

Spring 2013

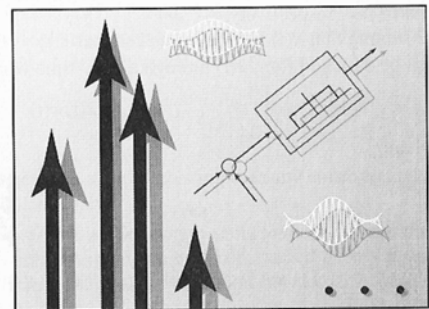
信號與系統 Signals and Systems

Chapter SS-8 Communication Systems

Feng-Li Lian

NTU-EE

Feb13 – Jun13



Figures and images used in these lecture notes are adopted from
“Signals & Systems” by Alan V. Oppenheim and Alan S. Willsky, 1997

Flowchart

Feng-Li Lian © 2013
NTUEE-SS8-Comm-2

Introduction

[\(Chap 1\)](#)

LTI & Convolution

[\(Chap 2\)](#)

Bounded/Convergent

Periodic

FS

[\(Chap 3\)](#)

CT
DT

Aperiodic

FT

CT
DT

[\(Chap 4\)](#)

[\(Chap 5\)](#)

Unbounded/Non-convergent

LT

CT

[\(Chap 9\)](#)

zT

DT

[\(Chap 10\)](#)

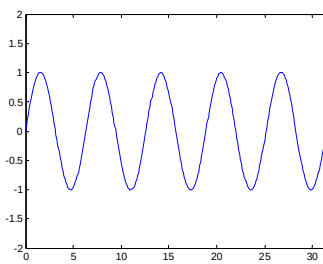
Time-Frequency [\(Chap 6\)](#)

CT-DT [\(Chap 7\)](#)

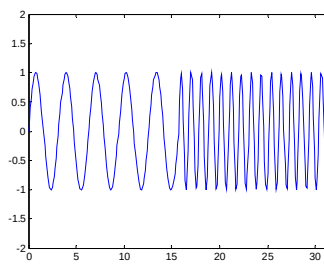
Communication [\(Chap 8\)](#)

Control [\(Chap 11\)](#)

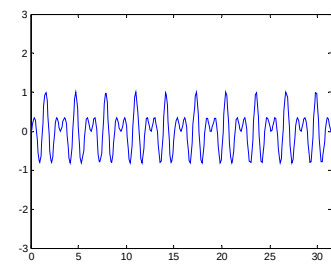
Digital
Signal
Processing [\(dsp-8\)](#)



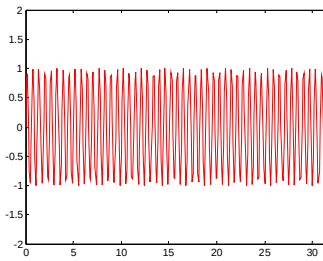
$$x(t) = \sin(t)$$



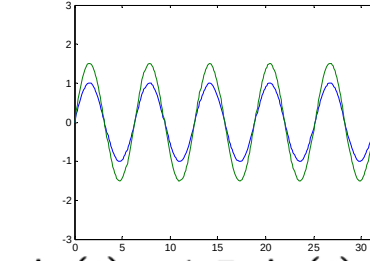
$$\sin(2t) \quad \sin(6t)$$



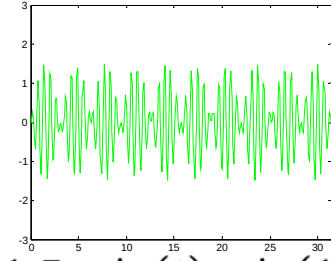
$$z_1 = 1 \cdot \sin(t) \cdot \sin(5t)$$



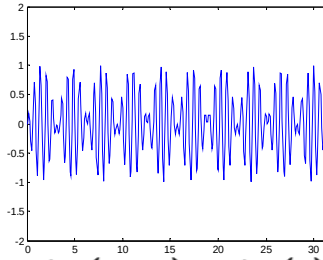
$$c(t) = \sin(10t)$$



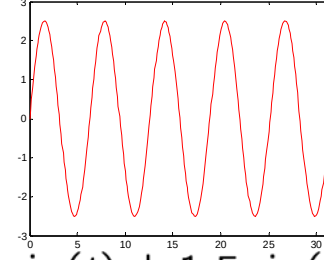
$$\sin(t) \quad 1.5 \sin(t)$$



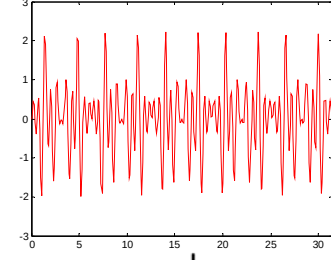
$$z_2 = 1.5 \cdot \sin(t) \cdot \sin(10t)$$



$$\sin(10t) \cdot \sin(t)$$



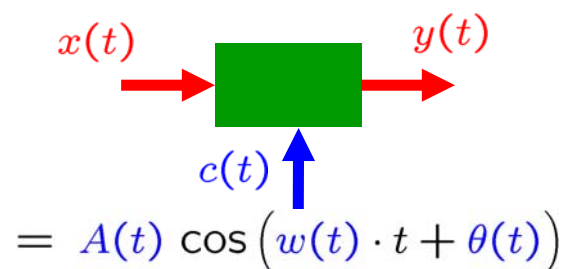
$$\sin(t) + 1.5 \sin(t)$$



$$z_1 + z_2$$

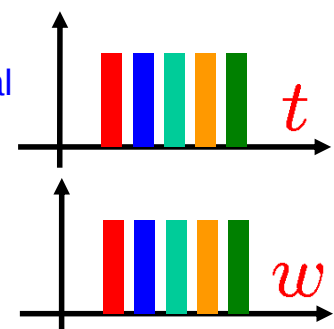
■ Modulation & Demodulation:

- M: Embedding an information-bearing signal into a second signal
- D: Extracting the information-bearing signal
- Methods:
 - > Amplitude Modulation (AM)
 - > Frequency Modulation (FM)



■ Multiplexing & Demultiplexing:

- Simultaneous transmission of more than one signal with overlapping spectra over the same channel
- Methods:
 - > Time-Division Multiplexing (TDM)
 - > Frequency-Division Multiplexing (FDM)

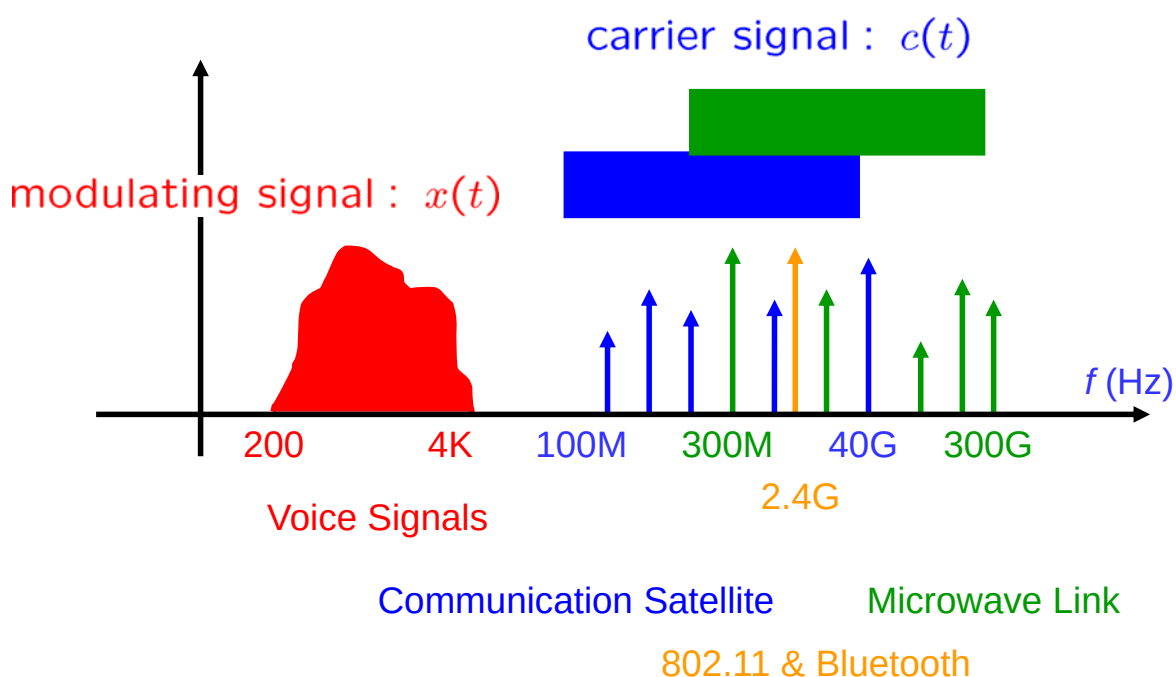


- Complex Exponential & Sinusoidal
Amplitude Modulation & Demodulation
- Frequency-Division Multiplexing
- Single-Sideband Sinusoidal Amplitude Modulation
- Amplitude Modulation with a Pulse-Train Carrier
- Pulse-Amplitude Modulation
- Sinusoidal Frequency Modulation
- Discrete-Time Modulation

Complex Exponential & Sinusoidal Amplitude Modulation & Demodulation

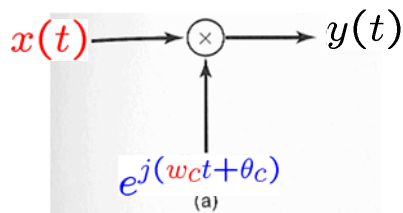
Feng-Li Lian © 2013
NTU-EE-SS8-Comm-6

▪ Signal Frequency Characteristics:



modulated signal : $y(t) = x(t) c(t)$

AM with a Complex Exponential Carrier:

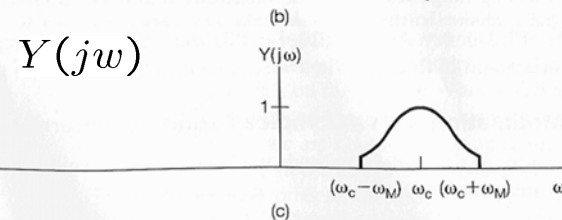
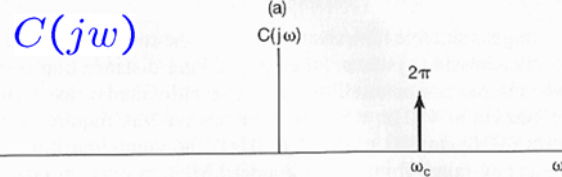
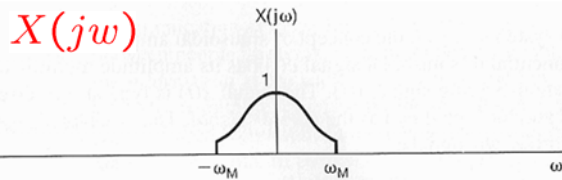


ω_c : carrier frequency

$$c(t) = e^{j(\omega_c t + \theta_c)}$$

$$y(t) = x(t) c(t) = x(t) e^{j\omega_c t}$$

$$\theta_c = 0$$

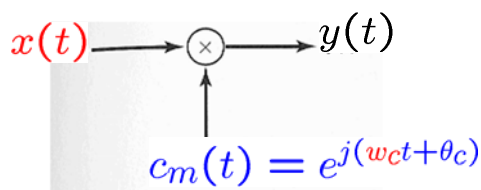


$$C(j\omega) = 2\pi \delta(\omega - \omega_c)$$

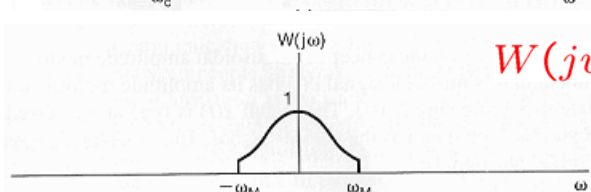
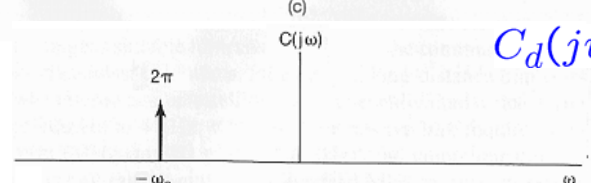
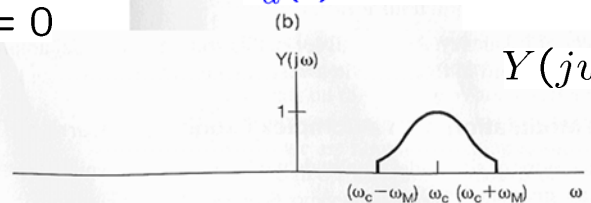
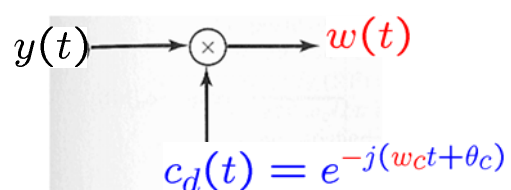
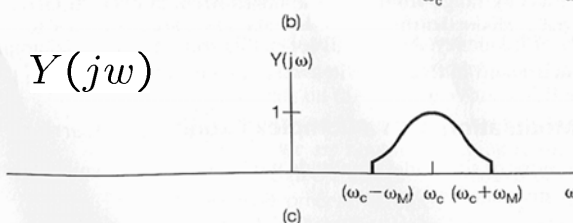
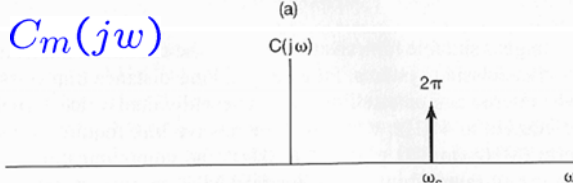
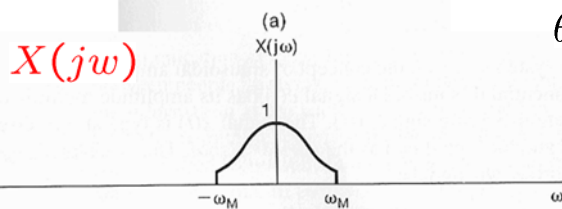
$$Y(j\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\theta) C(j(\omega - \theta)) d\theta$$

$$Y(j\omega) = X(j(\omega - \omega_c))$$

AM with a Complex Exponential Carrier:



