從信號與系統到控制

單元：離散摺積-4

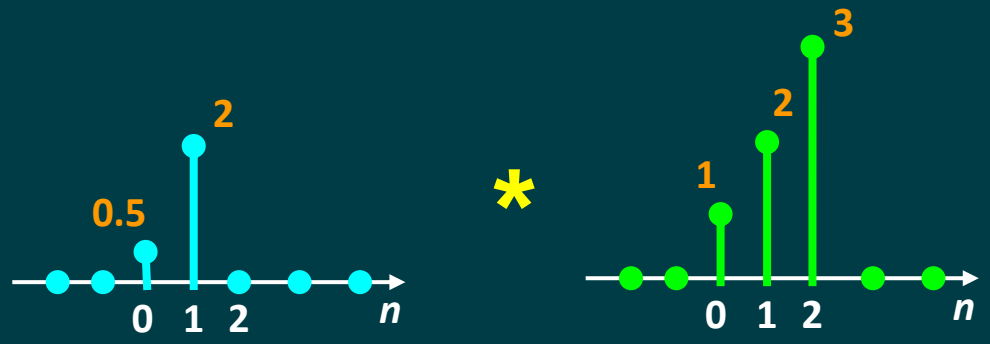
離散摺積計算-以輸出時間觀點

授課老師：連　豐　力

單元學習目標與大綱

- 離散摺積計算範例
- 以輸入信號時間軸為觀點
- 以輸出信號時間軸為觀點

離散摺積計算

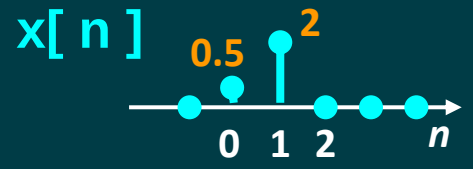


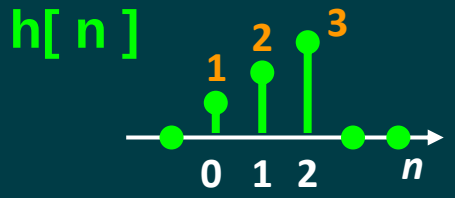
$$x[n] * h[n] = y[n]$$

$$= \sum_{k=-\infty}^{+\infty} x[k]\, h[n-k]$$

離散摺積計算-以輸出時間觀點

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]\,h[n-k]$$





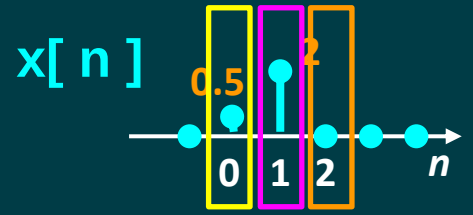
n = 0

$$y[0] = \sum_{k=-\infty}^{+\infty} x[k]\,h[0-k]$$

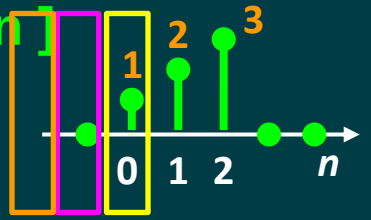
$$= \ldots + x[-1]\,h[1] + x[0]\,h[0] + x[1]\,h[-1] + x[2]\,h[-2] + \ldots$$

離散摺積計算-以輸出時間觀點

$$= \ \ldots \ + \ x[-1]\,h[1] \ + \ x[0]\,h[0]$$
$$+ \ x[1]\,h[-1] \ + \ x[2]\,h[-2] \ + \ \ldots$$

