Outline

- Representation of Aperiodic Signals: the Discrete-Time Fourier Transform
- The Fourier Transform for Periodic Signals
- Properties of Discrete-Time Fourier Transform
  - The Convolution Property
  - The Multiplication Property
  - Duality
- Systems Characterized by Linear Constant-Coefficient Difference Equations
5.6. Given that \( x[n] \) has Fourier transform \( X(e^{j\omega}) \), express the Fourier transforms of the following signals in terms of \( X(e^{j\omega}) \). You may use the Fourier transform properties listed in Table 5.1.

(a) \( x_1[n] = x[1 - n] + x[-1 - n] \)
(b) \( x_2[n] = \frac{x[n] - x[-n] + x[n]}{2} \)
(c) \( x_3[n] = (n - 1)^2 x[n] \)
Problem 5.7 (p.401) – Properties of FT [SS5:53]

5.7. For each of the following Fourier transforms, use Fourier transform properties (Table 5.1) to determine whether the corresponding time-domain signal is (i) real, imaginary, or neither and (ii) even, odd, or neither. Do this without evaluating the inverse of any of the given transforms.

(a) \( X_1(e^{j\omega}) = e^{-j\omega} \sum_{k=1}^{10} \sin k\omega \)

(b) \( X_2(e^{j\omega}) = j \sin(\omega) \cos(5\omega) \)

(c) \( X_3(e^{j\omega}) = A(\omega) + e^{jB(\omega)} \) where

\[
A(\omega) = \begin{cases} 
1, & 0 \leq |\omega| \leq \frac{\pi}{8} \\
0, & \frac{\pi}{8} < |\omega| \leq \frac{\pi}{2}
\end{cases}
\]

and \( B(\omega) = -\frac{3\omega}{2} + \pi. \)
5.8. Use Tables 5.1 and 5.2 to help determine $x[n]$ when its Fourier transform is

$$X(e^{j\omega}) = \frac{1}{1 - e^{-j\omega}} \left(\frac{\sin \frac{3}{2} \omega}{\sin \frac{\omega}{2}}\right) + 5\pi\delta(\omega), \quad -\pi < \omega \leq \pi$$
5.9. The following four facts are given about a real signal \( x[n] \) with Fourier transform \( X(e^{j\omega}) \):

1. \( x[n] = 0 \) for \( n > 0 \).
2. \( x[0] > 0 \).
3. \( \Im\{X(e^{j\omega})\} = \sin \omega - \sin 2\omega \).
4. \( \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(e^{j\omega})|^2 \, d\omega = 3 \).

Determine \( x[n] \).
5.10. Use Tables 5.1 and 5.2 in conjunction with the fact that

\[ X(e^{j\theta}) = \sum_{n=-\infty}^{\infty} x[n] \]

to determine the numerical value of

\[ A = \sum_{n=0}^{\infty} n \left( \frac{1}{2} \right)^n. \]
5.11. Consider a signal $g[n]$ with Fourier transform $G(e^{j\omega})$. Suppose

$$g[n] = x_{(2)}[n],$$

where the signal $x[n]$ has a Fourier transform $X(e^{j\omega})$. Determine a real number $\alpha$ such that $0 < \alpha < 2\pi$ and $G(e^{j\omega}) = G(e^{j(\omega - \alpha)})$. 