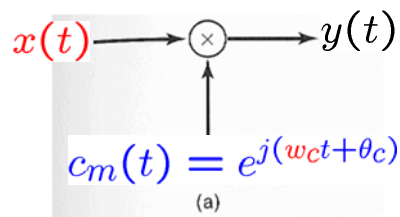
$$\theta_c = 0$$



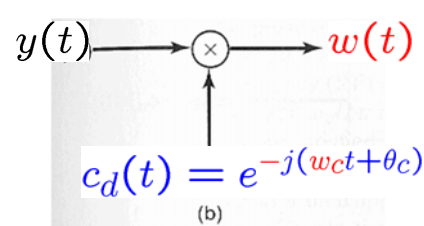
$$C_d(j\omega)$$

$$W(j\omega)$$

AM with a Complex Exponential Carrier:



$$\theta_c = 0$$



$$\begin{aligned} y(t) &= x(t) c_m(t) \\ &= x(t) e^{j\omega_c t} \end{aligned}$$

$$\begin{aligned} w(t) &= y(t) c_d(t) \\ &= y(t) e^{-j\omega_c t} \\ &= x(t) e^{j\omega_c t} e^{-j\omega_c t} \end{aligned}$$

$$\Rightarrow w(t) = x(t)$$

$$Y(j\omega) = X(j(\omega - \omega_c))$$

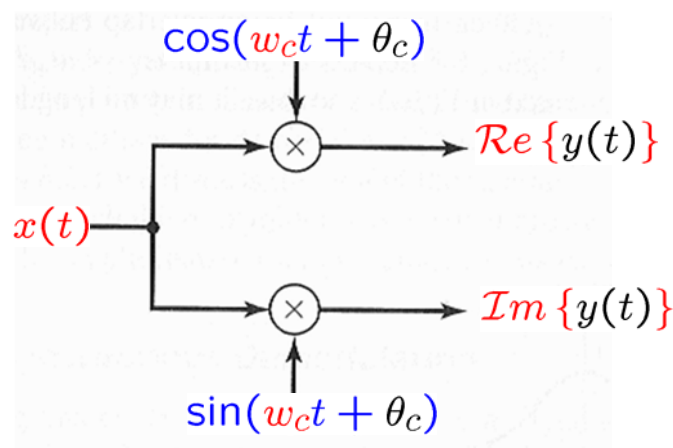
$$W(j\omega) = Y(j(\omega + \omega_c))$$

$$\Rightarrow W(j\omega) = X(j\omega)$$

AM with Sinusoidal Carriers:

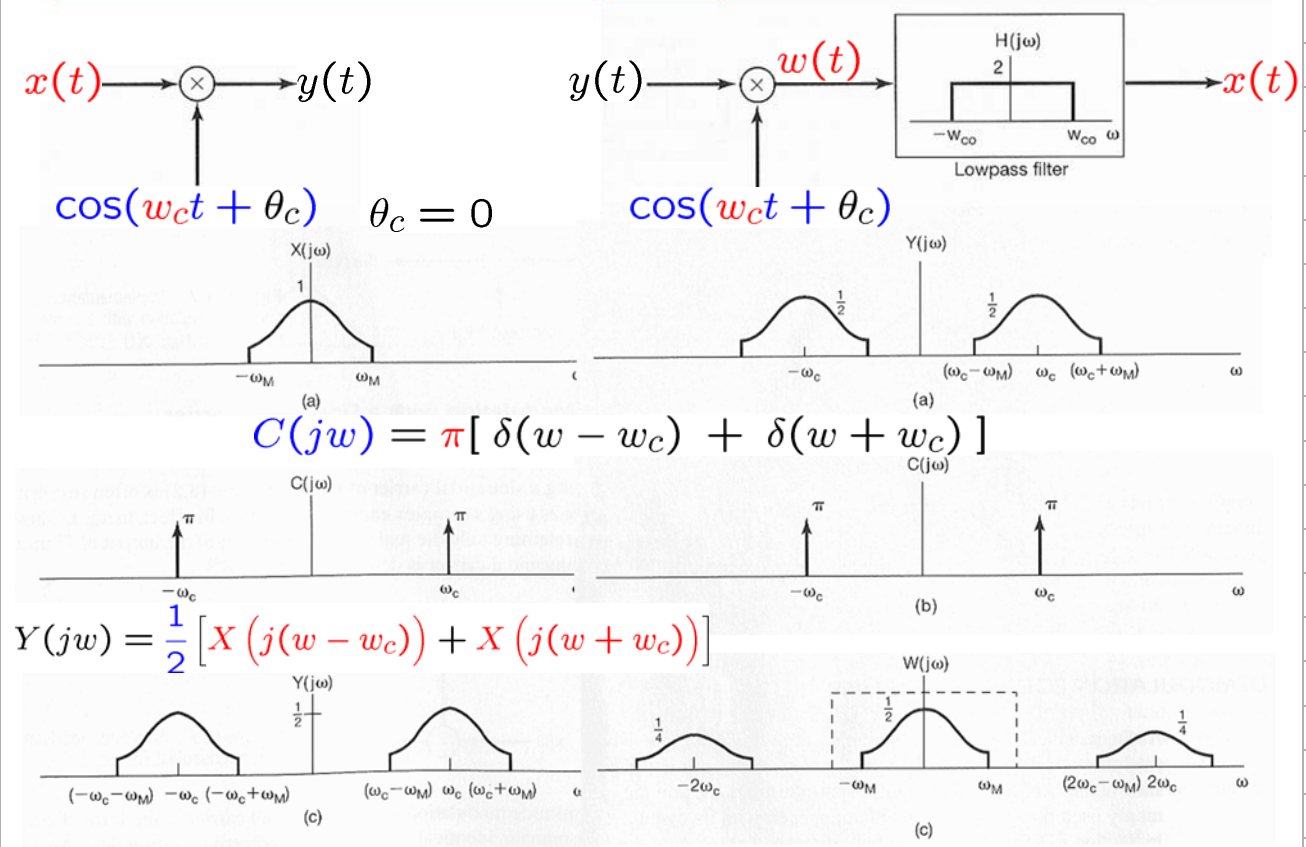
$$c(t) = e^{j\omega_c t} = \cos(\omega_c t) + j \sin(\omega_c t)$$

$$\Rightarrow y(t) = x(t) \cos(\omega_c t) + j x(t) \sin(\omega_c t)$$

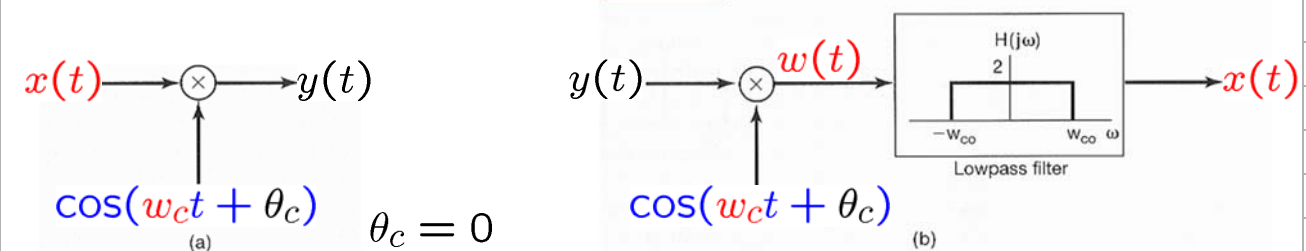


phase difference of $c_1(\cdot), c_2(\cdot)$?

AM with a Sinusoidal Carrier:



AM with a Sinusoidal Carrier:



$$y(t) = x(t) \cos(w_c t)$$

$$w(t) = y(t) \cos(w_c t)$$

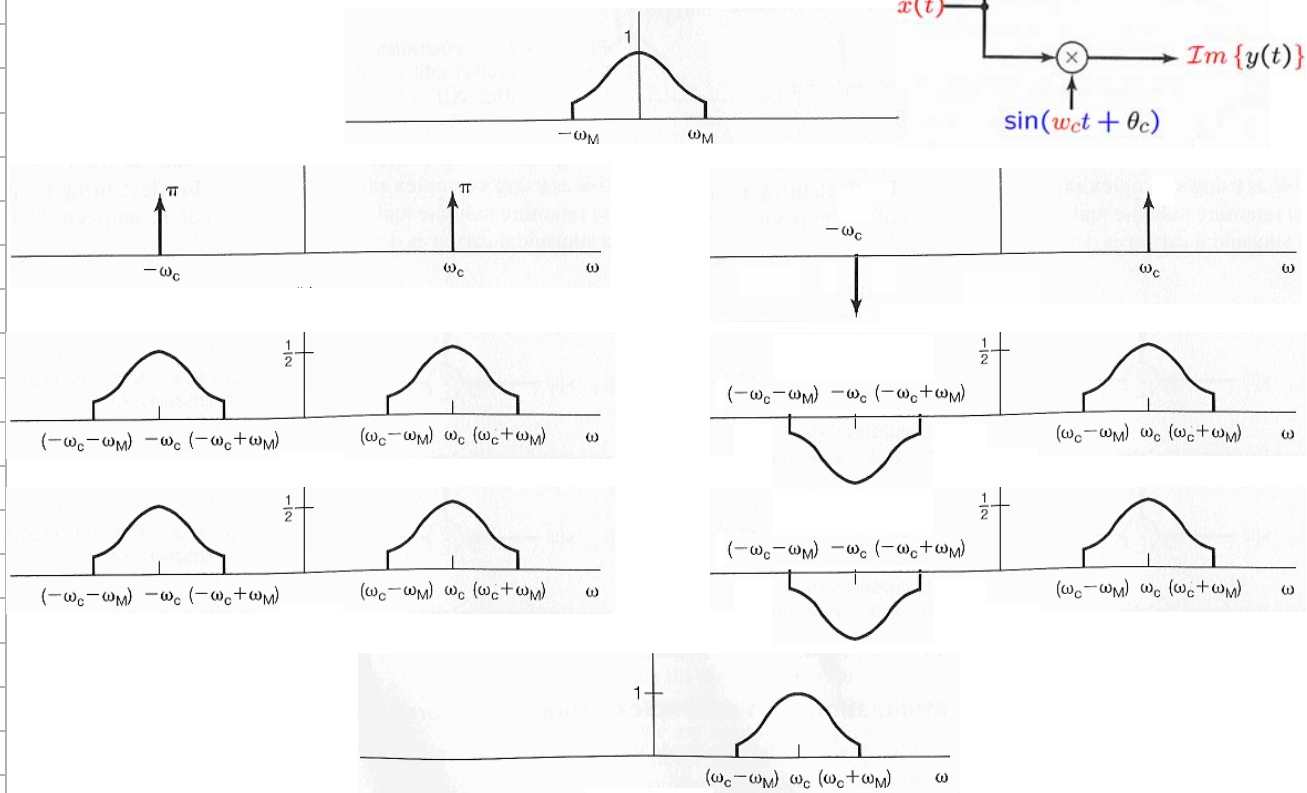
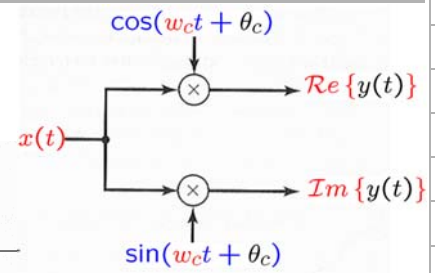
$$\Rightarrow w(t) = x(t) \cos^2(w_c t)$$

$$= x(t) \left[\frac{1}{2} + \frac{1}{2} \cos(2w_c t) \right]$$

$$= \frac{1}{2} x(t) + \frac{1}{2} x(t) \cos(2w_c t)$$

$$c(t) = e^{j\omega_c t} = \cos(\omega_c t) + j \sin(\omega_c t)$$

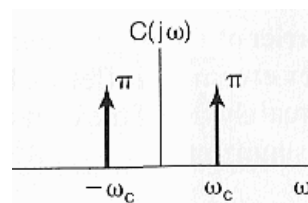
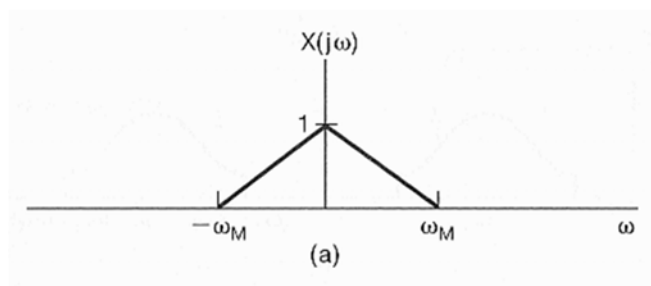
$$y(t) = x(t) \cos(\omega_c t) + j x(t) \sin(\omega_c t)$$



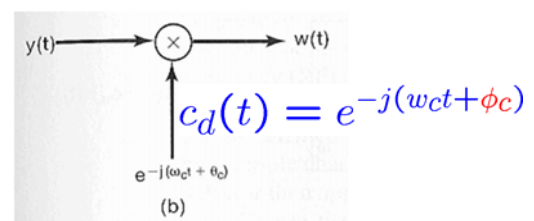
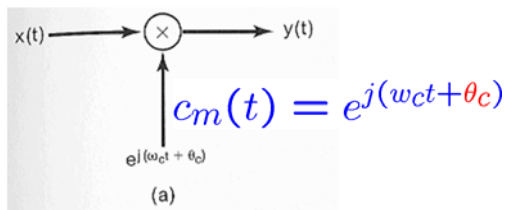
Complex Exponential & Sinusoidal Amplitude Modulation

Overlapping of AM with a Sinusoidal Carrier:

- If $\omega_c < \omega_M$,



■ Not Synchronized in Phase:



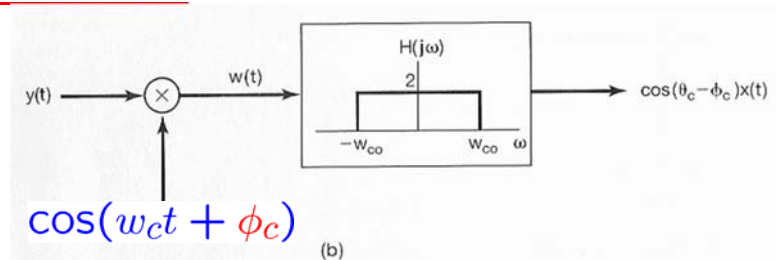
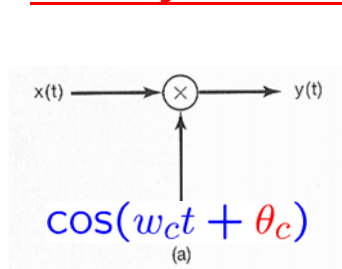
$$\theta_c \neq \phi_c$$

$$\begin{aligned} y(t) &= x(t) c_m(t) \\ &= x(t) e^{j(\omega_c t + \theta_c)} \end{aligned}$$

$$\begin{aligned} w(t) &= y(t) c_d(t) \\ &= y(t) e^{-j(\omega_c t + \phi_c)} \\ &= x(t) e^{j(\theta_c - \phi_c)} \end{aligned}$$

$$\Rightarrow \text{ONLY } |x(t)| = |w(t)|$$

■ Not Synchronized in Phase:



$$y(t) = x(t) \cos(\omega_c t + \theta_c) \quad w(t) = y(t) \cos(\omega_c t + \phi_c)$$

$$\Rightarrow w(t) = x(t) \cos(\omega_c t + \theta_c) \cos(\omega_c t + \phi_c)$$

$$= x(t) \left[\frac{1}{2} \cos(\theta_c - \phi_c) + \frac{1}{2} \cos(2\omega_c t + \theta_c + \phi_c) \right]$$