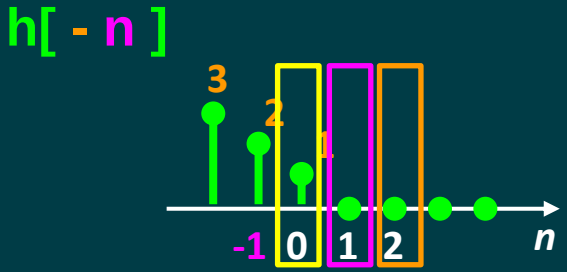


$$= \ \ldots + \ 0 * 2 \ + \ 0.5 * 1 \ + \ 2 * 0 \ + \ 0 * 0 \ + \ldots$$



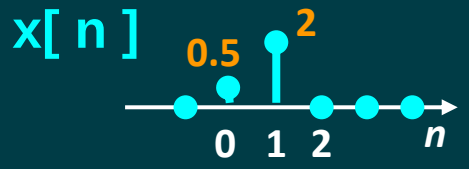
$$= \ 0.5$$

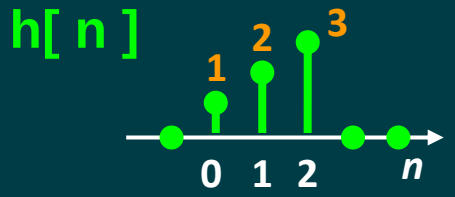
$$= \sum_{n=-\infty}^{+\infty} x[n]\,h[-n]$$



離散摺積計算-以輸出時間觀點

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]\,h[n-k]$$





n = 1

$$y[1] = \sum_{k=-\infty}^{+\infty} x[k]\,h[1-k]$$

$$= \ldots + x[-1]\,h[2] + x[0]\,h[1] + x[1]\,h[0] + x[2]\,h[-1] + \ldots$$

離散摺積計算-以輸出時間觀點

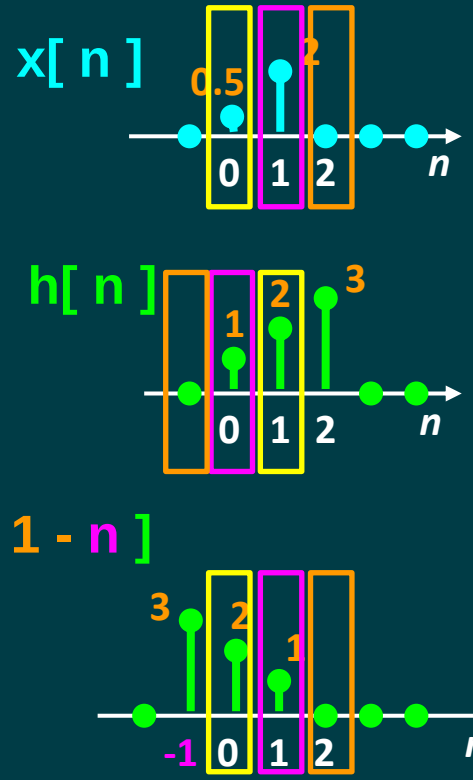
$$= \ \ldots \ + \ x[-1]\,h[2] \ + \ x[0]\,h[1]$$
$$+ \ x[1]\,h[0] \ + \ x[2]\,h[-1] \ + \ \ldots$$

$$= \ \ldots \ + \ 0 * 3 \ + 0.5 * 2 + 2 * 1 + 0 * 0 + \ldots$$

$$= \ 1 \ + \ 2$$

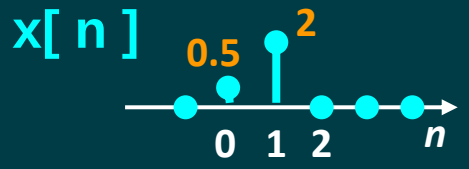
$$= \ 3$$

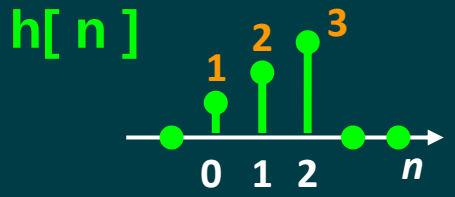
$$= \sum_{n=-\infty}^{+\infty} x[n]\,h[1-n]$$



離散摺積計算-以輸出時間觀點

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]\,h[n-k]$$





$n = 2$

$$y[2] = \sum_{k=-\infty}^{+\infty} x[k]\,h[2-k]$$

$$= \ldots + x[-1]\,h[3] + x[0]\,h[2] + x[1]\,h[1] + x[2]\,h[0] + \ldots$$

離散摺積計算-以輸出時間觀點

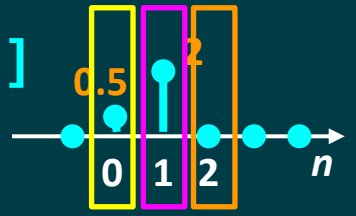
$$= \ \ldots \ + \ x[-1]\,h[3] \ + \ x[0]\,h[2]$$
$$+ \ x[1]\,h[1] \ + \ x[2]\,h[0] \ + \ \ldots$$

