Problem 1.1, 1.2 (p. 57) – x+jy [SS1:28]

1.1. Express each of the following complex numbers in Cartesian form \((x + jy)\): \(\frac{1}{2} e^{j\pi/2}, e^{j\pi/2}, e^{-j\pi/2}, e^{j5\pi/4}, \sqrt{2} e^{j\pi/4}, \sqrt{2} e^{j9\pi/4}, \sqrt{2} e^{-j\pi/4}\).

1.2. Express each of the following complex numbers in polar form \((re^{j\theta})\), with \(-\pi < \theta \leq \pi\): 5, -2, -3j, \(\frac{1}{2} - j\sqrt{3}/2\), 1 + j, \((1 - j)^2\), \(j(1 - j)\), \(1 + j)\), \((1 + j)/\sqrt{2}\), \((\sqrt{2} + j\sqrt{2})/2\).
Problem 1.9, 1.10, 1.11 (pp. 57-58) – Fundamental Period [SS1:21]

1.9. Determine whether or not each of the following signals is periodic. If a signal is periodic, specify its fundamental period.
   (a) \( x_1(t) = je^{j10t} \)  
   (b) \( x_2(t) = e^{(1+1)i} \)  
   (c) \( x_3[n] = e^{j7\pi n} \)  
   (d) \( x_4[n] = 3e^{j3\pi(n+1/2)i/5} \)  
   (e) \( x_5[n] = 3e^{j3\pi(5n+1/2)} \)

1.10. Determine the fundamental period of the signal \( x(t) = 2\cos(10t + 1) - \sin(4t - 1) \).

1.11. Determine the fundamental period of the signal \( x[n] = 1 + e^{j4\pi n/7} - e^{j2\pi n/5} \).

Problem 1.25, 1.26 (p. 61) – Fundamental Period [SS1:21]

1.25. Determine whether or not each of the following continuous-time signals is periodic. If the signal is periodic, determine its fundamental period.
   (a) \( x(t) = 3\cos(4t + \frac{\pi}{3}) \)  
   (b) \( x(t) = e^{j(\pi t - 1)} \)  
   (c) \( x(t) = [\cos(2t - \frac{\pi}{3})]^2 \)  
   (d) \( x(t) = \delta(t)[\cos(4\pi t)u(t)] \)  
   (e) \( x(t) = \delta(t)[\sin(4\pi t)u(t)] \)  
   (f) \( x(t) = \sum_{n=-\infty}^{\infty} e^{-(2t-n)}u(2t-n) \)

1.26. Determine whether or not each of the following discrete-time signals is periodic. If the signal is periodic, determine its fundamental period.
   (a) \( x[n] = \sin\left(\frac{6\pi}{7}n + 1\right) \)  
   (b) \( x[n] = \cos\left(\frac{n}{8} - \pi\right) \)  
   (c) \( x[n] = \cos\left(\frac{\pi}{8}n^2\right) \)  
   (d) \( x[n] = \cos\left(\frac{\pi}{2}n\right)\cos\left(\frac{\pi}{4}n\right) \)  
   (e) \( x[n] = 2\cos\left(\frac{\pi}{4}n\right) + \sin\left(\frac{\pi}{8}n\right) - 2\cos\left(\frac{\pi}{2}n + \frac{\pi}{6}\right) \)
**Problem 1.12 (p. 58) – δ[n] & u[n] [SS1:57]**

1.12. Consider the discrete-time signal

\[ x[n] = 1 - \sum_{k=3}^{\infty} \delta[n - 1 - k]. \]

Determine the values of the integers \( M \) and \( n_0 \) so that \( x[n] \) may be expressed as

\[ x[n] = u[Mn - n_0]. \]

**Problem 1.13 (p. 58) – δ(t) & integral [SS1:57]**

1.13. Consider the continuous-time signal

\[ x(t) = \delta(t + 2) - \delta(t - 2). \]

Calculate the value of \( E_{\infty} \) for the signal

\[ y(t) = \int_{-\infty}^{t} x(\tau) d\tau. \]
Problem 1.14 (p. 58) – impulse train & derivative [SS1:57]

1.14. Consider a periodic signal

\[ x(t) = \begin{cases} 
1, & 0 \leq t \leq 1 \\
-2, & 1 < t < 2 
\end{cases} \]

with period \( T = 2 \). The derivative of this signal is related to the “impulse train”

\[ g(t) = \sum_{k = -\infty}^{\infty} \delta(t - 2k) \]

with period \( T = 2 \). It can be shown that

\[ \frac{dx(t)}{dt} = A_1 g(t - t_1) + A_2 g(t - t_2) \]

Determine the values of \( A_1, t_1, A_2, \) and \( t_2 \).

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2. (10%) Consider the signals:

\[ x(t) = \cos\left(\frac{2\pi t}{3}\right) + 2\sin\left(\frac{16\pi t}{3}\right) \]
\[ y(t) = \sin(\pi t) \]

a) (5%) Show that \( z(t) = x(t)y(t) \) is periodic, and find the fundamental period of \( z(t) \).
b) (5%) Write \( z(t) \) as a linear combination of harmonically related complex exponentials. That is, find a number \( T \) and complex numbers \( c_k \) such that:

\[ z(t) = \sum_{k} c_k e^{j(2\pi/T)t} \]

4. (8%) Let \( V \) be the set of all periodic discrete-time signals with fundamental period \( N \). Consider the functions \( \phi_k[n] = e^{j\frac{2\pi k}{N} n} \), \( k = 0, \pm 1, \pm 2, \ldots \).

a) Show that

\[ \sum_{n=-N}^{N} \phi_k[n] = \begin{cases} 
N, & k = 0, \pm N, \pm 2N, \ldots \\
0, & \text{otherwise} 
\end{cases} \quad (4\%) \]

b) For any element \( x[n] \) in \( V \), is it true that \( x[n] \) can be generated by the linear combinations of the functions \( \phi_k[n] \), i.e., \( x[n] = \sum_{k\neq N} d_k \phi_k[n] \) with some coefficients \( d_k \)? If your answer is yes, determine the coefficients \( d_k \). If your answer is no, explain why. (4%)