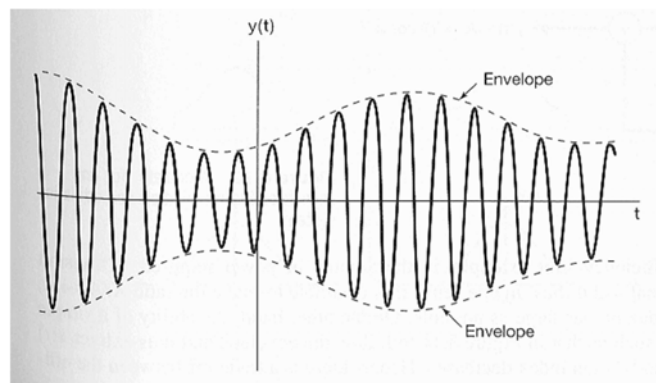
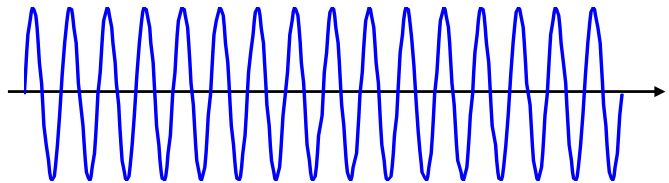
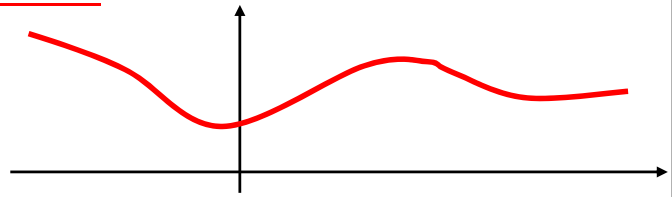
$$= \frac{1}{2} \cos(\theta_c - \phi_c) x(t) + \frac{1}{2} x(t) \cos(2\omega_c t + \theta_c + \phi_c)$$

■ Asynchronous Demodulation:

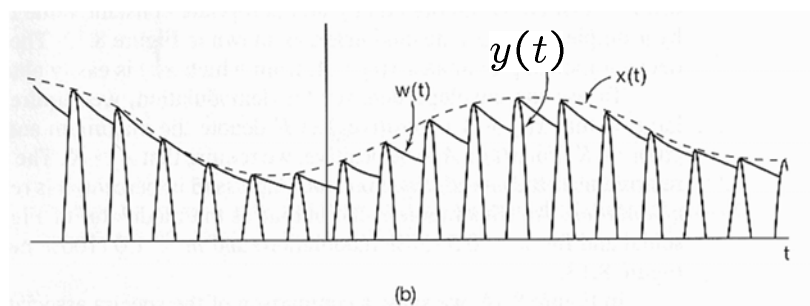
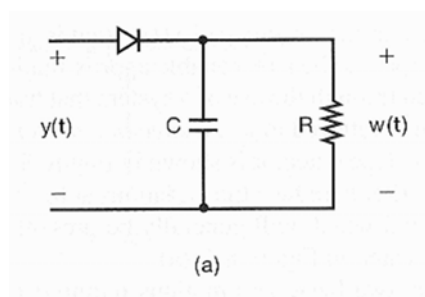
- $w_c \gg w_M$
- $x(t) > 0, \forall t$
 - In audio transmission over a RF channel
 - > w_M : 15 - 20 Hz
 - > $w_c/2\pi$: 500kHz – 2 MHz

$$y(t) = x(t) \cos(w_c t + \theta_c)$$

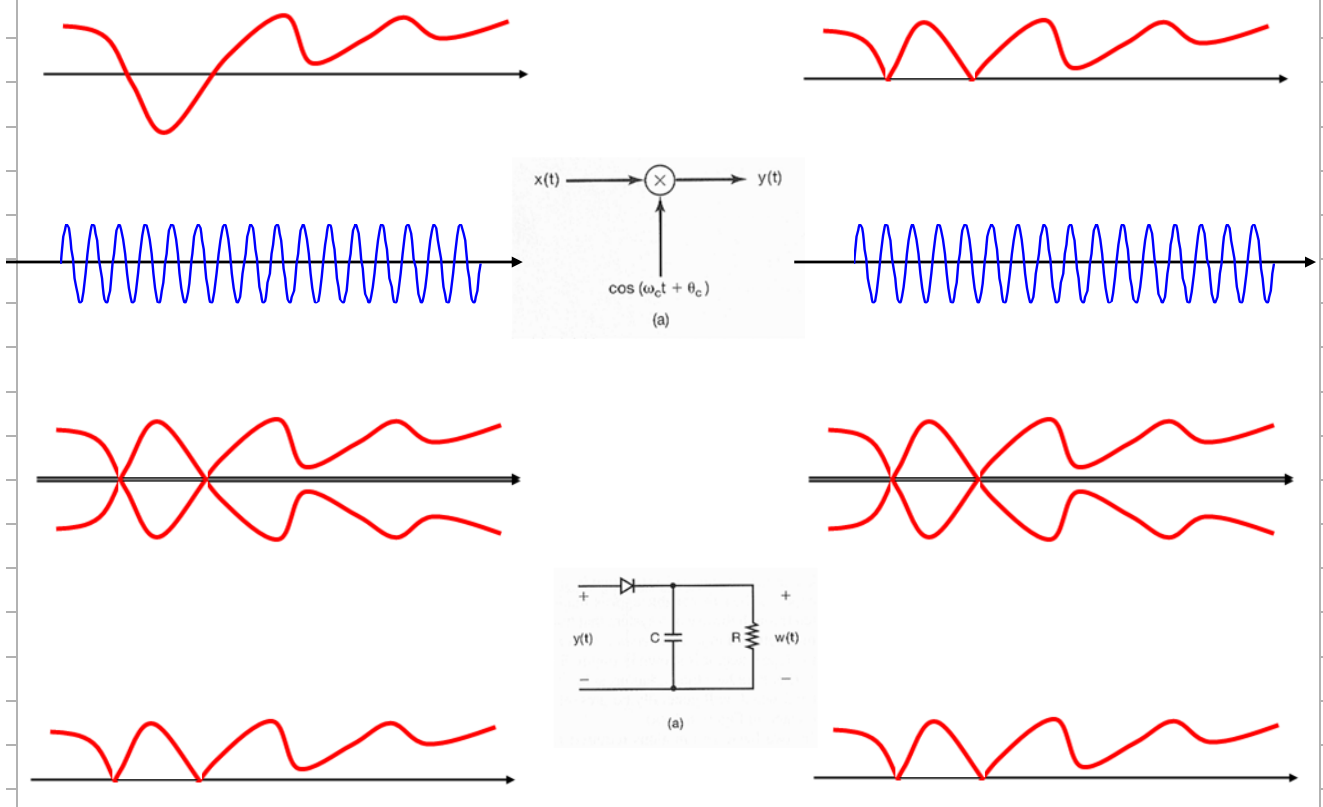
$$\approx x(t)$$



■ Envelope Detector:



Asynchronous Demodulation:



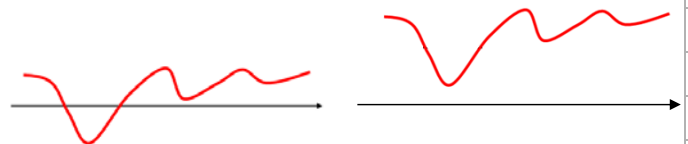
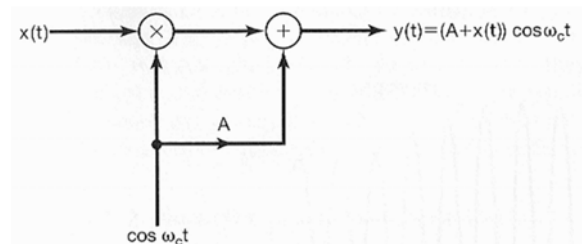
Asynchronous Demodulation:

- $\omega_c \gg \omega_M$

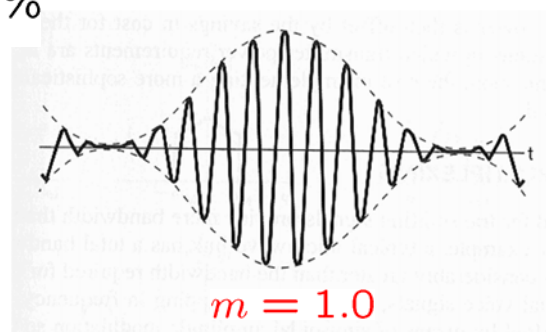
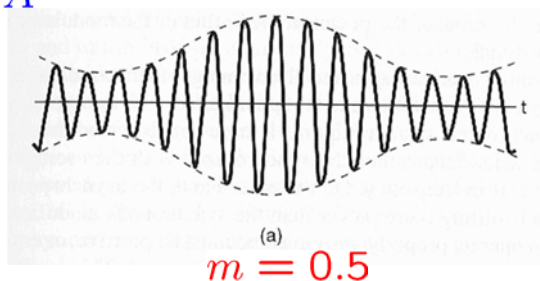
- $x(t) > 0, \forall t$

If not, $x(t) \rightarrow x(t) + A > 0$

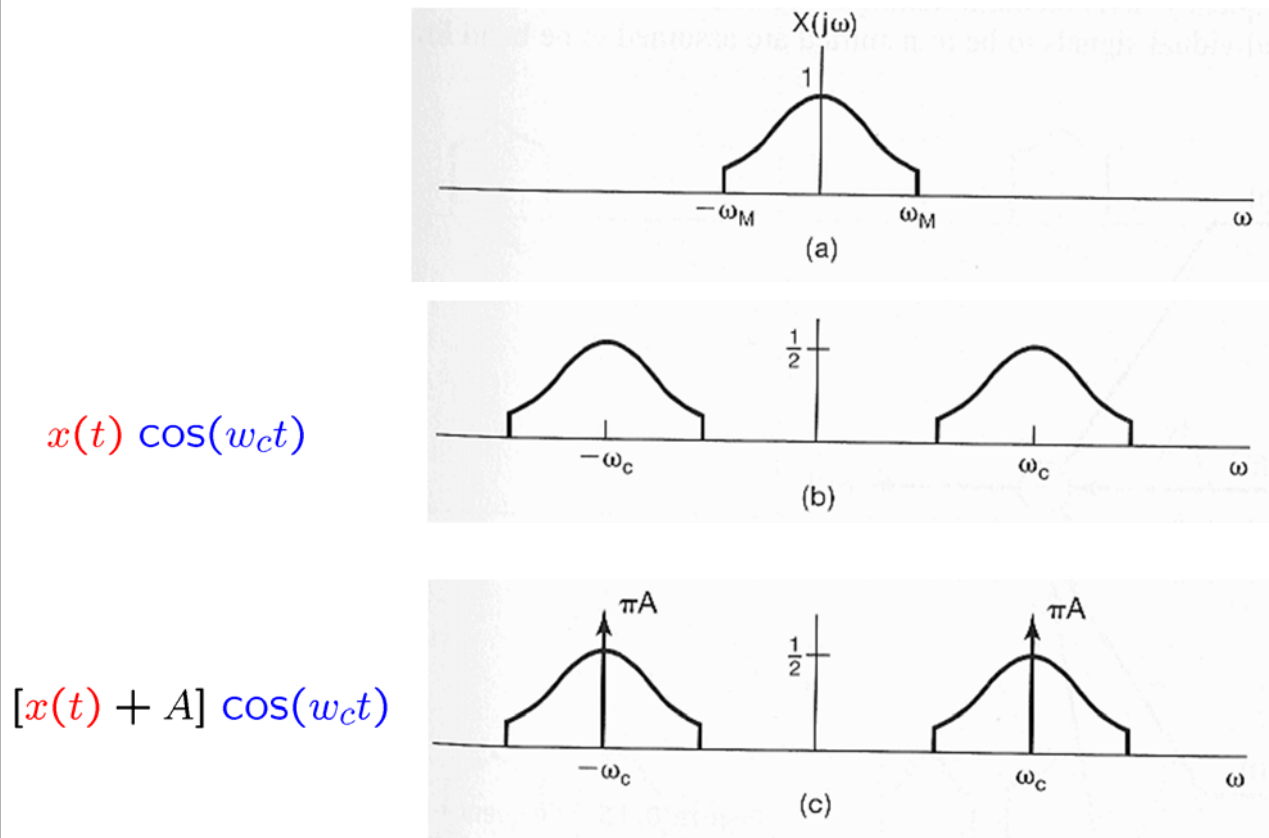
$$A \geq K, |x(t)| \leq K$$



- $\frac{K}{A}$: modulation index m , in %



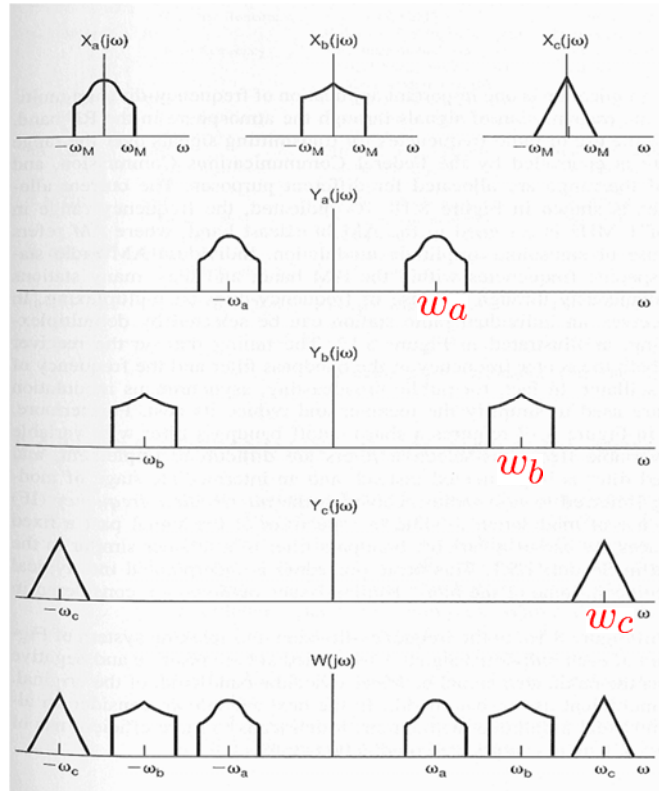
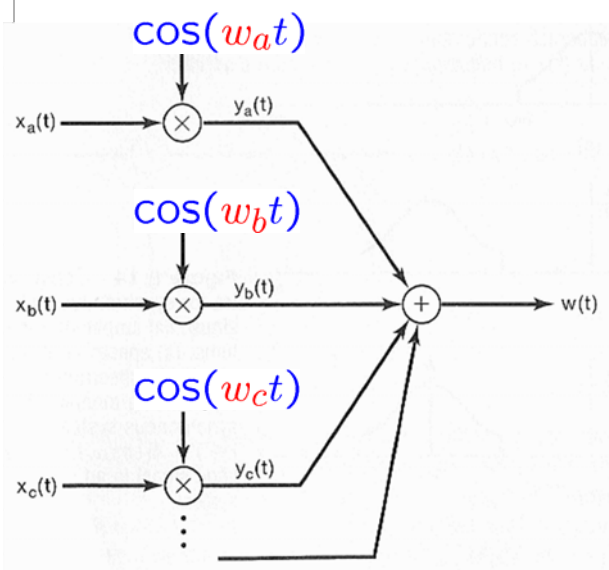
■ Synchronous & Asynchronous Demodulation:



