Outline

- Representation of Aperiodic Signals: the Discrete-Time Fourier Transform
- The Fourier Transform for Periodic Signals
- Properties of Discrete-Time Fourier Transform
- The Convolution Property
- The Multiplication Property
- Duality
- Systems Characterized by Linear Constant-Coefficent Difference Equations
9. (10%) Answer the following questions.
   a) (2%) Consider the following periodic signal \( p(t) \).

   Find the Fourier transform \( P(j\omega) \) of \( p(t) \).

   b) (3%) Consider the following operation.

   Assume that the Fourier transform \( X(j\omega) \) of one signal \( x(t) \) is as follows.
Assume that \( w_m < \frac{\pi}{T} \). Sketch the Fourier transform \( Y(jw) \) of \( y(t) \) and express \( Y(jw) \) in the form: \( Y(jw) = A \sum_{k=-\infty}^{\infty} X(jw+kB) \). What are the value of \( A \) and \( B \)?

c) \( 3\% \) Let \( r[n] = y(nT) \) and assume that:

\[
Y(jw) = \sum_{n=-\infty}^{\infty} y(nT)e^{-jwnT}, \quad \text{and} \\
R(e^{jw}) = \sum_{n=-\infty}^{\infty} r[n]e^{-jwn}.
\]

Show that \( R(e^{jw}) = Y(\frac{jw}{T}) \) and sketch the graph of \( R(e^{jw}) \).

d) \( 2\% \) Consider the following operation:

\[
\begin{array}{cccc}
& y(t) & H(j\omega) & w(t) \\
\downarrow & C & & \\
\downarrow & -\omega_0 & \omega_0 & \omega \\
\end{array}
\]

where \( H(j\omega) \) an ideal lowpass filter with a magnitude \( C \) in the passband between \(-\omega_0\) and \( \omega_0 \). Find the value of \( C \) and the range of \( \omega_0 \), such that \( w(t) = x(t) \). Justify your answer.
1. (24 %) The response $y[n]$ of a discrete-time system is related to the input $x[n]$ by

$$y[n] = \frac{1}{4} (x[n-1] + 2x[n] + x[n+1]).$$

a) Is it a linear time-invariant system? Justify your answer. (3%)
b) Is the system causal? Justify your answer. (3%)
c) Is the system stable? Justify your answer. (3%)
d) Is the system a recursive filter? Justify your answer. (3%)
e) Determine the impulse response $h[n]$. (3%)
f) Determine the step response of the system. (3%)
g) Determine the frequency response $H(e^{j\omega})$ of the system by the eigenfunction approach and by Fourier transform. (3%)
h) Is it possible to find another LTI system with impulse response $\tilde{h}[n] \neq h[n]$ such that $\tilde{H}(e^{j\omega}) = H(e^{j\omega})$? Justify your answer. (3%)
5. [12] Consider a discrete-time signal \( x[n] \) given by \( x[n] = a^n \) with \(|a| < 1\) and a continuous-time signal \( y(t) \) given by \( y(t) = \frac{1}{5 - 4 \cos(2\pi t)} \) with period \( T = 1 \).

   (a) Derive the discrete-time Fourier transform (DT-FT) of \( x[n] \) using the DT-FT analysis equation. [6]

   (b) Use the concept of duality to determine the Fourier series coefficients of \( y(t) \). [6]

6. [8] Consider the discrete-time Fourier transform pair: \( x[n] \xleftarrow{DT-FT} X(e^{j\omega}) \). If another signal \( y[n] \) is defined by the following relationship:

   \[
   y[n] = \begin{cases} 
   x[n/m], & \text{if } n \text{ is a multiple of } m \\
   0, & \text{if } n \text{ is not a multiple of } m 
   \end{cases}
   \]

   Show that \( Y(e^{j\omega}) = X(e^{jm\omega}) \), where \( Y(e^{j\omega}) \) is the Fourier transform of \( y[n] \).
8. [16] Use some properties or derivations to answer the following questions.

(a) \( x(t) = te^{-2|t-1|} \), what is \( X(j\omega) \)? [4]

(b) \( x[n] = (\sin(\frac{\pi n}{8}) - 2\cos(\frac{\pi n}{4})) * \left( \frac{\sin(\frac{\pi n}{6})}{\pi n} + \frac{\sin(\frac{\pi n}{2})}{\pi n} \right) \), what is \( X(e^{j\omega}) \)? [6]

Note that * denotes convolution.

(c) \( x(t) = \left( \frac{\sin(\pi t)}{\pi t} \right) \left( \frac{\sin(2\pi t)}{2\pi t} \right) \), what is \( X(j\omega) \)? [6]