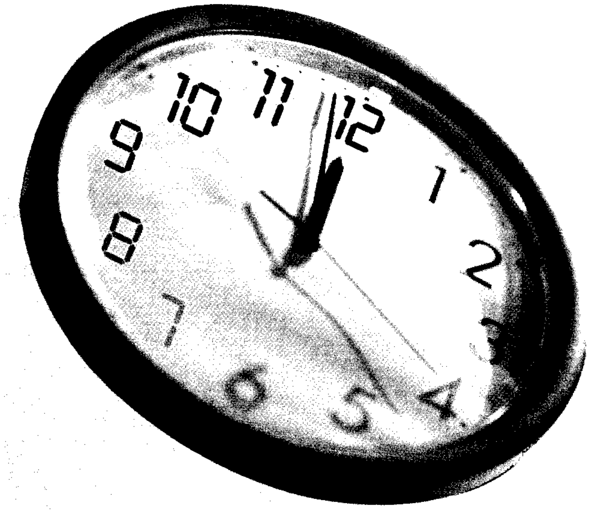
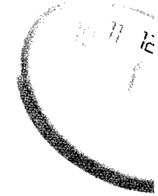




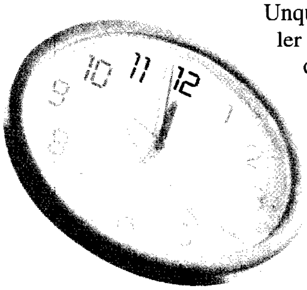
# *Technique for Enhancing the Performance of* Discretized Controllers


Daniel Raviv
and Eddy W. Djaja


Digital control design through discretizing an analog controller has been the topic of much controversy [1]. The problem is as follows: given a process $G_p(s)$, sensor $H(s)$ and a presumably well designed analog controller $G_c(s)$ (Fig. 1a), find a digital controller $D(z)$ (Fig. 1b) which produces closed-loop behavior similar to the analog system both in the time and frequency domains.

At relatively low sampling rates, having a discretized controller without zero order hold (ZOH) compensation may lead to a degraded performance of the closed-loop system (and may cause instability). An appropriate design that compensates for the ZOH effects is desired.

## The Proposed Method



Unquestionably, analog control design followed by controller discretization is far more convenient than direct digital control design for the main reason that the sampling period value T affects the design process only at the final phase and not up front [2]. Our goal in this article is to share with the readers a simple and highly practical ZOH compensation technique that we have successfully applied to many "benchmark" problems studied by us and other researchers, and to other design problems. We have found is that adding this compensation to a discretized controller, which maintains closed-loop stability, results in noticeably improved performance.



The method is based on adding a pole-zero pair in the Z-plane to the digital controller obtained through some suitable discretization methods, such as Tustin (bilinear) transformation, with or without prewarping. The additional pole and zero partially compensate for the low and mid-frequencies phase and gain frequency response effects contributed by the ZOH.

As is well known, the contributions of the ZOH and $G_p(s)$ to the exact discrete time pulse transfer function are not separable. Yet, generally speaking, regardless of the $G_p(s)$ effect, the ZOH causes a delay of approximately $T/2$ as shown intuitively in Fig. 2.

It has been shown that digital controller based closed-loop systems can be improved using a predictor algorithm [9]. The input signal to the


*The authors (eddy.djaja@mail.publix.com and ravivd@acc.fau.edu) are with the Robotics Center and Electrical Engineering Department, Florida Atlantic University, Boca Raton, Florida. Raviv is also affiliated with the National Institute of Standards and Technology, Gaithersburg, Maryland.*



0272-1708/99/$10.00©1999IEEE

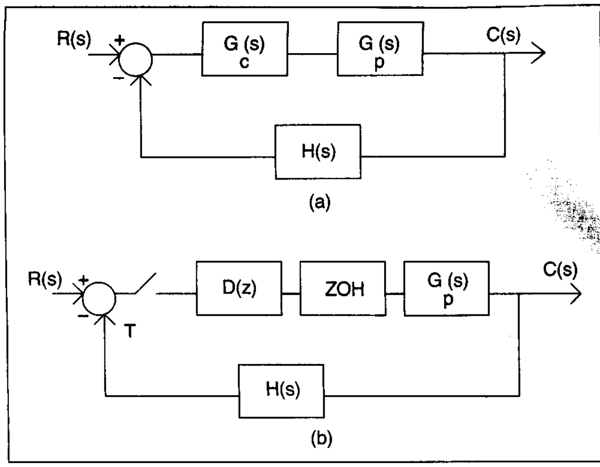



*Fig. 1. (a) The analog closed-loop control system, (b) The digital closed-loop control system.*

plant is a predicted one based on previous samples. However the compensator for the delay is controller and sampling time dependent.