$$= \ \ldots \ + \ 0 * 0 \ + 0.5 * 3 + 2 * 2 + 0 * 1 + \ldots$$

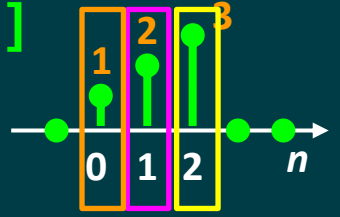
$$= \ 1.5 \ + \ 4$$

$$= \ 5.5 \qquad = \sum_{n=-\infty}^{+\infty} x[n]\,h[2-n]$$

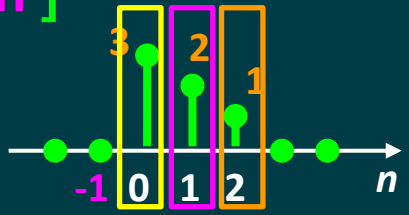
x[n]



h[n]

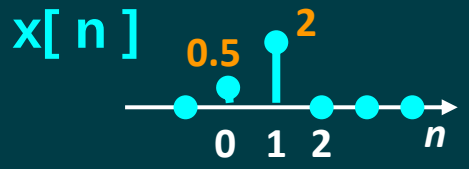


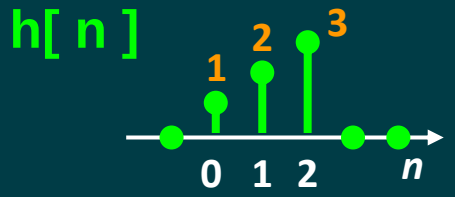
h[2 - n]



離散摺積計算-以輸出時間觀點

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]\,h[n-k]$$





$n = 3$

$$y[3] = \sum_{k=-\infty}^{+\infty} x[k]\,h[3-k]$$

$$= \ldots + x[-1]\,h[4] + x[0]\,h[3] + x[1]\,h[2] + x[2]\,h[1] + \ldots$$

離散摺積計算-以輸出時間觀點

$= \ldots + x[-1]\,h[4] + x[0]\,h[3]$

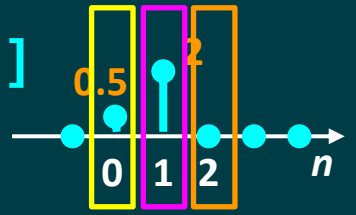
$\quad + x[1]\,h[2] + x[2]\,h[1] + \ldots$

$= \ldots + 0 * 0 + 0.5 * 0 + 2 * 3 + 0 * 2 + \ldots$

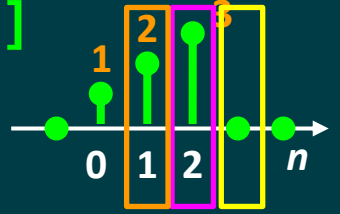
$= 6$

$$= \sum_{n=-\infty}^{+\infty} x[n]\,h[3-n]$$

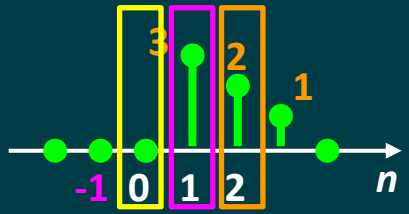
x[n]



h[n]



h[3 - n]



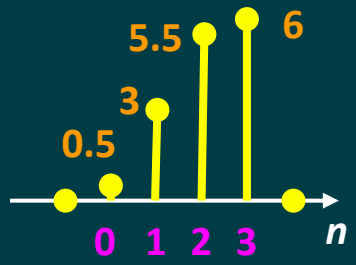
離散摺積計算-以輸出時間觀點

$y[0] = 0.5 * 1$

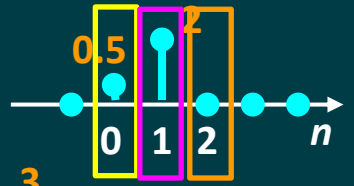
$y[1] = 0.5 * 2 + 2 * 1$

$y[2] = 0.5 * 3 + 2 * 2$

$y[3] = 2 * 3$

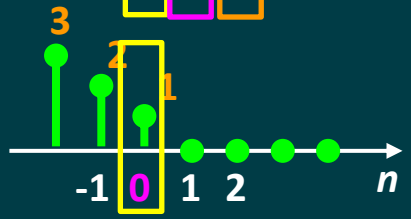


x[n]

