Outline

- Representation of Aperiodic Signals: the Discrete-Time Fourier Transform
- The Fourier Transform for Periodic Signals
- Properties of Discrete-Time Fourier Transform
- The Convolution Property
- The Multiplication Property
- Duality
- Systems Characterized by Linear Constant-Coefficient Difference Equations
5.13. An LTI system with impulse response $h_1[n] = \left( \frac{1}{3} \right)^n u[n]$ is connected in parallel with another causal LTI system with impulse response $h_2[n]$. The resulting parallel interconnection has the frequency response

$$H(e^{j\omega}) = \frac{-12 + 5e^{-j\omega}}{12 - 7e^{-j\omega} + e^{-j2\omega}}.$$ 

Determine $h_2[n]$. 
5.14. Suppose we are given the following facts about an LTI system $S$ with impulse response $h[n]$ and frequency response $H(e^{j\omega})$:

1. $(\frac{1}{4})^nu[n] \rightarrow g[n]$, where $g[n] = 0$ for $n \geq 2$ and $n < 0$.
2. $H(e^{j\pi/2}) = 1$.
3. $H(e^{j\omega}) = H(e^{j(\omega - \pi)})$.

Determine $h[n]$. 
5.19. Consider a causal and stable LTI system $S$ whose input $x[n]$ and output $y[n]$ are related through the second-order difference equation

$$y[n] - \frac{1}{6} y[n - 1] - \frac{1}{6} y[n - 2] = x[n].$$

(a) Determine the frequency response $H(e^{j\omega})$ for the system $S$.
(b) Determine the impulse response $h[n]$ for the system $S$. 
5.20. A causal and stable LTI system $S$ has the property that

$$
\left( \frac{4}{5} \right)^n u[n] \rightarrow n \left( \frac{4}{5} \right)^n u[n].
$$

(a) Determine the frequency response $H(e^{j\omega})$ for the system $S$.
(b) Determine a difference equation relating any input $x[n]$ and the corresponding output $y[n]$. 