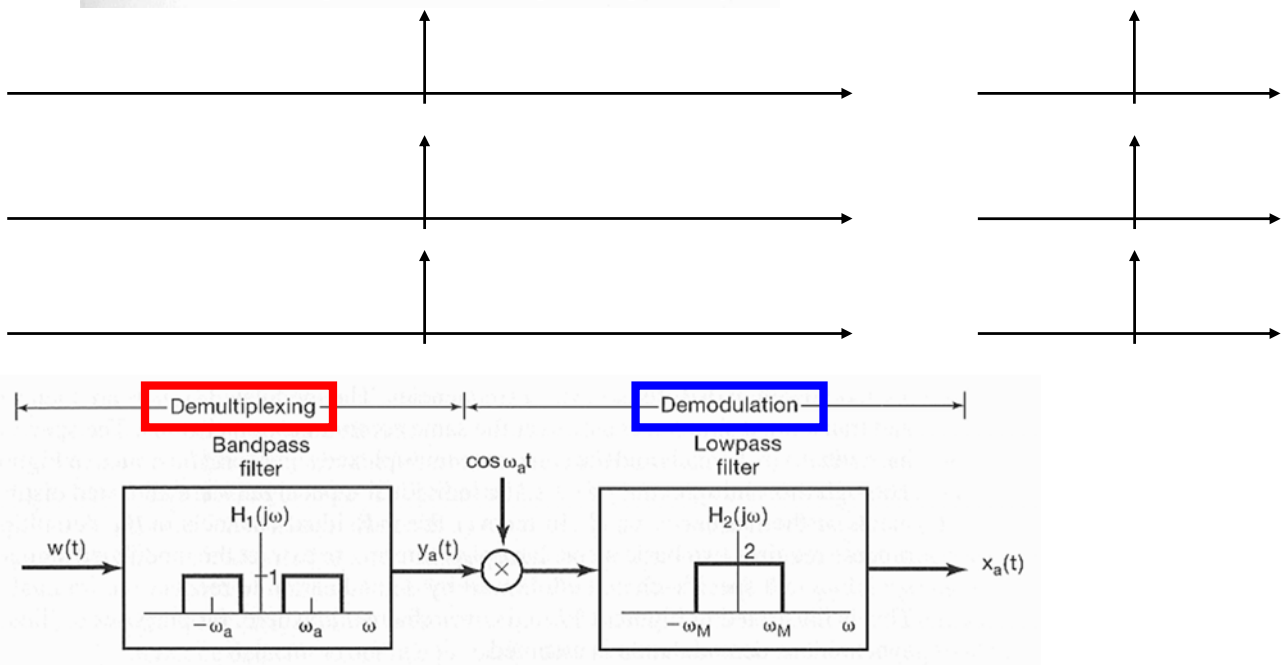
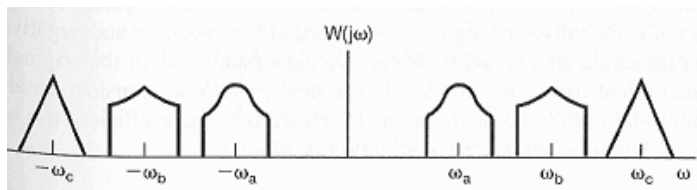
Outline

- Complex Exponential & Sinusoidal Amplitude Modulation & Demodulation
- Frequency-Division Multiplexing
- Single-Sideband Sinusoidal Amplitude Modulation
- Amplitude Modulation with a Pulse-Train Carrier
- Pulse-Amplitude Modulation
- Sinusoidal Frequency Modulation
- Discrete-Time Modulation

FDM Using Sinusoidal AM:



Demultiplexing and Demodulation:



Frequency range	Designation	Typical uses	Propagation method	Channel features
30–300 Hz	ELF (extremely low frequency)	Macrowave, submarine communication	Megametric waves	Penetration of conducting earth and seawater
0.3–3 kHz	VF (voice frequency)	Data terminals, telephony	Copper wire	
3–30 kHz	VLF (very low frequency)	Navigation, telephone, telegraph, frequency and timing standards	Surface ducting (ground wave)	Low attenuation, little fading, extremely stable phase and frequency, large antennas
30–300 kHz	LF (low frequency)	Industrial (power line) communication, aeronautical and maritime long-range navigation, radio beacons	Mostly surface ducting	Slight fading, high atmospheric pulse
0.3–3 MHz	MF (medium frequency)	Mobile, AM broadcasting, amateur, public safety	Ducting and ionospheric reflection (sky wave)	Increased fading, but reliable
3–30 MHz	HF (high frequency)	Military communication, aeronautical mobile, international fixed, amateur and citizen's band, industrial	Ionospheric reflecting sky wave, 50–400 km layer altitudes	Intermittent and frequency-selective fading, multipath
30–300 MHz	VHF (very high frequency)	FM and TV broadcast, land transportation (taxis, buses, railroad)	Sky wave (ionospheric and tropospheric scatter)	Fading, scattering, and multipath
0.3–3 GHz	UHF (ultra high frequency)	UHF TV, space telemetry, radar, military	Transhorizon tropospheric scatter and line-of-sight relaying	
3–30 GHz	SHF (super high frequency)	Satellite and space communication, common carrier (CC), microwave	Line-of-sight ionosphere penetration	Ionospheric penetration, extraterrestrial noise, high directly
30–300 GHz	EHF (extremely high frequency)	Experimental, government, radio astronomy	Line of sight	Water vapor and oxygen absorption
10^3 – 10^7 GHz	Infrared, visible light, ultraviolet	Optical communications	Line of sight	

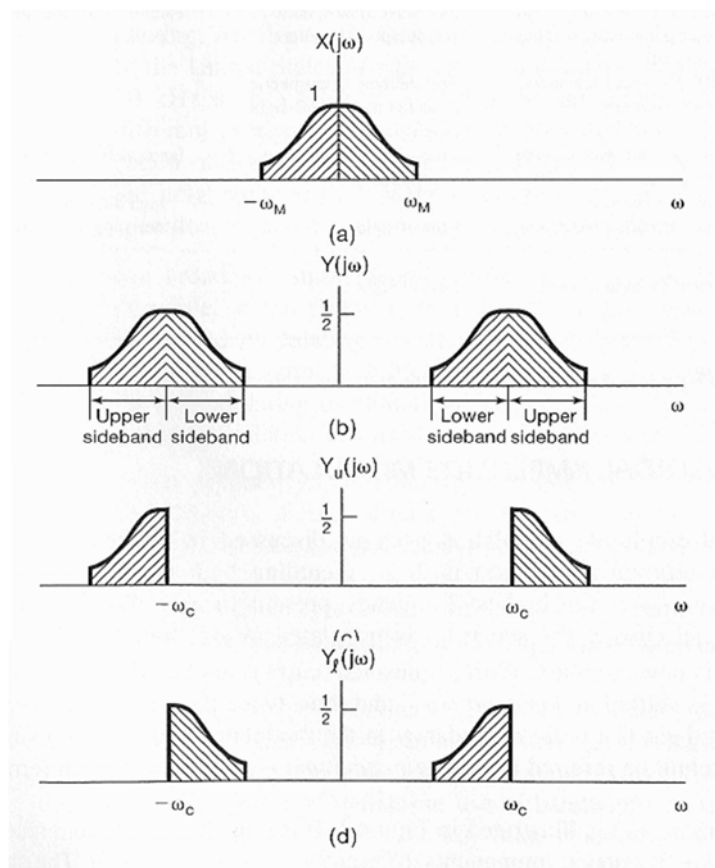
Outline

- Complex Exponential & Sinusoidal Amplitude Modulation & Demodulation
- Frequency-Division Multiplexing
- Single-Sideband Sinusoidal Amplitude Modulation
- Amplitude Modulation with a Pulse-Train Carrier
- Pulse-Amplitude Modulation
- Sinusoidal Frequency Modulation
- Discrete-Time Modulation

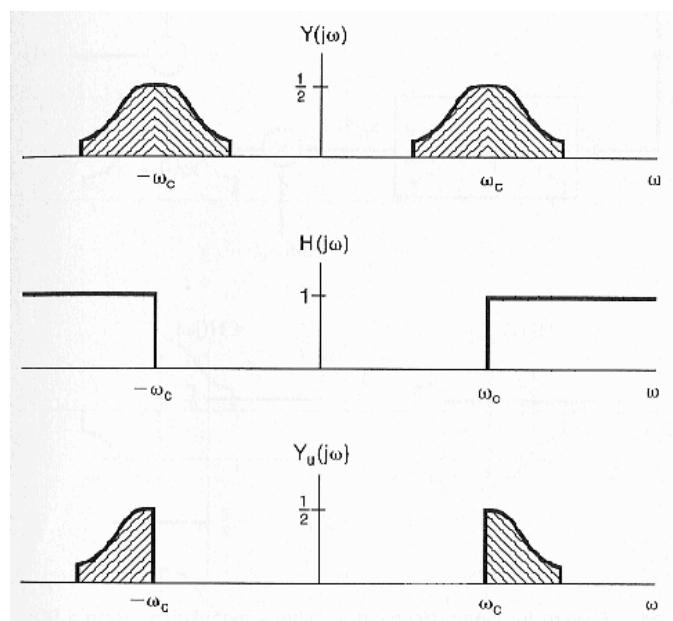
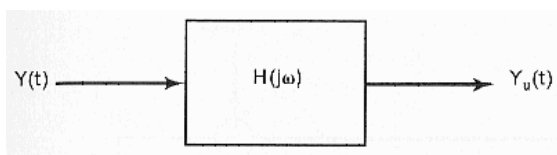
SSB Modulation:

upper sidebands

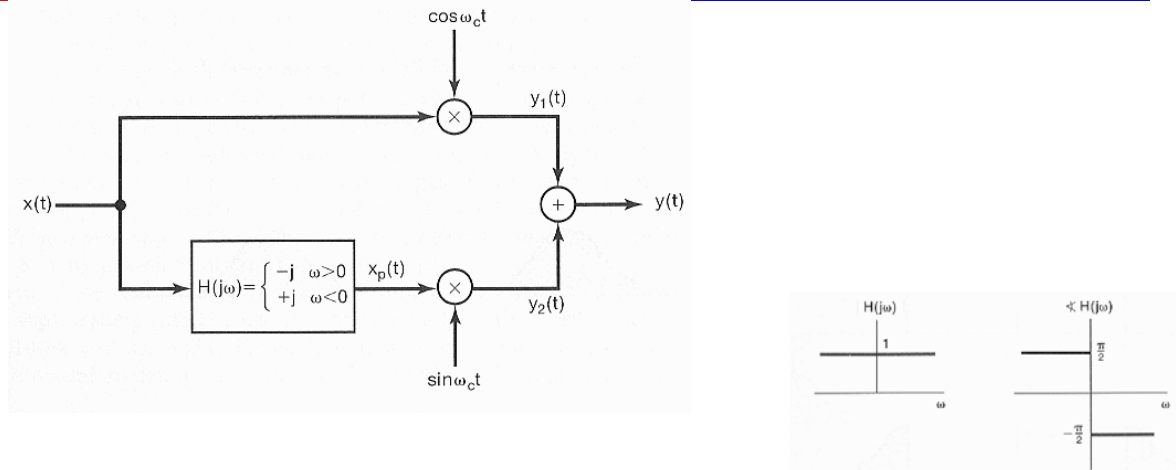
lower sidebands



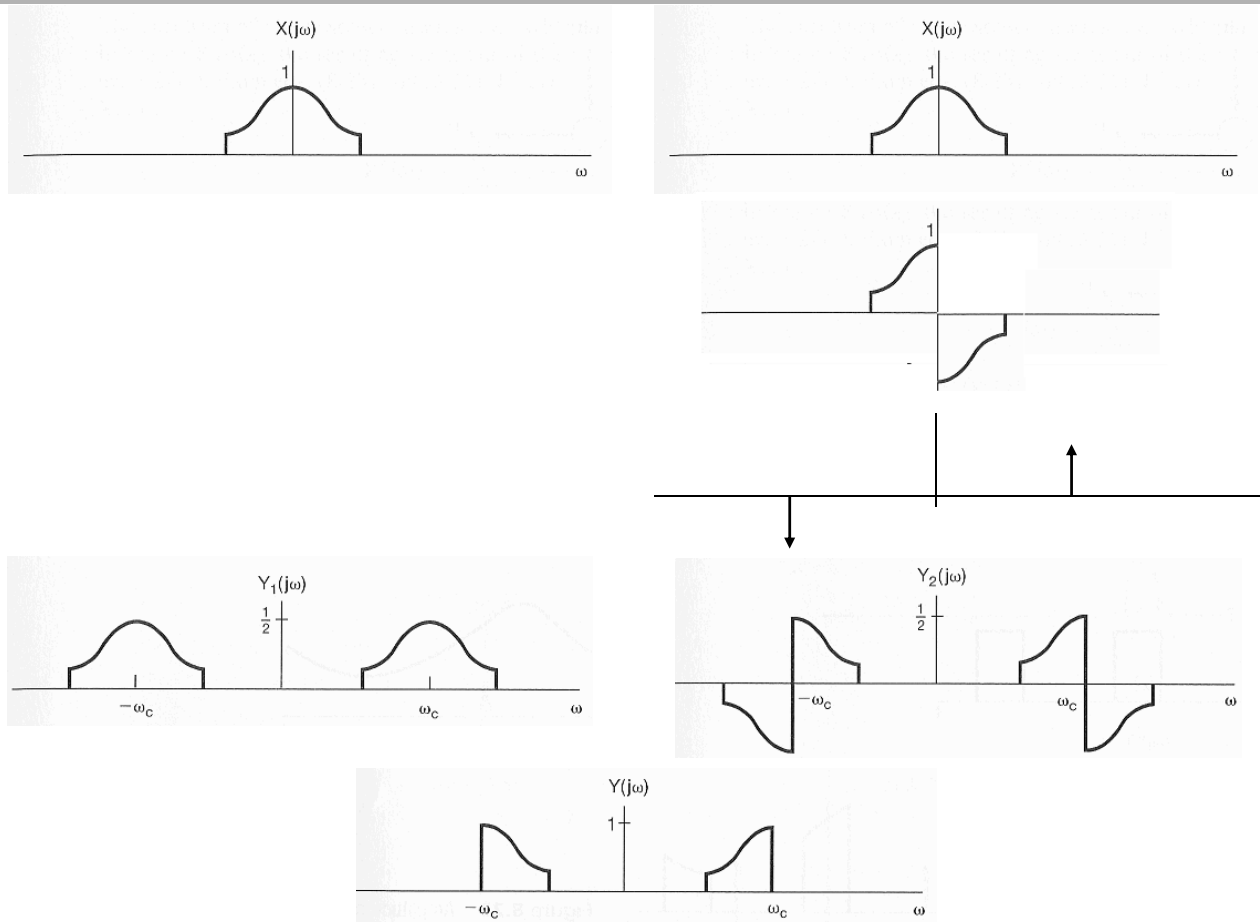
Retain Upper Sidebands Using Ideal Highpass Filter