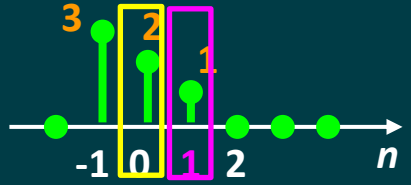


h[n]

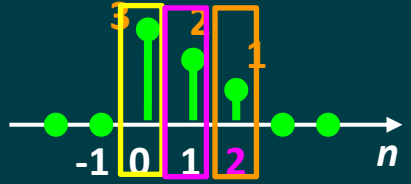
h[0 - n]



h[1 - n]

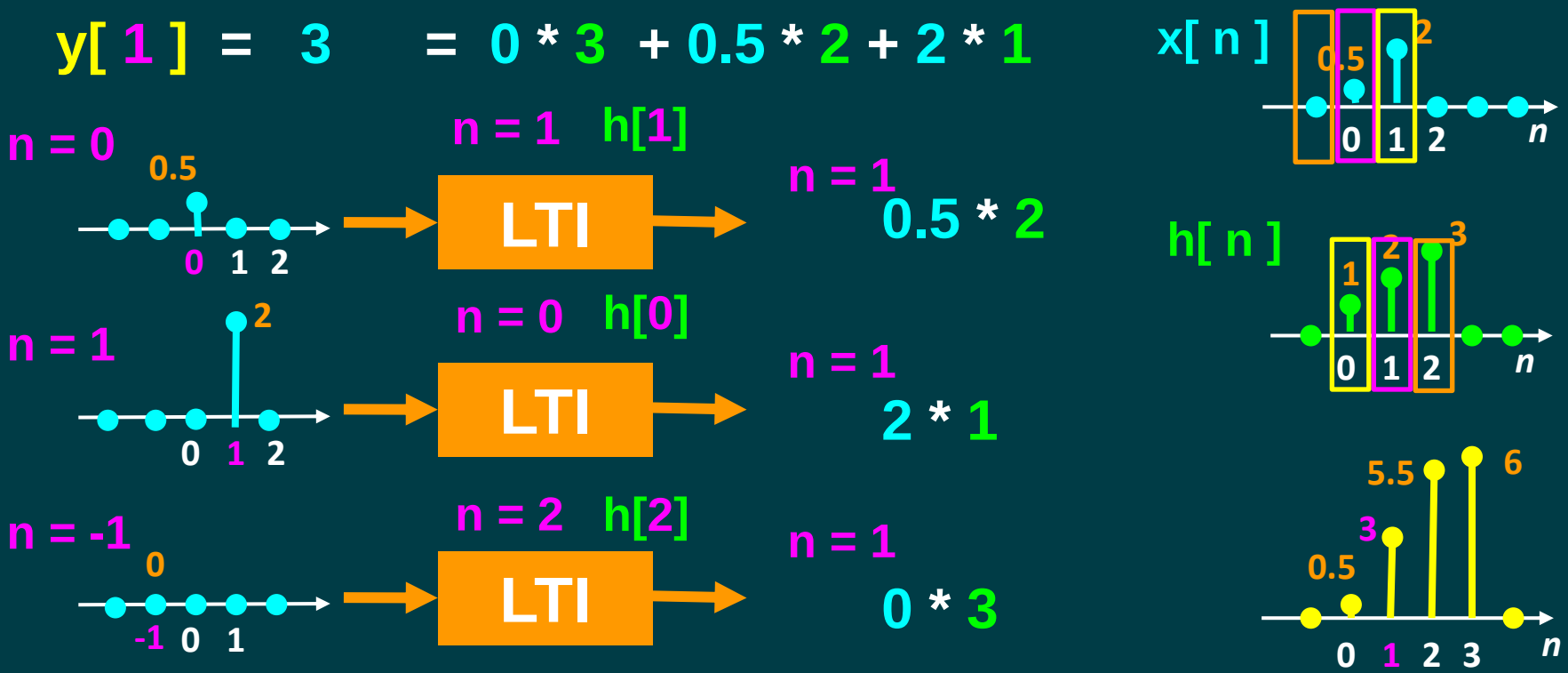


h[2 - n]