A pole-zero compensation

$$C(z)=\frac{2z}{z+1} \tag{1}$$

shown in Fig. 3 provides a phase of $\omega T/2$ which exactly cancels the frequency phase response of the ZOH as obtained from $(1-e^{-sT})/s$. The ZOH magnitude response is canceled at frequencies for which

$$\tan\frac{\omega T}{2}\approx\frac{\omega T}{2}.$$

It is interesting to note that this cancellation (up to a scale factor of $1/T$) is the inverse of the Tustin transformation of the first order Pade approximation to the so-called ZOH transfer function [3]:

$$\frac{1-e^{-sT}}{s}\approx\frac{T}{1+\frac{sT}{2}} \tag{2}$$

Then:

$$\left.\frac{T}{1+\frac{sT}{2}}\right|_{s=\frac{2}{T}\frac{z-1}{z+1}}=\frac{T}{2}\frac{z+1}{z}. \tag{3}$$

Higher order Pade approximations of the ZOH are usually discussed in the literature. However, they correspond to more complicated (i.e., of higher degree) digital controllers. The suggested approximation (along with the bilinear transformation) compensates exactly for the ZOH phase lag.

The method does not guarantee a stable closed-loop system since it is independent of the discretization method and the sampling rate. However one can investigate the effects of the sampling rate by applying the polynomial root locus [4]. For a given stable analog closed-loop system, a necessary condition for the proposed method to be used is that the characteristic polynomial of the discretized system be such that

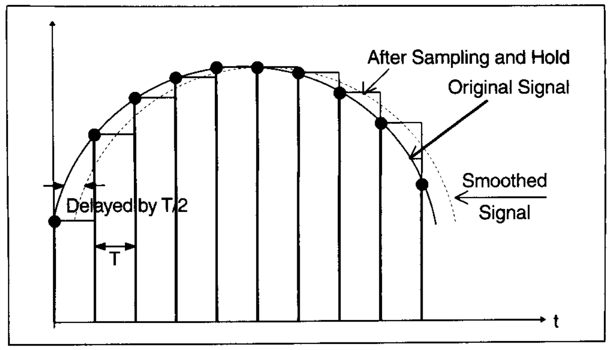



*Fig. 2. A reconstructed signal using ZOH and its smoothed approximation.*

$$1+\left(\frac{2z}{z+1}\right)D'(z)(1-z^{-1})Z\left\{\frac{G_p(s)}{s}\right\}=0, \tag{4}$$

where $D'(z)$, an arbitrary discretized version of $G_c(s)$, has all the polynomial roots inside the unit circle.

In cases where the proposed compensation (1) causes closed-loop instability, a modified ZOH compensation of the following form is to be considered:

$$C'(z)=\frac{2(z-\varepsilon)}{z+1-2\varepsilon}, \tag{5}$$

where $\varepsilon$ is a small positive constant. For closed-loop stability, the characteristic polynomial implied by:

$$1+\left(\frac{2(z-\varepsilon)}{z+1-2\varepsilon}\right)D'(z)(1-z^{-1})Z\left\{\frac{G_p(s)}{s}\right\}=0 \tag{6}$$

must have all roots inside the unit circle. Note that this modified compensation preserves the dc gain of the controller. Fig. 4 il-