Retain Lower Sidebands Using Phase-Shift Network



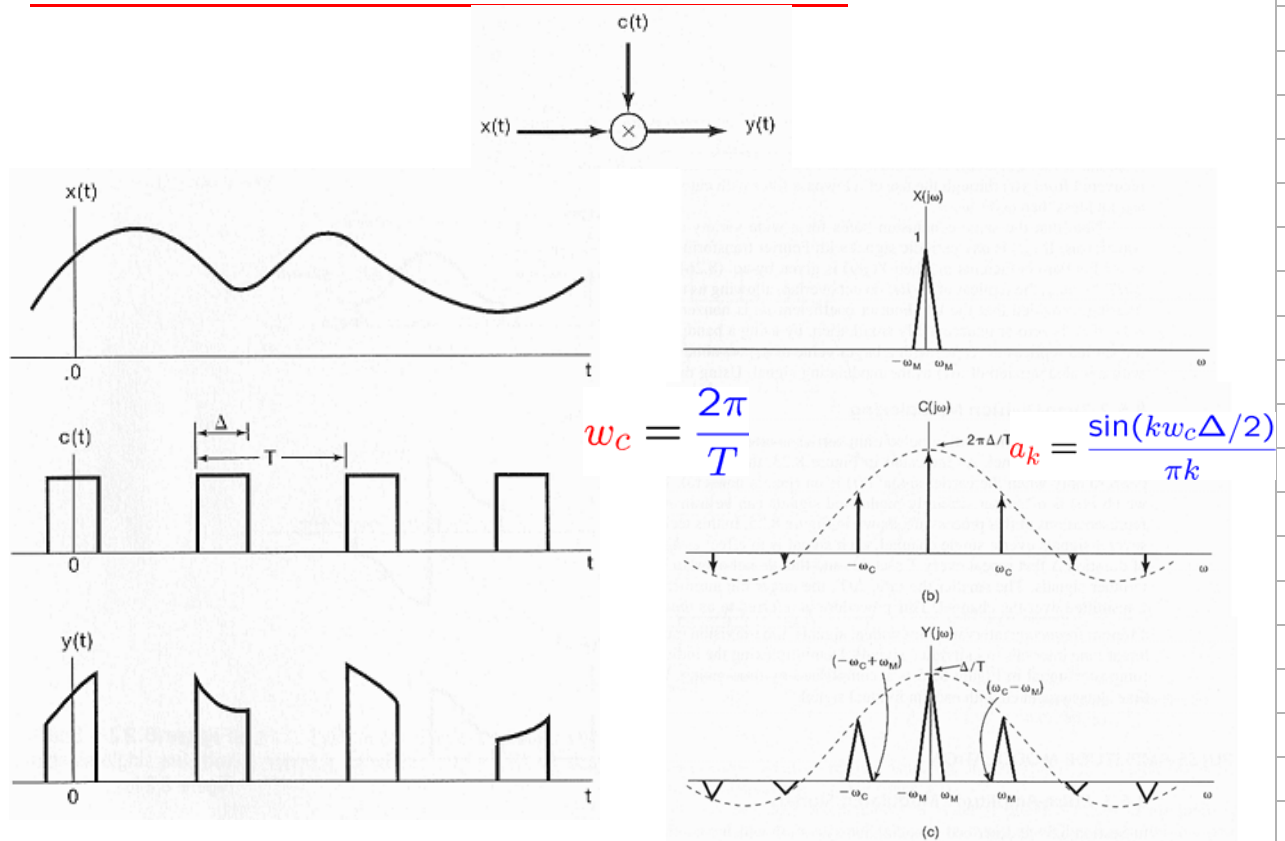
- Retain Lower Sidebands $H(j\omega) = \begin{cases} -j, & \omega > 0 \\ +j, & \omega < 0 \end{cases}$
- Retain Upper Sidebands $H(j\omega) = \begin{cases} +j, & \omega > 0 \\ -j, & \omega < 0 \end{cases}$



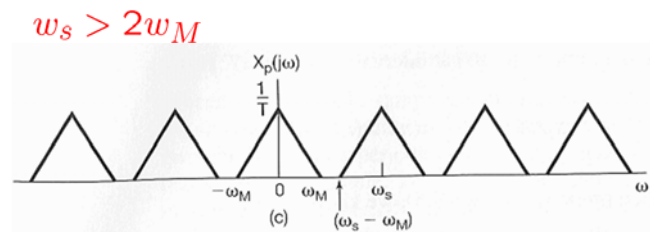
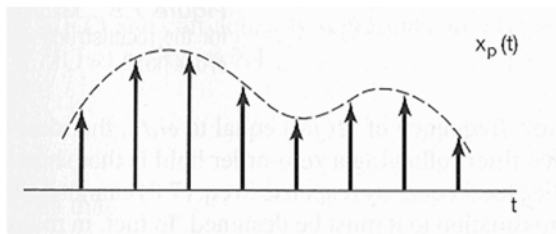
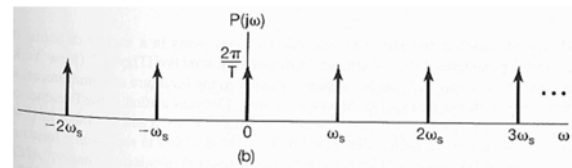
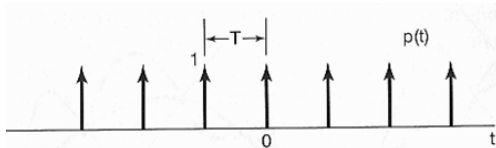
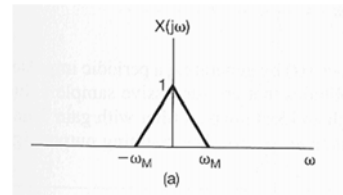
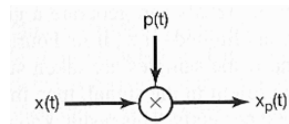
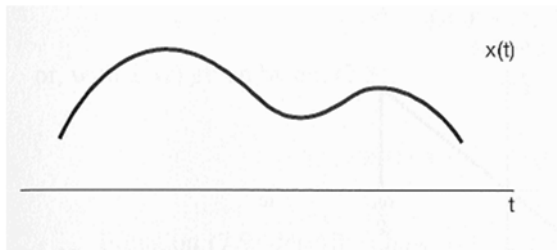
- Complex Exponential & Sinusoidal Amplitude Modulation & Demodulation
- Frequency-Division Multiplexing
- Single-Sideband Sinusoidal Amplitude Modulation
- **Amplitude Modulation with a Pulse-Train Carrier**
- Pulse-Amplitude Modulation
- Sinusoidal Frequency Modulation
- Discrete-Time Modulation

Amplitude Modulation with a Pulse-Train Carrier

■ Modulation of a Pulse-Train Carrier:

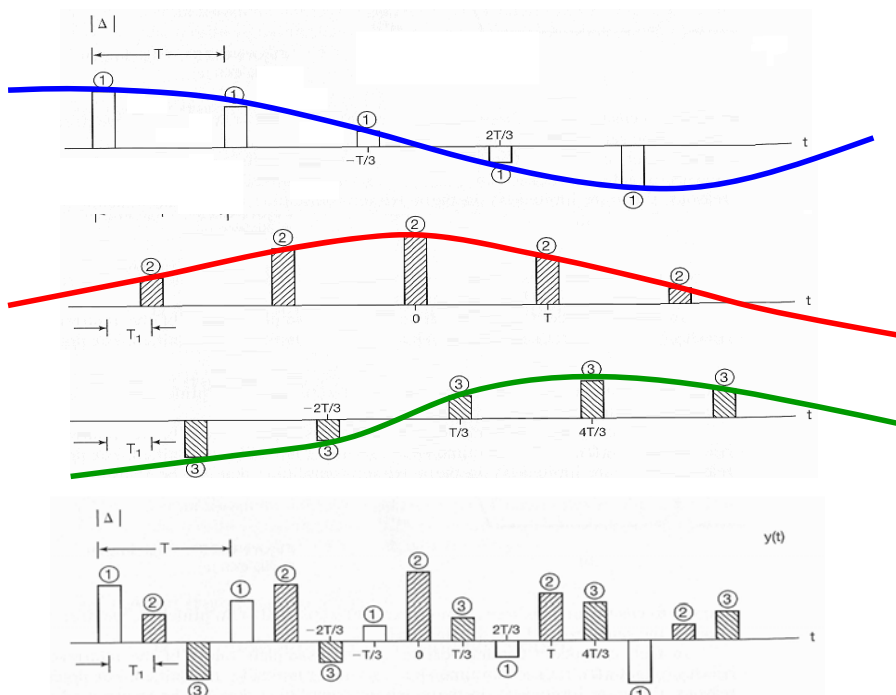
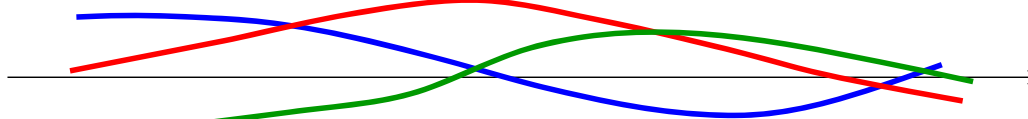


■ Impulse-Train Sampling:

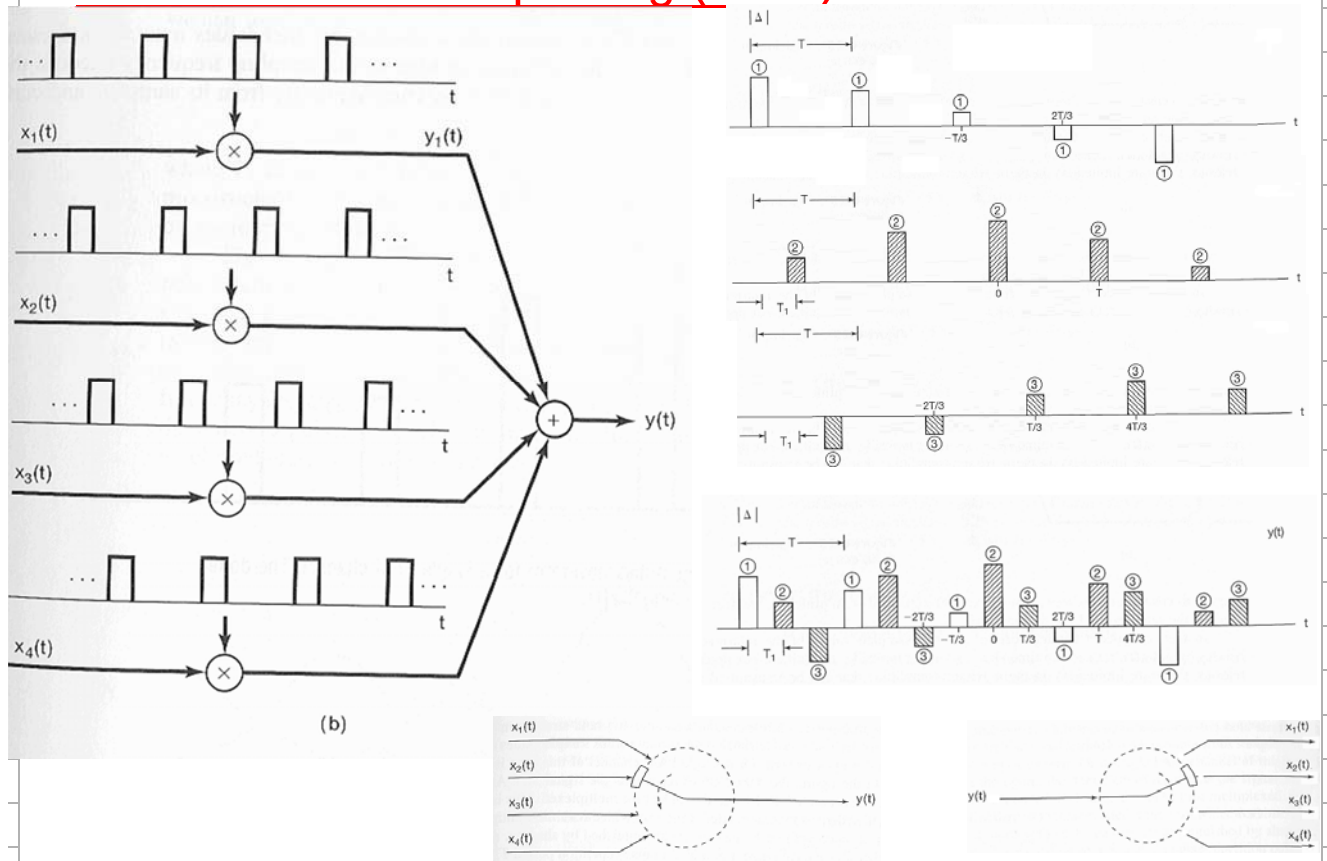


Amplitude Modulation with a Pulse-Train Carrier

■ Time-Division Multiplexing (TDM):



