Outline

- Representation of Aperiodic Signals: the Discrete-Time Fourier Transform
- The Fourier Transform for Periodic Signals
- Properties of Discrete-Time Fourier Transform
  - The Convolution Property
  - The Multiplication Property
  - Duality
- Systems Characterized by Linear Constant-Coefficient Difference Equations
5.12. Let

\[ y[n] = \left( \frac{\sin \frac{\pi}{4} n}{\pi n} \right)^2 \ast \left( \frac{\sin \omega_c n}{\pi n} \right), \]

where \( \ast \) denotes convolution and \( |\omega_c| \leq \pi \). Determine a stricter constraint on \( \omega_c \) which ensures that

\[ y[n] = \left( \frac{\sin \frac{\pi}{4} n}{\pi n} \right)^2. \]
5.15. Let the inverse Fourier transform of $Y(e^{j\omega})$ be

$$y[n] = \left(\frac{\sin \omega_c n}{\pi n}\right)^2,$$

where $0 < \omega_c < \pi$. Determine the value of $\omega_c$ which ensures that

$$Y(e^{j\pi}) = \frac{1}{2}.$$
5.16. The Fourier transform of a particular signal is

\[ X(e^{j\omega}) = \sum_{k=0}^{3} \frac{(1/2)^k}{1 - \frac{1}{4}e^{-j(\omega - \pi/2)k}}. \]

It can be shown that

\[ x[n] = g[n]q[n], \]

where \( g[n] \) is of the form \( \alpha^n u[n] \) and \( q[n] \) is a periodic signal with period \( N \).

(a) Determine the value of \( \alpha \).

(b) Determine the value of \( N \).

(c) Is \( x[n] \) real?
5.17. The signal $x[n] = (-1)^n$ has a fundamental period of 2 and corresponding Fourier series coefficients $a_k$. Use duality to determine the Fourier series coefficients $b_k$ of the signal $g[n] = a_n$ with a fundamental period of 2.
5.18. Given the fact that

\[
d^n \leftrightarrow \frac{1 - a^2}{1 - 2a \cos \omega + a^2}, \quad |a| < 1,
\]

use duality to determine the Fourier series coefficients of the following continuous-time signal with period \( T = 1 \):

\[
x(t) = \frac{1}{5 - 4 \cos(2\pi t)}.
\]