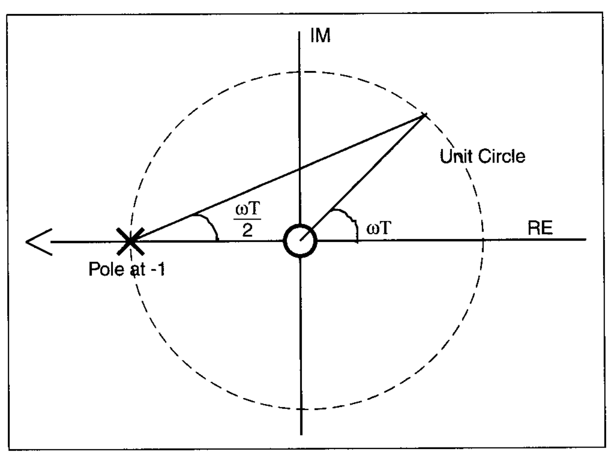



*Fig. 3. The location of pole and zero of ZOH compensator in the Z-domain.*

| Table 1 | | | |
|---|---|---|---|
| | D′(z) | Multiplier | D(z) |
| T = 0.01 s | $\frac{0.0130z-0.0120}{z-0.9990}$ | $\frac{2z}{z+1}$ | $\frac{0.0260z^2-0.0240z}{z^2+0.0010z-0.9990}$ |
| T = 0.05 s | $\frac{0.0150z-0.0100}{z-0.9950}$ | $\frac{2z}{z+1}$ | $\frac{0.0299z^2-0.0200z}{z^2+0.0050z-0.9950}$ |
| T = 0.1 s | Unstable $\frac{0.0174z-0.0075}{z-0.9900}$ | $\frac{2z}{z+1}$ | Unstable $\frac{0.0348z^2-0.0150z}{z^2+0.0100z-0.9900}$ |
| T = 0.1 s | Unstable $\frac{0.0174z-0.0075}{z-0.9900}$ | $\frac{2(z-0.2)}{(z+0.6)}$ | $\frac{0.0348z^2-0.0219z+0.0030}{z^2-0.3900z-0.5940}$ |

lustrates the effect of the new pole-zero pair of the ZOH compensation.

The paper by Hori, et al. [10], targets matched pole-zero models for ZOH compensation. Based on the frequency domain in the paper, it does not seem to improve the phase of the system (compared to other methods shown).

## Summary of the Design Procedure

1. Select a suitable $S$ to $Z$ transformation, e.g., Tustin (bilinear) transformation, and sampling time $T$ to discretize the existing analog controller $G_c(s)$ to obtain $D'(z)$.

2. Multiply the result of step (1) by $C(z) = 2z/(z+1)$ to obtain: $D(z) = C(z)D'(z)$.

2a. Match the open-loop dc gain of the digital control system to the analog system.

3. Check the closed-loop stability in Z-domain using equation (4). If stable, observe the closed-loop system performance.

4. If in step (3) the closed-loop system becomes unstable, then:

4a. Try $C'(z) = 2(z-\varepsilon)/(z+1-2\varepsilon)$ for small $\varepsilon$>0 to obtain $D(z) = C'(z)D'(z)$; use $\varepsilon$ as small as possible to guarantee better performance at higher frequencies.

4b. Match the open-loop dc gain of the analog and discretized systems.

4c. Check the closed-loop stability in Z domain using (6). If stable, observe the closed-loop system performance.

5. If the closed-loop system in step (4c) is still unstable, use other methods.

Note that this method is applicable for cases where (4) and (6) produce stable poles).

## Examples

### a. Lag Compensator [5]

Given the process

$$G_p(s) = \frac{4\times10^6}{s(s+20)(s+200)};\ H(s)=1.$$

The analog controller $G_c(s) = \frac{1}{80}\frac{(s+8)}{(s+0.1)}$ was designed using a Bode diagram to satisfy the following design specifications: (a) velocity error constant, $K_v$ at least 1000 $s^{-1}$, (b) attenuation of all sinusoidal inputs of frequency above 400 rad/s by at least 16, (c) steady-state error of (up to) 1% for sinusoidal inputs with frequencies less than 1 rad/s.

Table 1 is a summary of the discretization results for three different sampling rates. The discretized controller $D'(z)$, using Tustin transformation, is listed in the second column. The third column is the ZOH compensation and the fourth column is the complete controller $D(z)$ using the proposed ZOH compensation. Note that in the third row, with or without the ZOH compensator, the closed-loop system is unstable. The problem is overcome by using the modified compensator shown in the fourth row.