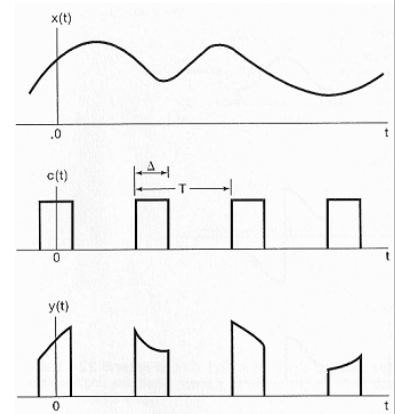
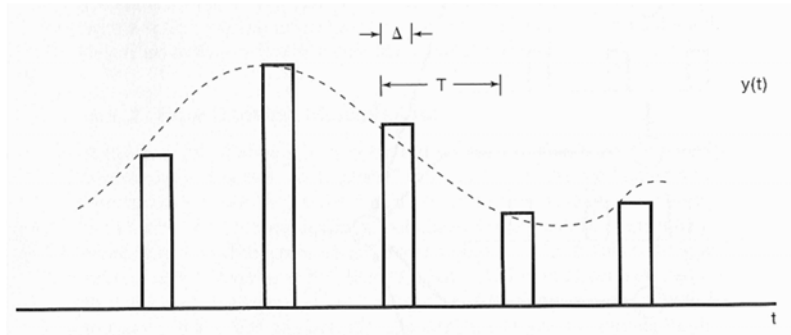
Time-Division Multiplexing (TDM):



Outline

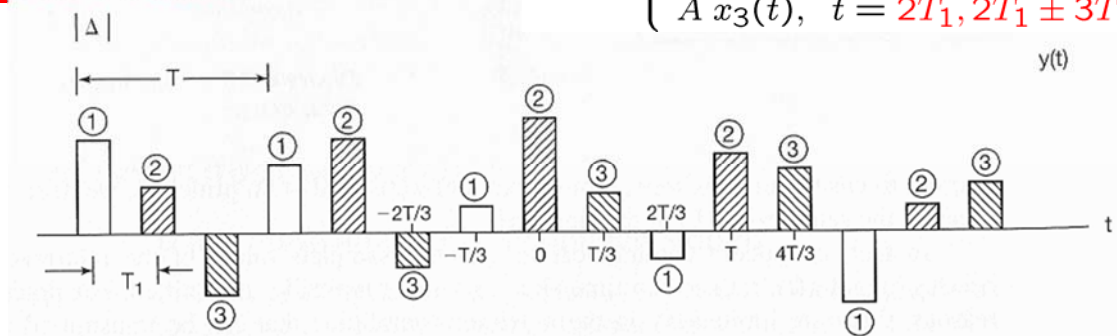
- Complex Exponential & Sinusoidal Amplitude Modulation & Demodulation
- Frequency-Division Multiplexing
- Single-Sideband Sinusoidal Amplitude Modulation
- Amplitude Modulation with a Pulse-Train Carrier
- Pulse-Amplitude Modulation
- Sinusoidal Frequency Modulation
- Discrete-Time Modulation

Pulse-Amplitude Modulated Signals:

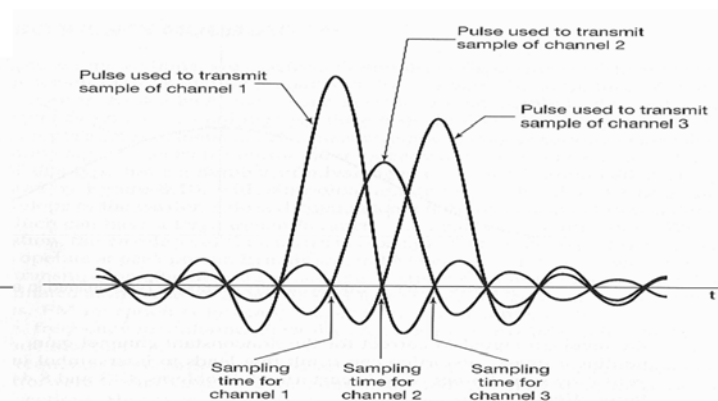
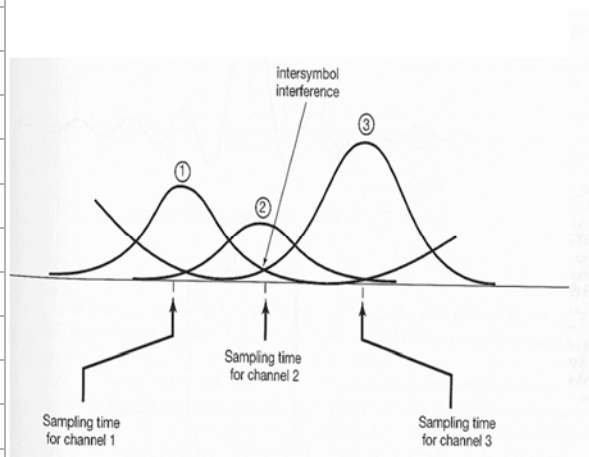
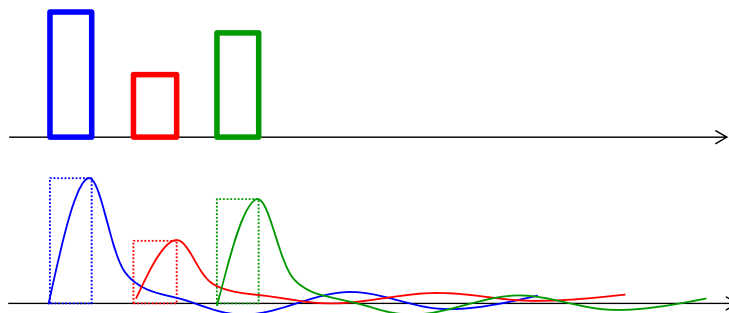


TDM-PAM:

$$y(t) = \begin{cases} A x_1(t), & t = 0, \pm 3T_1, \dots, \\ A x_2(t), & t = T_1, T_1 \pm 3T_1, \dots, \\ A x_3(t), & t = 2T_1, 2T_1 \pm 3T_1, \dots, \end{cases}$$



Intersymbol Interference in PAM Systems:

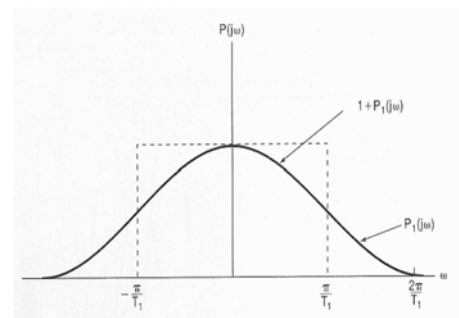
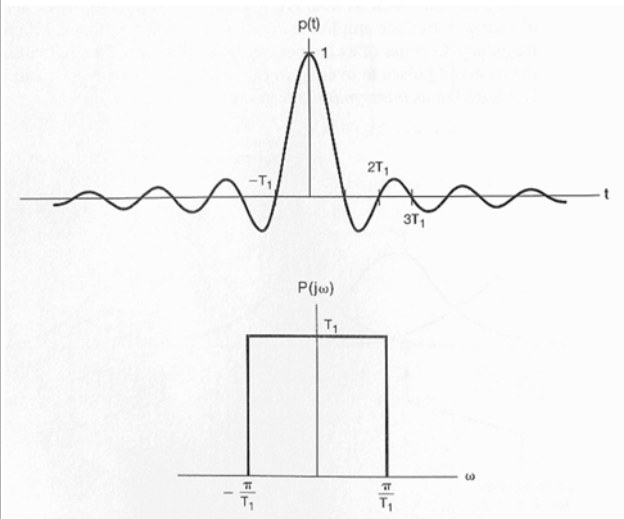


Avoiding Intersymbol Interference in PAM Systems:

$$p(t) = \frac{T_1 \sin(\pi t / T_1)}{\pi t}$$

$$p(\pm T_1) = 0, p(\pm 2T_1) = 0, p(\pm 3T_1) = 0, \dots$$

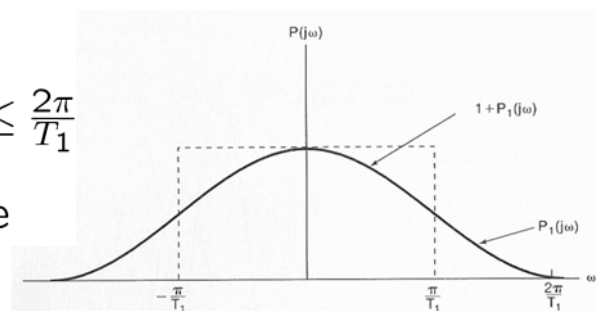
Zero-Crossing at kT_1



General Form of Band-Limited Pulses with Time-Domain Zero-Crossing at kT_1 , $k \in \mathbb{Z}$:

Problem 8.42

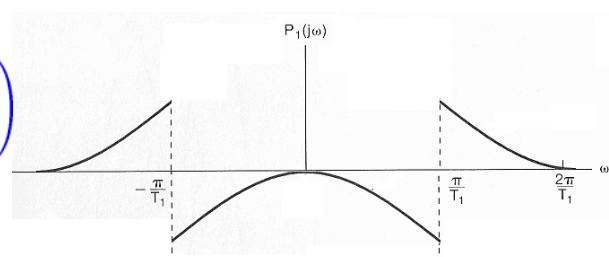
$$P(j\omega) = \begin{cases} 1 + P_1(j\omega) & |\omega| \leq \frac{\pi}{T_1} \\ P_1(j\omega) & \frac{\pi}{T_1} < |\omega| \leq \frac{2\pi}{T_1} \\ 0 & \text{otherwise} \end{cases}$$



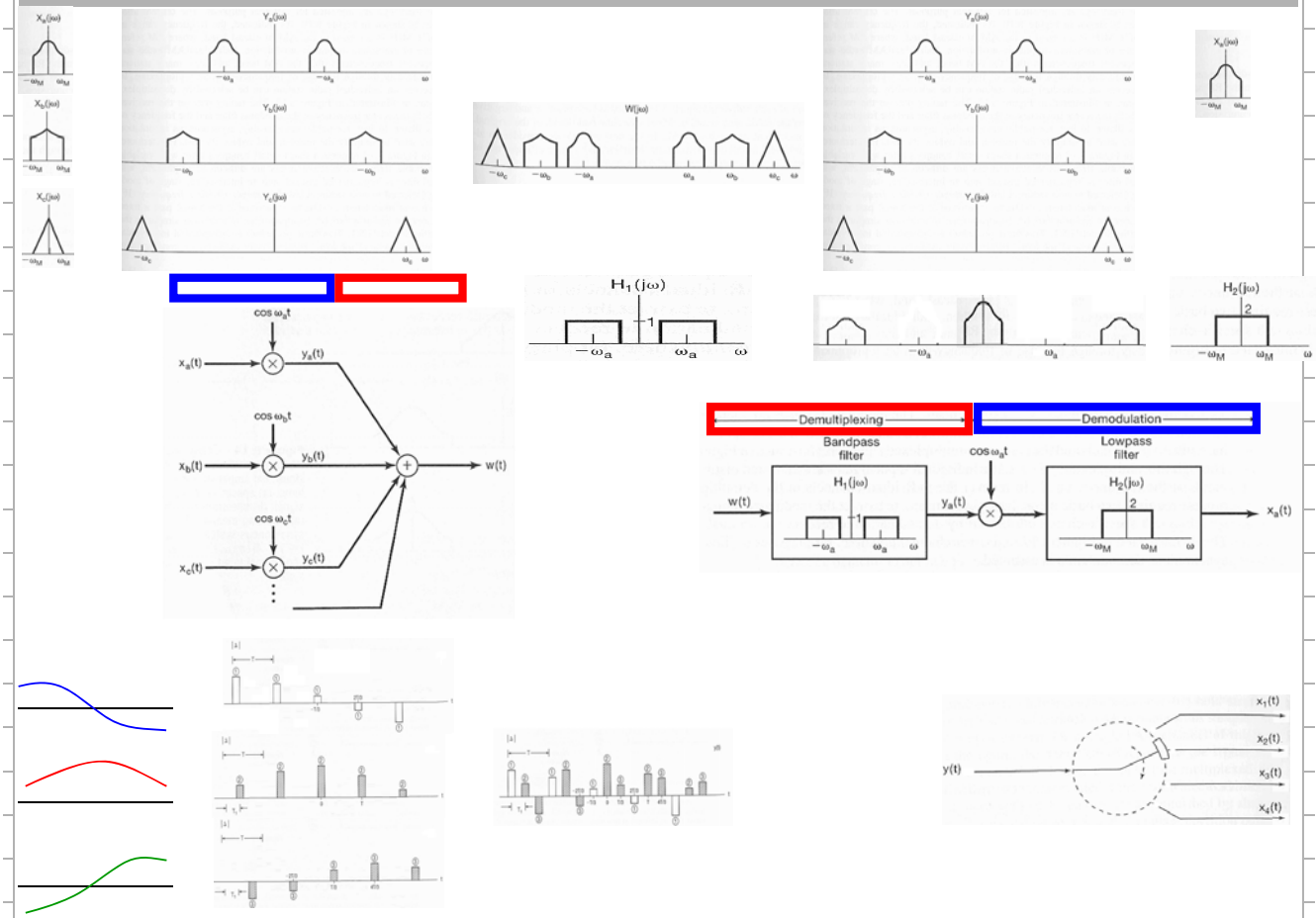
$P_1(j\omega)$: odd symmetry around π/T_1

$$P_1\left(-j\omega + j\frac{\pi}{T_1}\right) = -P_1\left(j\omega + j\frac{\pi}{T_1}\right)$$

$$0 \leq \omega \leq \frac{\pi}{T_1}$$



$\Rightarrow p(t)$ has zero crossing at $\pm T_1, \pm 2T_1, \dots$ i.e., $p(\pm kT_1) = 0$