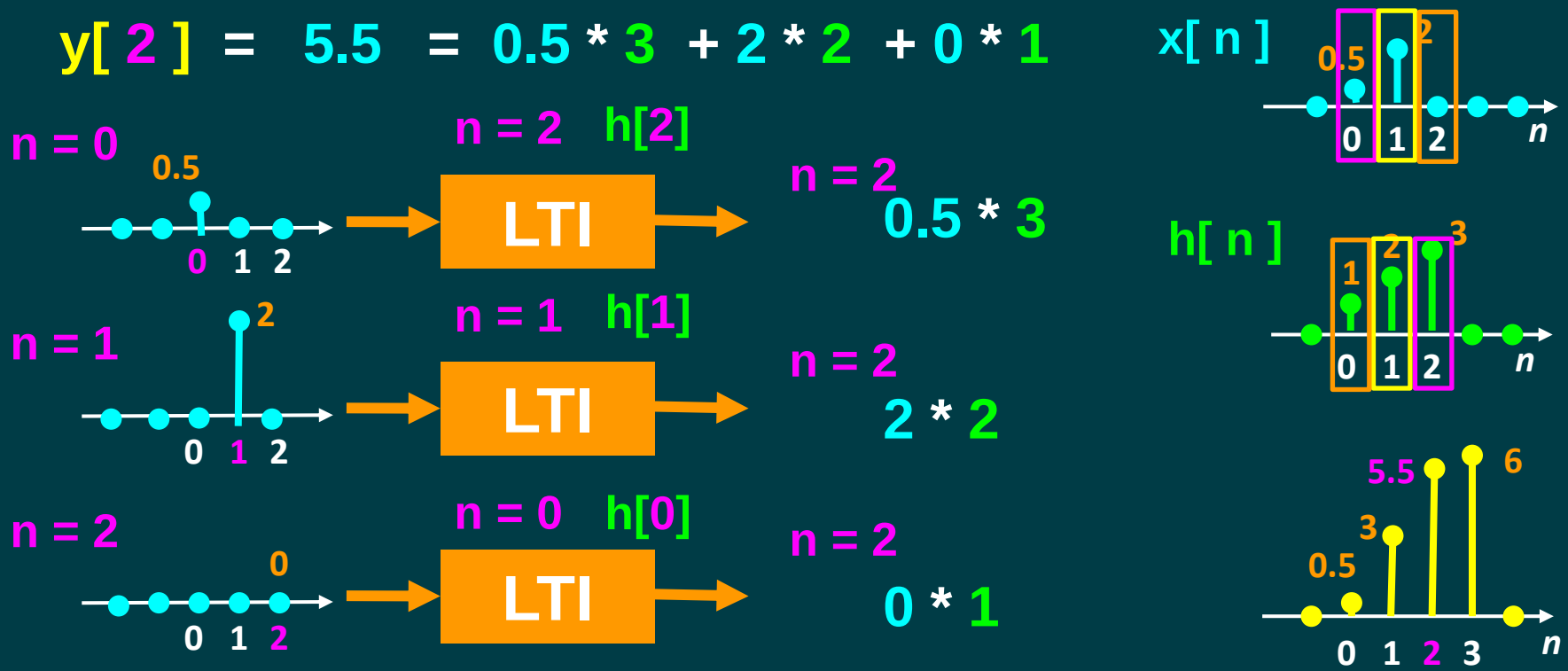
離散摺積計算-以輸出時間觀點

y[1] = 3 = 0 * 3 + 0.5 * 2 + 2 * 1



離散摺積計算-以輸出時間觀點

$y[2] = 5.5 = 0.5 * 3 + 2 * 2 + 0 * 1$



參考文獻

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  Signals & Systems,
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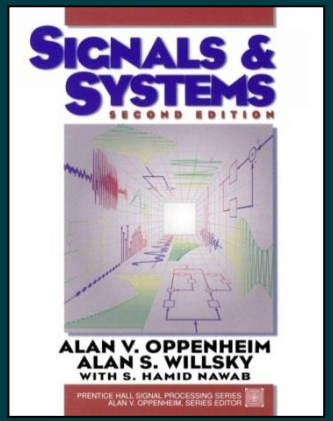

- **SciLab**:
  Open source software for numerical computation
  http://www.scilab.org/