Fig. 5a depicts the closed-loop step responses of the analog system and the digital control system using Tustin with and without ZOH compensation for $T = 0.01$ s. The ZOH compensation design method performance matches the analog system's performance very well, while without the compensation, the system has slightly greater overshoot. When reducing the sampling rate to $T = 0.05$ s, the system with no compensation has very poor performance in the time domain as shown in Fig. 5b. However, with the ZOH

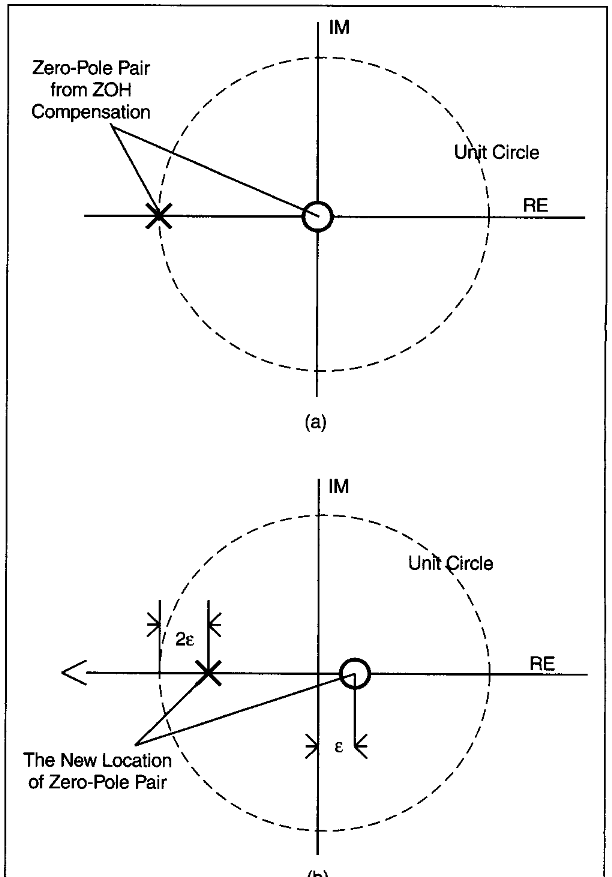



*Fig. 4. (a) Unmodified ZOH compensator, (b) Modified ZOH compensator.*

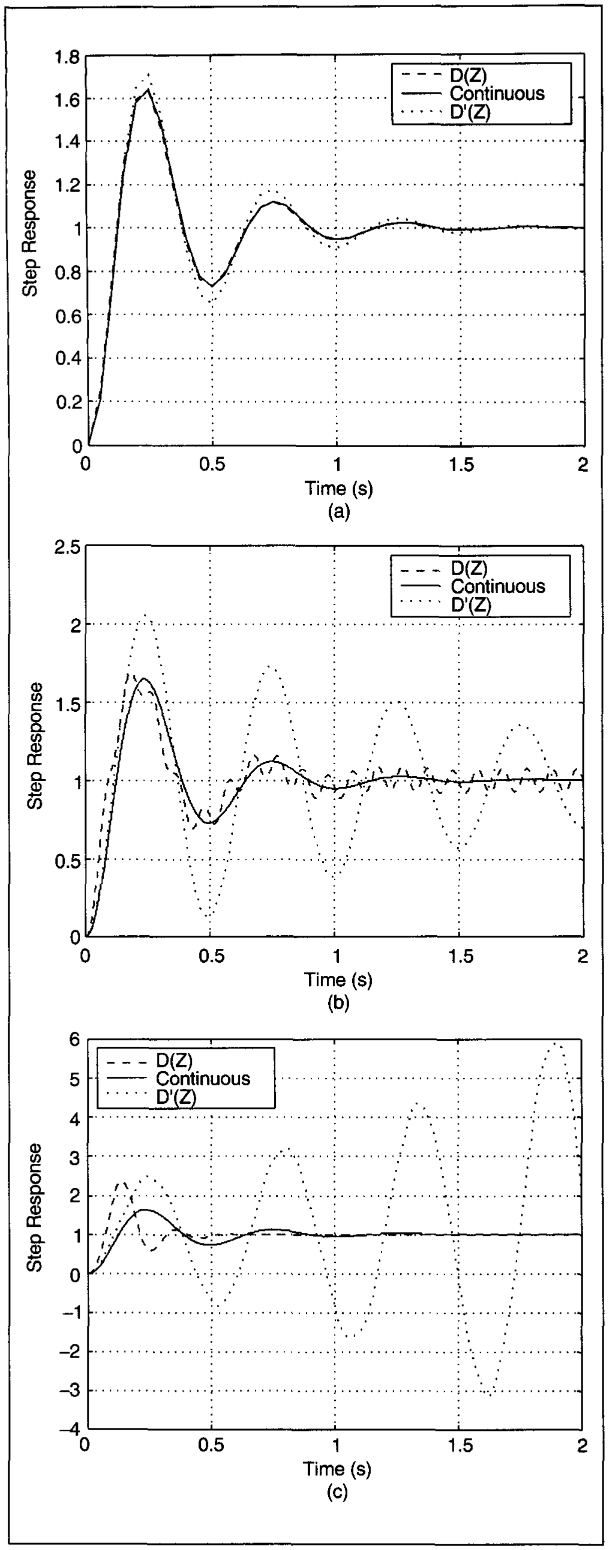



*Fig. 5. Lag Compensation Example. (a) Closed-loop step response with T= 0.01 s; (b) Closed-loop step response of with T=0.05 s; (c) Closed-loop step response with T=0.1 s.*

compensation, the system performance is close to that of the analog system.

Reducing the sampling rates even further to $T = 0.1$ s results in an unstable closed-loop system with and without compensation as shown in Table 1. Using the modified ZOH compensation with $\varepsilon = 0.2$, the closed-loop system becomes stable. The step response shown in Fig. 5c demonstrates relatively good performance, while the Tustin transformation has made the closed-loop system unstable.

### b. Lead-Lag Compensation [5]

Given

