Outline

- Representation of Aperiodic Signals: the Continuous-Time Fourier Transform
- The Fourier Transform for Periodic Signals
- Properties of Continuous-Time Fourier Transform
- The Convolution Property
- The Multiplication Property
- Systems Characterized by Linear Constant-Coefficient Differential Equations
Problem 4.6 (p.335) – Properties of FT [SS4:27]

4.6. Given that $x(t)$ has the Fourier transform $X(j\omega)$, express the Fourier transforms of the signals listed below in terms of $X(j\omega)$. You may find useful the Fourier transform properties listed in Table 4.1.

(a) $x_1(t) = x(1 - t) + x(-1 - t)$
(b) $x_2(t) = x(3t - 6)$
(c) $x_3(t) = \frac{d^2}{dt^2} x(t - 1)$
**Problem 4.7 (p.335) – Rea, Imaginary, Even, Odd [SS4:35]**

**4.7.** For each of the following Fourier transforms, use Fourier transform properties (Table 4.1) to determine whether the corresponding time-domain signal is (i) real, imaginary, or neither and (ii) even, odd, or neither. Do this without evaluating the inverse of any of the given transforms.

(a) \( X_1(j\omega) = u(\omega) - u(\omega - 2) \)

(b) \( X_2(j\omega) = \cos(2\omega) \sin(\omega/2) \)

(c) \( X_3(j\omega) = A(\omega)e^{jB(\omega)}, \) where \( A(\omega) = (\sin 2\omega)/\omega \) and \( B(\omega) = 2\omega + \frac{\pi}{2} \)

(d) \( X(j\omega) = \sum_{k=-\infty}^{\infty} (\frac{1}{2})^{|k|} \delta(\omega - \frac{k\pi}{4}) \)
4.8. Consider the signal

\[ x(t) = \begin{cases} 
0, & t < -\frac{1}{2} \\
 t + \frac{1}{2}, & -\frac{1}{2} \leq t \leq \frac{1}{2}. \\
1, & t > \frac{1}{2}
\end{cases} \]

(a) Use the differentiation and integration properties in Table 4.1 and the Fourier transform pair for the rectangular pulse in Table 4.2 to find a closed-form expression for \( X(j\omega) \).

(b) What is the Fourier transform of \( g(t) = x(t) - \frac{1}{2} \)?
4.9. Consider the signal

\[ x(t) = \begin{cases} 
0, & |t| > 1 \\
(t + 1)/2, & -1 \leq t \leq 1 
\end{cases} \]

(a) With the help of Tables 4.1 and 4.2, determine the closed-form expression for \( X(j\omega) \).
(b) Take the real part of your answer to part (a), and verify that it is the Fourier transform of the even part of \( x(t) \).
(c) What is the Fourier transform of the odd part of \( x(t) \)?
3. (8 %) Suppose a continuous-time signal $x(t)$ is an odd signal. Show that the Fourier transform $X(j\omega)$ of $x(t)$ is given by

$$X(j\omega) = -2j\int_{0}^{\infty} x(t)\sin \omega t \, dt.$$
7. [12] Let $X(j\omega)$ be the Fourier transform of a signal $x(t)$. Assume that another signal $g(t)$ has the same shape as that of $X(j\omega)$, i.e., $g(t) = X(jt)$.

(a) Show that the Fourier transform $G(j\omega)$ of $g(t)$ has the same shape as $2\pi x(-t)$. [6]

(b) Using the result of Part (a), find the Fourier transform of a signal $x(t) = e^{iQt}$, where $Q$ is a real number. Justify your answer. [6]