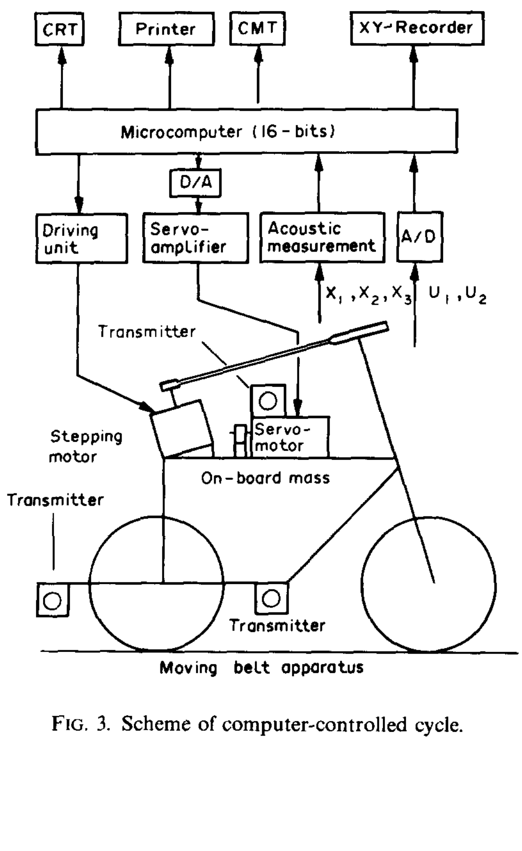

FIG. 3. Scheme of computer-controlled cycle.

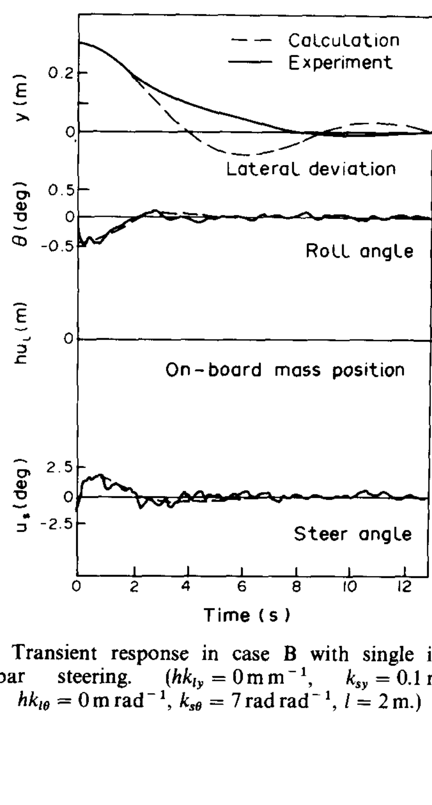

FIG. 4. Transient response in case B with single input of handlebar steering. ($hk_{ly} = 0\,\mathrm{m\,m^{-1}}$, $k_{sy} = 0.1\,\mathrm{rad\,m^{-1}}$, $hk_{l\theta} = 0\,\mathrm{m\,rad^{-1}}$, $k_{s\theta} = 7\,\mathrm{rad\,rad^{-1}}$, $l = 2\,\mathrm{m}$.)

From these results, it can be said that the automatically tracking and balancing robot bicycle runs successfully with the operations of handlebar steering and upper body leaning. Moreover according to the consistency of the experimental results with the theoretical ones, the model of cycle presented in this paper is simple but good enough to represent the real bicycle system, at least for these experimental conditions, e.g. at relatively low speeds 2–10 km h^{-1}.

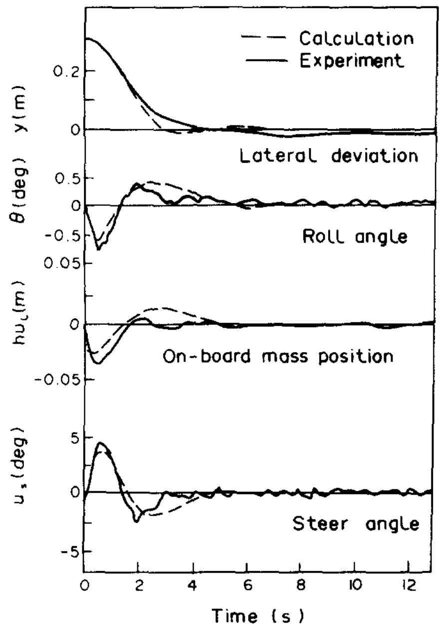

FIG. 5. Transient response in case C with double inputs. ($hk_{ly} = 0.1\,\mathrm{m\,m^{-1}}$, $k_{sy} = 0.1\,\mathrm{rad\,m^{-1}}$, $hk_{l\theta} = 0\,\mathrm{m\,rad^{-1}}$, $k_{s\theta} = 7\,\mathrm{rad\,rad^{-1}}$, $l = 2\,\mathrm{m}$.)