$$G_p(s) = \frac{1000}{s\left(1+\frac{s}{10}\right)\left(1+\frac{s}{250}\right)}; \quad H(s)=1,$$

the analog controller

$$G_c(s) = \frac{\left(1+\frac{s}{4.5}\right)\left(1+\frac{s}{10}\right)}{\left(1+\frac{s}{0.1}\right)\left(1+\frac{s}{110}\right)}$$

has been designed using a Bode diagram to meet several specifications: (a) phase margin of at least 50°, (b) velocity error constant, $K_v$ at least 1000 $s^{-1}$, (c) attenuation of the input noise at 60 Hz and above by a factor of 100, and (d) steady state error for frequencies less than 1 rad/s less than 1%. Using Tustin transformation applied to $G_c(s)$ provided with $T = 0.01$ s, the following digital controller is obtained:

$$D'(z) = \frac{0.6597z^2 - 1.2897z + 0.6300}{1.00z^2 - 1.2893z + 0.2900}.$$

With the ZOH compensation of $\frac{2z}{z+1}$, the digital controller becomes

$$D(z) = \frac{1.3194z^3 - 2.5793z^2 + 1.26z}{z^3 - 0.2893z^2 - 0.9993z + 0.2900}.$$

Fig. 6 depicts the closed-loop step response of the analog system and the digital control system using the Tustin transformation with and without ZOH compensation. The effect of the ZOH compensation is evident.

### c. Katz's example [6]

Given

$$G_p(s) = \frac{863.3}{s^2}$$

the analog controller

$$G_c(s) = 2940\frac{(s+29.4)}{(s+294)^2}$$

has been designed to meet the following closed-loop system specifications: (a) the maximum phase lag at $f = 3$ Hz should not