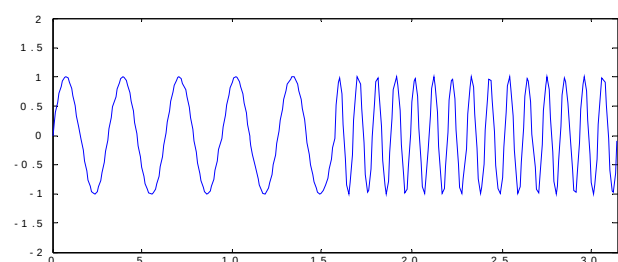
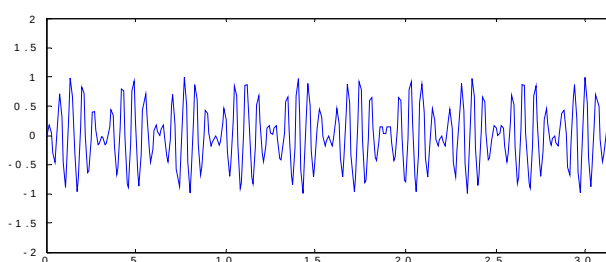
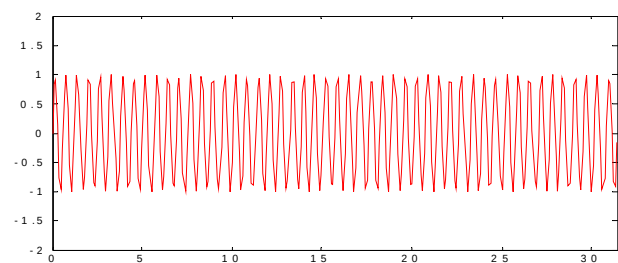
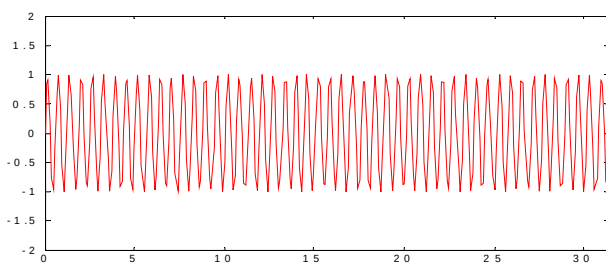
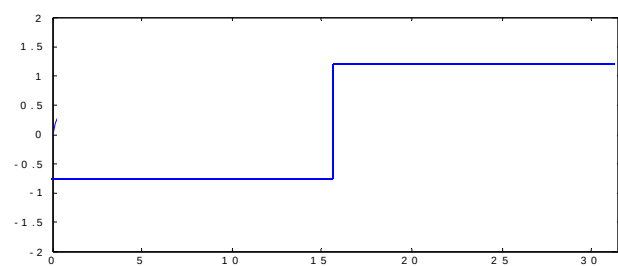
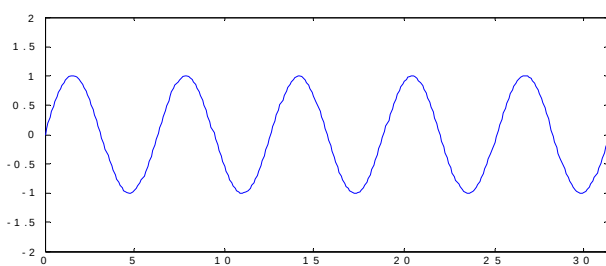
Outline

- Complex Exponential & Sinusoidal Amplitude Modulation & Demodulation
- Frequency-Division Multiplexing
- Single-Sideband Sinusoidal Amplitude Modulation
- Amplitude Modulation with a Pulse-Train Carrier
- Pulse-Amplitude Modulation
- Sinusoidal Frequency Modulation
- Discrete-Time Modulation

■ Frequency Modulation (FM):

- The **modulating signals** is used to control the **frequency** of a **sinusoidal carrier**
- With **sinusoidal AM**, the **peak amplitude** of the **envelope** of the **carrier** is directly dependent on the **amplitude** of the **modulating signal $x(t)$** , which can have a large dynamic range.
- With **FM**, the **envelope** of the **carrier** is **constant**
- An **FM transmitter** can always operate **at peak power** and **amplitude variations** introduced over a transmission channel due to **additive disturbances** or **fading** can be eliminated at the receiver
- **FM** generally requires **greater bandwidth** than does sinusoidal **AM**

Amplitude Modulation and Frequency Modulation



■ Angle Modulation:

$$c(t) = A \cos(w_c t + \theta_c) = A \cos \theta(t)$$

• Phase Modulation:

- Use the modulating signal $x(t)$ to vary the phase θ_c

$$y(t) = A \cos(\theta(t)) = A \cos(w_c t + \theta_c(t))$$

$$\theta_c(t) = \theta_0 + k_p x(t)$$

$$\frac{d\theta(t)}{dt} = w_c + k_p \frac{dx(t)}{dt}$$

• Frequency Modulation:

- Use the modulating signal $x(t)$ to vary the derivative of the angle

$$y(t) = A \cos(\theta(t)) \quad \frac{d\theta(t)}{dt} = w_c + k_f x(t)$$

■ Phase & Frequency Modulation:

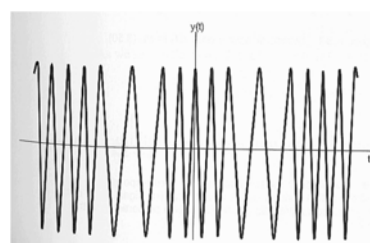
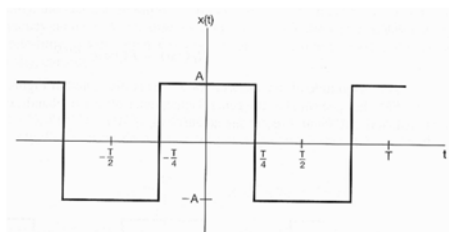
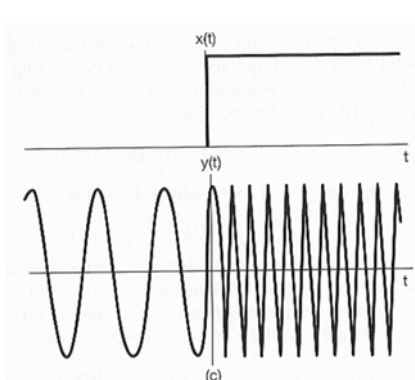
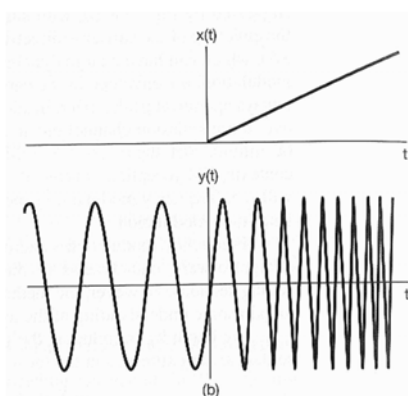
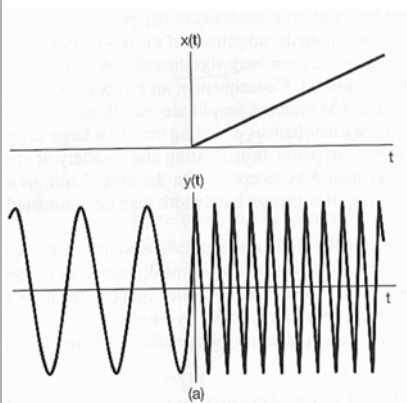
$$\frac{d\theta(t)}{dt} = w_c + k_p \frac{dx(t)}{dt}$$

$$\frac{d\theta(t)}{dt} = w_c + k_f x(t)$$

phase modulation

frequency modulation

frequency modulation



■ Instantaneous Frequency:

$$y(t) = A \cos(\theta(t)) \Rightarrow w_i = \frac{d\theta(t)}{dt}$$

- If $y(t)$ is truly sinusoidal:

$$\theta(t) = w_c t + \theta_0 \quad w_i = w_c$$

- Phase Modulation:

$$w_i = \frac{d\theta(t)}{dt} = w_c + k_p \frac{dx(t)}{dt}$$

- Frequency Modulation:

$$w_i = \frac{d\theta(t)}{dt} = w_c + k_f x(t)$$

■ Narrowband FM:

$$c(t) = A \cos(w_c t + \theta_c) = A \cos \theta(t)$$

- Frequency Modulation with $x(t) = A \cos(w_m t)$

- Instantaneous Frequency: $w_i = \frac{d\theta(t)}{dt} = w_c + k_f x(t)$

$$w_i(t) = \frac{d\theta(t)}{dt} = w_c + k_f A \cos(w_m t)$$

$$\Rightarrow w_c - k_f A \leq w_i(t) \leq w_c + k_f A$$

$$\Rightarrow \Delta w \triangleq k_f A$$

$$\Rightarrow w_i(t) = w_c + \Delta w \cos(w_m t)$$



■ Narrowband FM:

$$x(t) = A \cos(w_m t)$$

$$y(t) = \cos(\theta(t)) = \cos(w_c t + \theta_c(t))$$

$$\frac{d\theta(t)}{dt} = w_c + k_f x(t)$$

$$\Delta w \triangleq k_f A$$

$$\Rightarrow y(t) = \cos\left(w_c t + k_f \int x(t) dt\right)$$

$$= \cos\left(w_c t + \frac{\Delta w}{w_m} \sin(w_m t) + \theta_0\right)$$

$$= \cos\left(w_c t + \frac{\Delta w}{w_m} \sin(w_m t)\right)$$

let $\theta_0 = 0$

- Modulation Index for FM: $m \triangleq \frac{\Delta w}{w_m}$

- Which m is small $\Rightarrow$ narrowband FM

■ Narrowband FM:

$$\cos(A + B) = \cos(A) \cos(B) - \sin(A) \sin(B)$$

$$\Rightarrow y(t) = \cos(w_c t + m \sin(w_m t))$$

$$\text{or } y(t) = \cos(w_c t) \cos(m \sin(w_m t)) - \sin(w_c t) \sin(m \sin(w_m t))$$

- When m is sufficiently small ($\ll \pi/2$)

if $0 < \theta \ll 1$

$$\Rightarrow \cos(m \sin(w_m t)) \approx 1$$

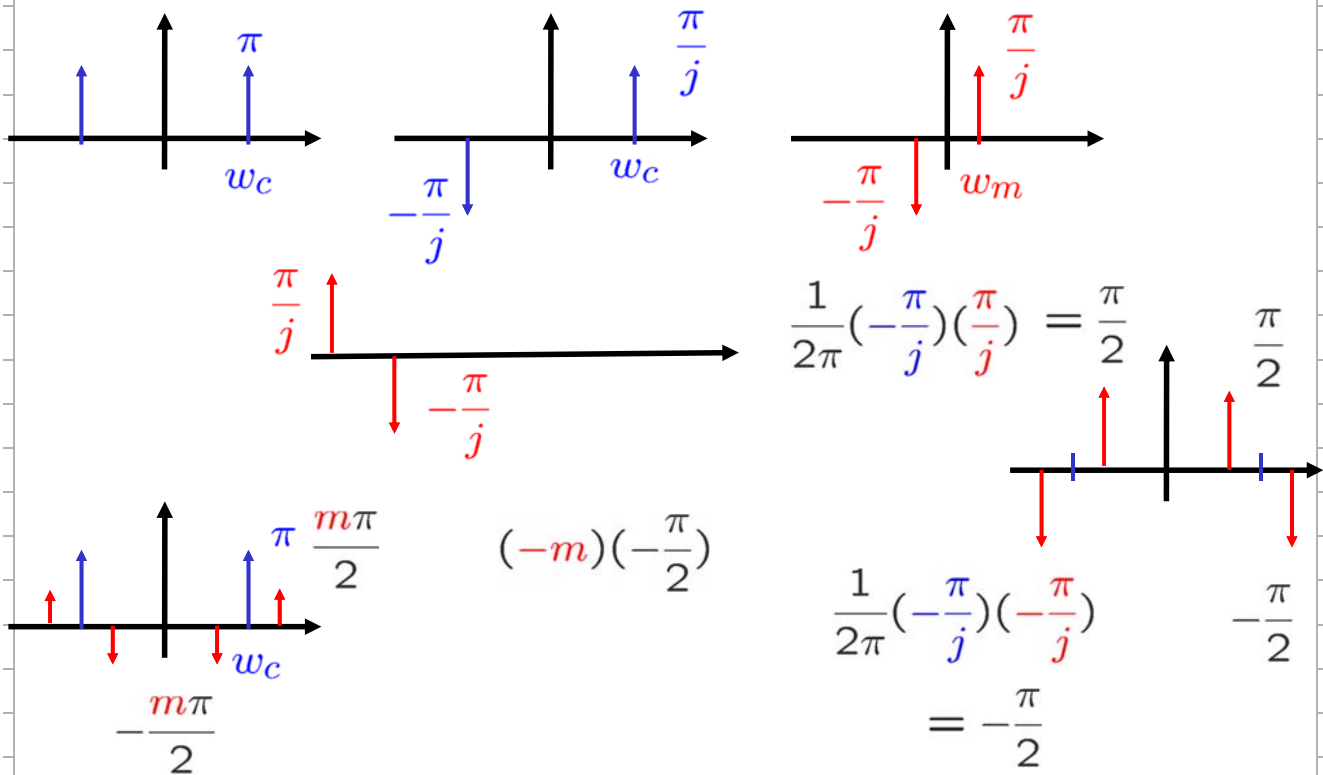
$$\sin(m \sin(w_m t)) \approx m \sin(w_m t)$$

$$\Rightarrow \begin{aligned} \cos(\theta) &\approx 1 \\ \sin(\theta) &\approx \theta \end{aligned}$$

$$\Rightarrow y(t) \approx \cos(w_c t) - m \sin(w_m t) \sin(w_c t)$$

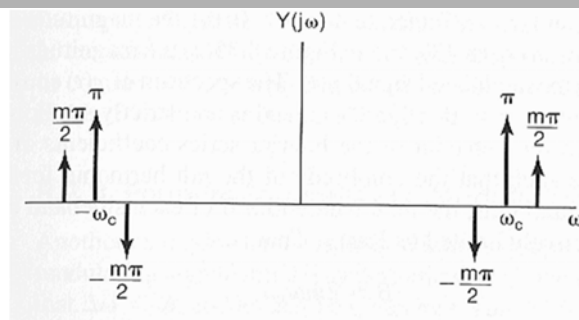
■ Narrowband FM:

$$\Rightarrow y(t) \approx \cos(w_c t) - m \sin(w_m t) \sin(w_c t)$$



■ Narrowband FM:

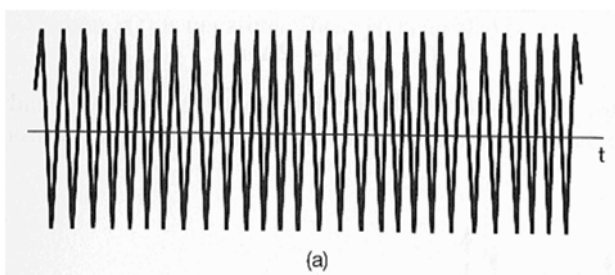
$$x(t) = A \cos(w_m t)$$



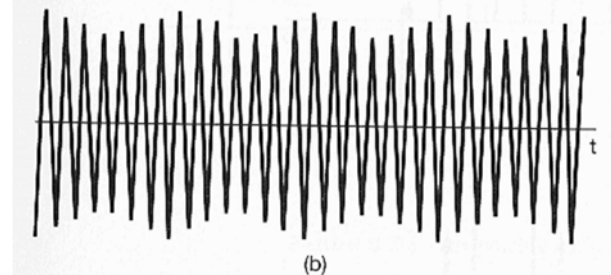
Approximate spectrum for narrowband FM

$$y(t) \approx \cos(w_c t) - m \sin(w_m t) \sin(w_c t)$$

$$y_2(t) = \cos(w_c t) + m \cos(w_m t) \cos(w_c t)$$



Narrowband FM



AM-Double Sideband/with carrier

Wideband FM:

- When m is large

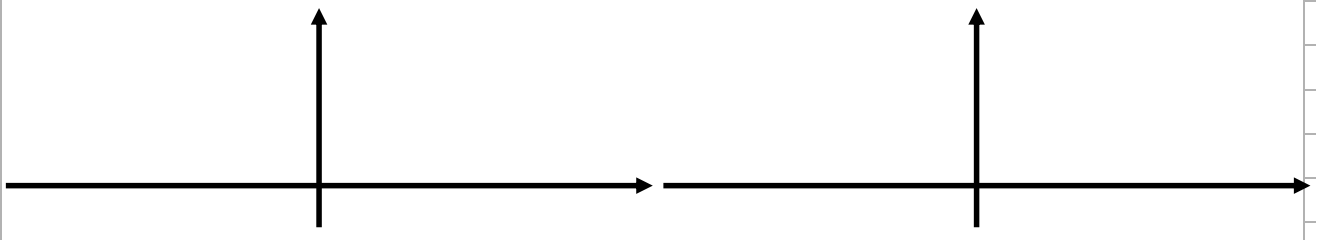
$$m \triangleq \frac{\Delta \omega}{\omega_m}$$

$$y(t) = \cos(\omega_c t) \cos(m \sin(\omega_m t)) - \sin(\omega_c t) \sin(m \sin(\omega_m t))$$

Periodic signals with fundamental frequency ω_m

$$\cos(m \sin(\omega_m t)) = J_0(m) + \sum_{n \text{ even}}^{\infty} 2J_n(m) \cos(n\omega_m t)$$

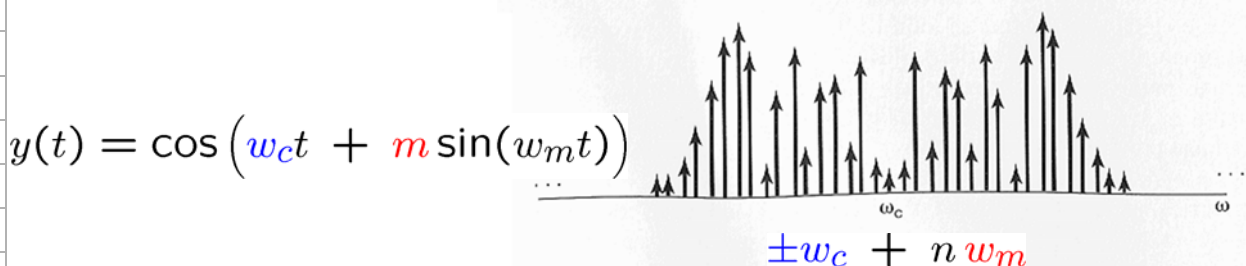
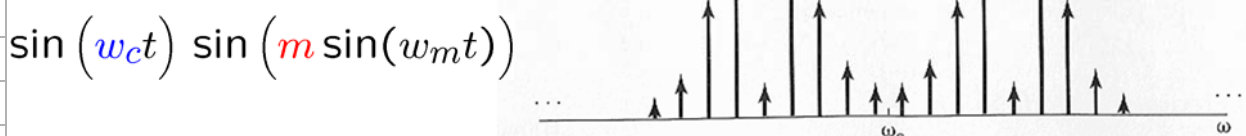
$$\sin(m \sin(\omega_m t)) = \sum_{n \text{ odd}}^{\infty} 2J_n(m) \sin(n\omega_m t)$$



Magnitude of Spectrum of Wideband FM:

$$\Delta \omega \triangleq k_f A$$

$$m \triangleq \frac{\Delta \omega}{\omega_m}$$



$$\Rightarrow B \approx 2 m \omega_m = 2 k_f A = 2 \Delta \omega$$

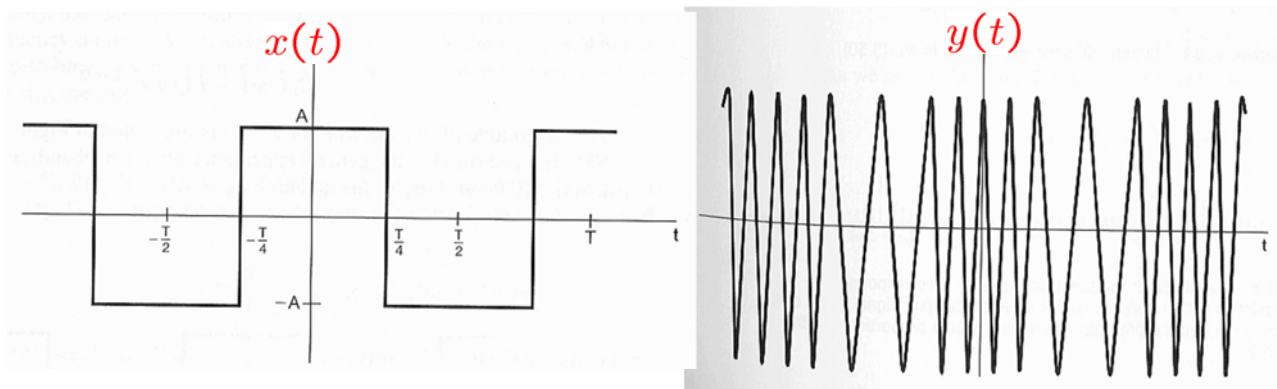
Periodic Square-Wave Modulating Signal:

$$\Delta\omega \triangleq k_f A$$

$$m \triangleq \frac{\Delta\omega}{\omega_m}$$

$$w_i(t) = w_c + k_f x(t) \quad k_f = 1 \Rightarrow \Delta\omega = A$$

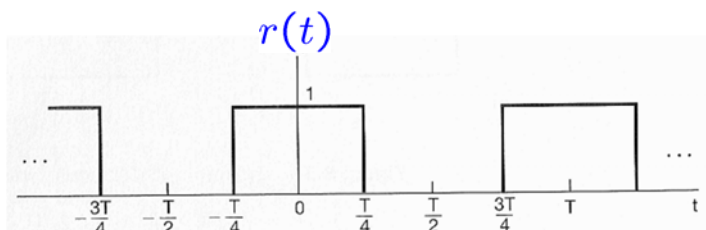
- When $x(t) > 0$, $w_i(t) = w_c + \Delta\omega$
- When $x(t) < 0$, $w_i(t) = w_c - \Delta\omega$



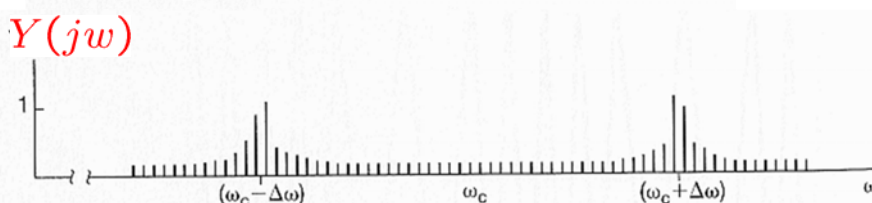
$$\Rightarrow y(t) = r(t) \cos((w_c + \Delta\omega)t) + r\left(t - \frac{T}{2}\right) \cos((w_c - \Delta\omega)t)$$

$$\Rightarrow Y(j\omega) = \frac{1}{2} [R(j\omega + j\omega_c + j\Delta\omega) + R(j\omega - j\omega_c - j\Delta\omega)] \\ + \frac{1}{2} [R_T(j\omega + j\omega_c - j\Delta\omega) + R_T(j\omega - j\omega_c + j\Delta\omega)]$$

$$R(j\omega) = \sum_{k=-\infty}^{\infty} \frac{2}{2k+1} (-1)^k \delta\left(\omega - \frac{2\pi(2k+1)}{T}\right) + \pi\delta(\omega)$$

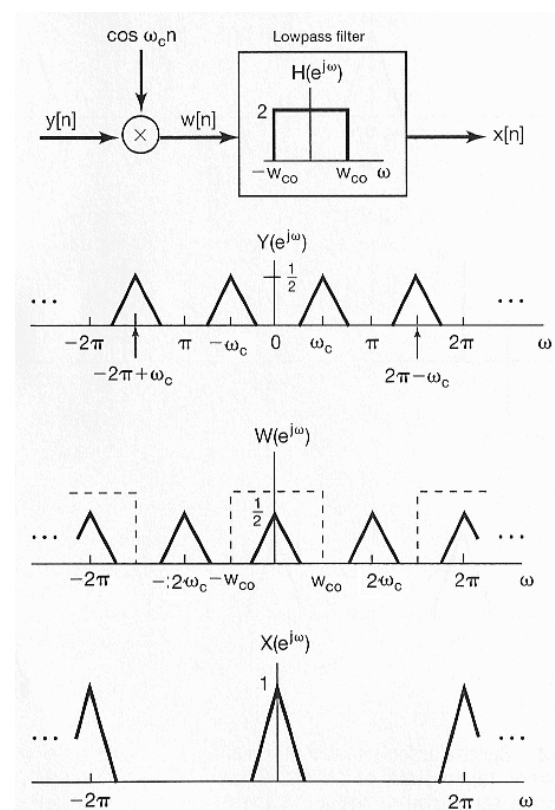
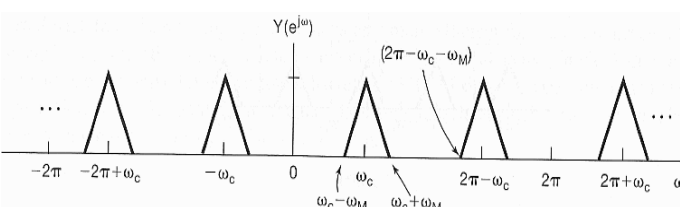
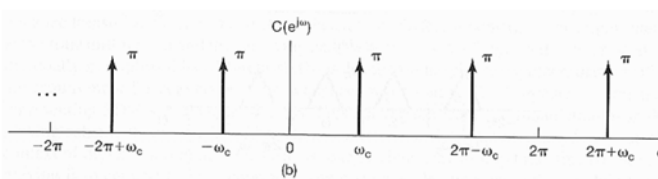
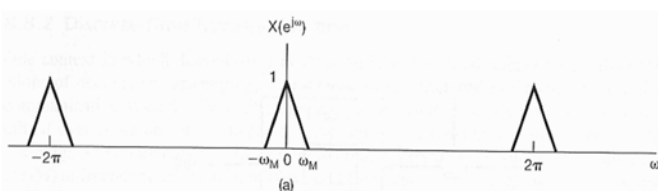
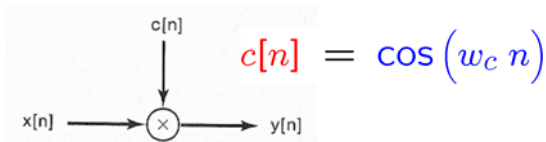


$$R_T(j\omega) = R(j\omega) e^{-j\omega T/2}$$



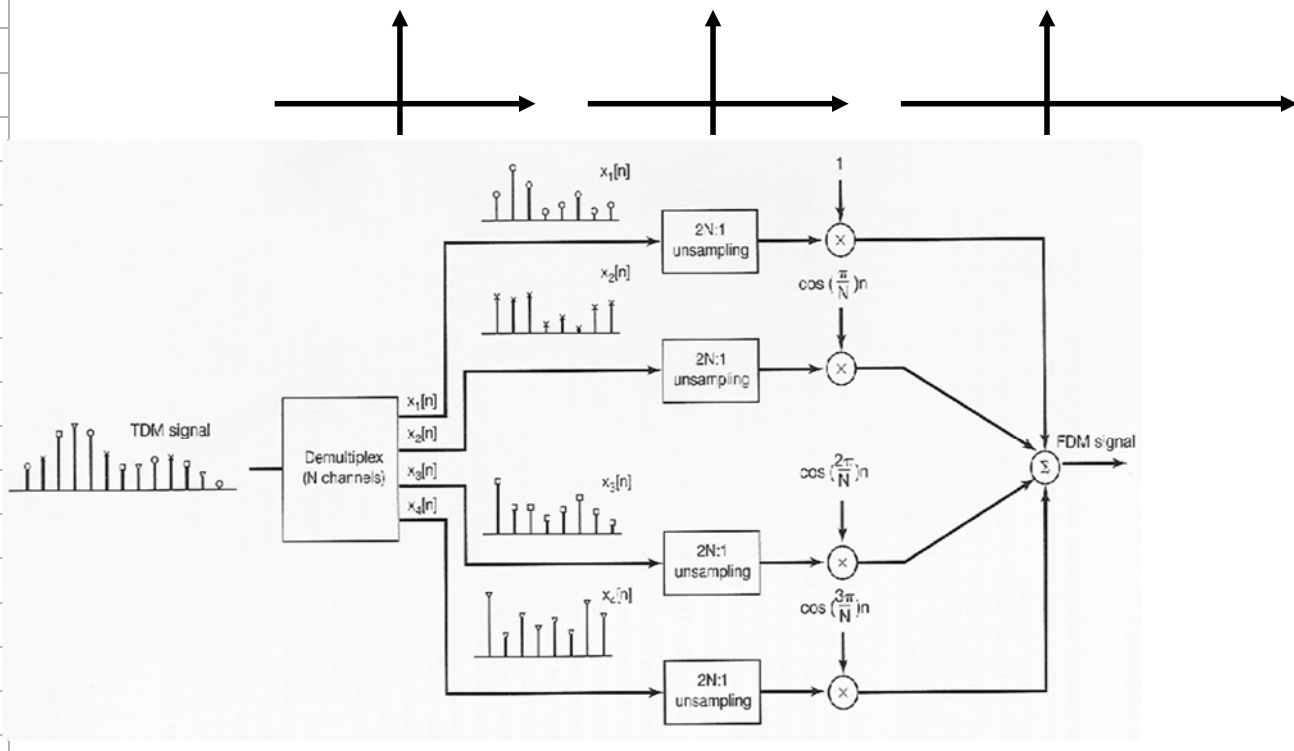
- Complex Exponential & Sinusoidal Amplitude Modulation & Demodulation
- Frequency-Division Multiplexing
- Single-Sideband Sinusoidal Amplitude Modulation
- Amplitude Modulation with a Pulse-Train Carrier
- Pulse-Amplitude Modulation
- Sinusoidal Frequency Modulation
- Discrete-Time Modulation

Discrete-Time Modulation

■ DT Sinusoidal AM:

Transmodulation or Transmultiplexing:

- TDM to FDM



Sampling of Discrete-Time Signals in Chap 7

Higher Equivalent Sampling Rate: Up-sampling

