Problem 2.1 (p.137) – Convolution Sum [SS2:12]

2.1. Let

\[ x[n] = \delta[n] + 2\delta[n - 1] - \delta[n - 3] \quad \text{and} \quad h[n] = 2\delta[n + 1] + 2\delta[n - 1]. \]

Compute and plot each of the following convolutions:
(a) \( y_1[n] = x[n] \ast h[n] \)
(b) \( y_2[n] = x[n + 2] \ast h[n] \)
(c) \( y_3[n] = x[n] \ast h[n + 2] \)
2.4. Compute and plot $y[n] = x[n] * h[n]$, where

$$x[n] = \begin{cases} 1, & 3 \leq n \leq 8 \\ 0, & \text{otherwise} \end{cases},$$

$$h[n] = \begin{cases} 1, & 4 \leq n \leq 15 \\ 0, & \text{otherwise} \end{cases}.$$

2.21. Compute the convolution $y[n] = x[n] * h[n]$ of the following pairs of signals:

(a) $x[n] = \alpha^n u[n]$, $h[n] = \beta^n u[n]$, $\alpha \neq \beta$
(b) $x[n] = h[n] = \alpha^n u[n]$
(c) $x[n] = (-\frac{1}{2})^n u[n-4]$, $h[n] = 4^n u[2 - n]$
(d) $x[n]$ and $h[n]$ are as in Figure P2.21.
Problem 2.7 (p.138) – Convolution Sum [SS2:12]

2.7. A linear system $S$ has the relationship

$$ y[n] = \sum_{k=-\infty}^{\infty} x[k]g[n-2k] $$

between its input $x[n]$ and its output $y[n]$, where $g[n] = u[n] - u[n - 4]$.

(a) Determine $y[n]$ when $x[n] = \delta[n - 1]$.
(b) Determine $y[n]$ when $x[n] = \delta[n - 2]$.
(c) Is $S$ LTI?
(d) Determine $y[n]$ when $x[n] = u[n]$.

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4. (10%) Consider a discrete-time, linear, time-invariant system that has unit sample response

$$ h[n] = \left(\frac{1}{2}\right)^{n}u[n] \quad \text{and input} \quad x[n]. $$

a) (3%) Evaluate and sketch the response $y_1[n]$ of the system if $x[n] = x_1[n] = \delta[n - d]$, for some integer $d > 0$. Justify your answer.

b) (3%) Evaluate and sketch the response $y_2[n]$ of the system if $x[n] = x_2[n] = u[n]$. Justify your answer.

c) (4%) Identify the relationship between $y_1[n]$ and $y_2[n]$. Justify your answer.

5. (8 %) Compute the convolution of the two sequences

$$ x[n] = \begin{cases} 
  a^n, & 0 \leq n \leq 4 \\
  0, & \text{otherwise}
\end{cases} $$

and

$$ h[n] = \begin{cases} 
  1, & 0 \leq n \leq 6 \\
  0, & \text{otherwise}
\end{cases} $$
4. [10] Consider three systems with the following input $i[n]$ – output $o[n]$ relationships:

- $S1: o[n] = \begin{cases} 
i[n/2], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$
- $S2: o[n] = i[n] + \frac{1}{2}i[n-1] + \frac{1}{4}i[n-2]$
- $S3: o[n] = i[2n]$

Suppose that these systems are interconnected in series as follows.

(a) Find the input-output relationship between $x[n]$ and $y[n]$. [6]
(b) Find the response of the overall system to the input $x[n] = \delta[n] - \delta[n-1]$. [4]