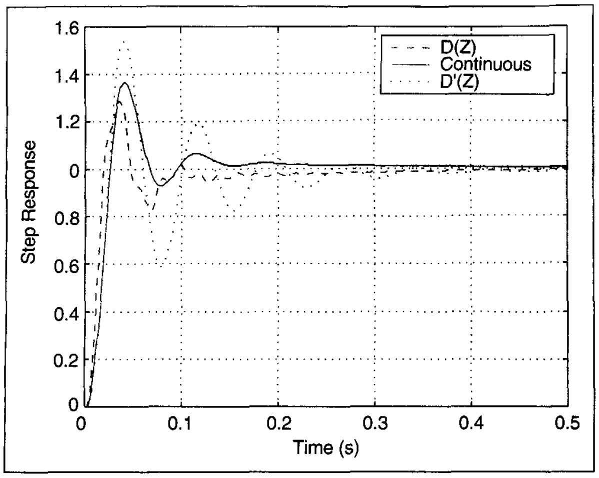



*Fig. 6. Closed-loop step response of Example (b). T = 0.01 s.*

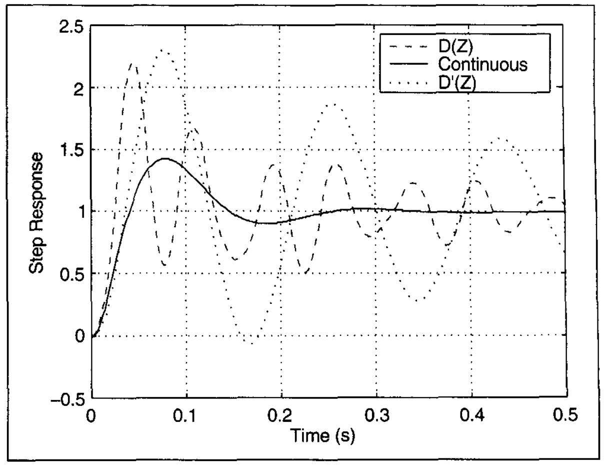



*Fig. 7. Closed-loop step response of Example (c). T = 0.03 s.*

be more than 13°, (b) at any given frequency the closed-loop gain should not exceed 5 dB beyond the closed-loop dc gain, and (c) maximum tracking error due to an input disturbance moment of 0.028 Nm should be 0.01 rad. Taking the Tustin transformation with pre-warping at the sampling time T = 0.03 s of $G_c(s)$, the following digital controller is obtained by [6]:

$$D'(z) = \frac{1.8958z^2 + 1.1685z - 0.7273}{z^2 + 1.1653z + 0.3395}.$$

(Note that Katz's pre-warping, which warps the frequency of both pole and zero [6], differs from the "regular" pre-warping, which warps at a single frequency.)

This was the only discretization method that resulted in a stable closed-loop system at such low sampling rate [3]. The same controller with the additional proposed ZOH compensation is:

$$D(z) = \frac{3.7916z^3 + 2.3369z^2 - 1.4546z}{z^3 + 2.1653z^2 + 1.5047z + 0.3395}.$$

Adding this ZOH compensation to the $D(z)$, the system output at the sampling instants is significantly closer to the original analog step response, shown in Fig. 7. The proposed method significantly improves the response compared to Tustin transformation with pre-warping. Even though the suggested compensator does not perform better than those obtained by Evans-Kennedy [7] and Keller-Anderson[2] methods, the simplicity of the proposed method is quite attractive. The methods in [2] and [7] are sophisticated and difficult to use.

### d. Rattan's Example [8]

Given

$$G_p(s) = \frac{10}{s(s+1)}$$

$$G_c(s) = \frac{1 + 0.416s}{1 + 0.319s},$$

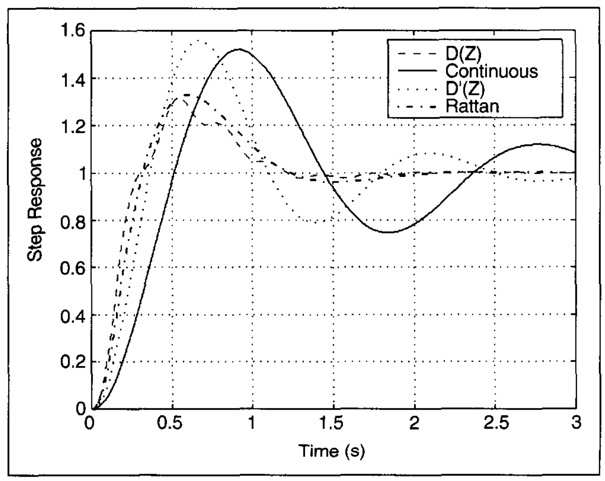



*Fig. 8. Closed-loop step response of Example (d). T = 0.15 s.*

a digital controller proposed by Rattan (no design specifications are available in his example) is:

$$D_{\text{Rattan}}(z) = \frac{3.436z - 2.191}{z + 0.2390}.$$

Using Tustin transformation to discretize the analog controller, the digital controller becomes:

$$D'(z) = \frac{2.294z - 1.5935}{z - 0.2991}.$$

Applying the modified ZOH compensation with $\varepsilon = 0.1$ to $D'(z)$:

$$D(z) = \frac{4.5888z^2 - 3.6459z + 0.3187}{z^2 + 0.5009z - 0.2393}.$$

In Rattan's example, the closed-loop frequency matching in the W-domain has shown better performance than Tustin and pre-warping methods at $T = 0.15$ s [8]. Fig. 8 shows that the step response of the closed-loop system using the modified ZOH compensation has slightly lower overshoot than Rattan's method and by far better than Tustin transformation without compensation.

## Conclusion

This paper has introduced a method to partially compensate for the ZOH effects of a closed-loop digital control system. By multiplying a given discretized controller in the Z-domain by a pole-zero pair, significant closed-loop performance improvement has been achieved in many design examples. We hope that practicing engineers find this method useful.

## Acknowledgments

While investigating active vision control techniques, we came across this compensation method. We would like to thank the National Science Foundation, Division of Information, Robotics and Intelligent Systems (Grant # IRI-9115939) for partial support of this research. The authors would like to thank Dr. Roth for his excellent suggestions, as well as the reviewers and the associate editor who suggested very constructive modifications of this article.

## References

[1] B.D.O. Anderson, "Controller design: moving from theory to practice," *IEEE Control Systems*, pp. 16-25, Aug. 1993.

[2] J.P. Keller and B.D.O. Anderson, "A new approach to the discretization of continuous-time controllers," *IEEE Trans. Automat. Contr.*, vol. 37, pp. 214-223, Feb. 1992.

[3] G.F. Franklin, J.D. Powell, and A.E. Naeini, *Feedback Control of Dynamic Systems*, Reading, MA: Addison Wesley, 1991.

[4] C.H. Wang, W.Y. Wang, and T.T. Lee, "Sampling-time effects of higher-order digitisations and their applications in digital redesign," *IEE Proc. Control Theory Appl.*, vol. 141, no. 2, March 1994.

[5] J.L. Bower and P.M. Schultheiss, *Introduction to Design of Servomechanisms*, New York: Wiley, 1958.

[6] P. Katz, *Digital Control Using Microprocessors*, Englewood Cliffs, NJ: Prentice-Hall, 1981.

[7] R.A. Kennedy and R.J. Evans, "Digital redesign of a continuous controller based on closed-loop performance," in *Proc. 29th Conf. Decision Control*, pp. 1898-1901, 1990.

[8] K.S. Rattan, "Digitalization of existing control systems," *IEEE Trans. Automat. Contr.*, vol. AC-29, pp. 282-285, March 1984.

[9] C.F. Moore, C.L. Smith, and P.W. Murrill, " Improved algorithm for direct digital control," *Instruments and Control Systems*, pp. 70-74, Jan. 1970.

[10] N. Hori, R. Cormier Jr., and K. Kanai, "On Matched Ple-Zero Models," *IEE Proceedings-D*, vol.139, no. 3, pp. 273-278, May 1992.

**Daniel Raviv** received the B.Sc. and M.Sc. degrees in electrical engineering from the Technion in Haifa, Israel, in 1980 and 1982, respectively, and the Ph.D. degree in Electrical Engineering and Applied Physics from Case Western Reserve University, Cleveland, Ohio, in 1987. He is currently an associate professor of Electrical Engineering at Florida Atlantic University, Boca Raton, Florida, and a visiting researcher at the National Institute of Standards and Technology (NIST), Gaithersburg, Maryland. His research interests are in vision-based autonomous navigation, computer vision, and inventive thinking.

**Eddy W. Djaja** received the B.Sc. and M.Sc. degrees in electrical engineering and computer engineering from Florida Atlantic University, Boca Raton, Florida, in 1994 and 1996, respectively. He is currently employed as a database administrator in the Information Systems Department, Publix Super Markets, Inc., Lakeland, Florida.