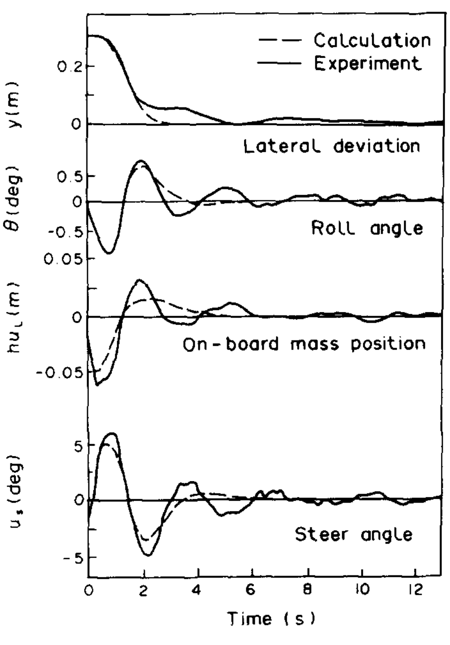

FIG. 6. Transient response in case C with double inputs. ($hk_{ly} = 0.2\,\mathrm{m\,m^{-1}}$, $k_{sy} = 0.1\,\mathrm{rad\,m^{-1}}$, $hk_{l\theta} = 0\,\mathrm{m\,rad^{-1}}$, $k_{s\theta} = 7\,\mathrm{rad\,rad^{-1}}$, $l = 2\,\mathrm{m}$.)

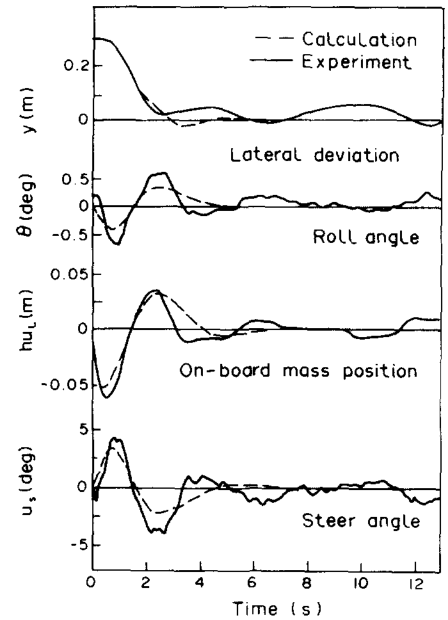

FIG. 7. Transient response in case A with double inputs, independently compensated. ($hk_{ly} = 0.2\,\mathrm{m\,m^{-1}}$, $k_{sy} = 0\,\mathrm{rad\,m^{-1}}$, $hk_{l\theta} = 0\,\mathrm{m\,rad^{-1}}$, $k_{s\theta} = 7\,\mathrm{rad\,rad^{-1}}$, $l = 2\,\mathrm{m}$.)

5. *Conclusions*

This paper shows one of the design approaches of automatically tracking and balancing a robot bicycle. According to the theoretical analysis of rider and cycle system, the experimental bicycle has been made, which is controlled by a robotical rider instead of a human rider. The experimental results of the lane change show a good running property and both qualitatively and quantitatively a good fit with the theoretical results, at relatively low speeds.

In the future, a driving robot realized with a microcomputer is expected to be one of the useful experimental methods for the researches of good handling qualities for vehicles, because of the flexibility of microcomputer control. For this purpose, data of real rider behaviour must be accumulated and applied to this experimental system, and also the rather perfect dynamical model of cycle, including fork assembly and tire properties, must be analyzed for comparing its results with experimental ones, over a wider range of speeds.

References

Van Lunteren, A. and H. G. Stassen (1969). On-line parameter estimation of the human transfer function in a man–bicycle system. *Proc. 4th IFAC Congress*, Warsaw, paper 70.3, p. 17.

Van Zytveld, P. J. (1975). A method for the automatic stabilization of an unmanned bicycle. Thesis, Department of Aeronautics and Astronautics, Stanford University.

Weir, D. H. and J. W. Zellner (1979). Lateral/directional motorcycle dynamics and rider control. SAE paper 780304.

Whipple, F. J. W. (1899). The stability of the motion of a bicycle. *Quart. J. Pure Appl. Math.*, **30**, 312.