從信號與系統到控制

單元：摺積操作性質 - 7
交換律 分配律 結合律 與 系統的關係

授課老師：連 豐 力
單元學習目標與大綱

• 交換律 分配律 結合律 之 操作法則 與

• 系統 輸入 與 輸出 之間的關係

• 多個系統 運作的關係
摺積計算操作 之 交換律

\[ x[n] \ast h[n] = h[n] \ast x[n] \]
\[ x(t) \ast h(t) = h(t) \ast x(t) \]

\[ y[n] = x[n] \ast h[n] \quad y[n] = h[n] \ast x[n] \]
\[ y(t) = x(t) \ast h(t) \quad y(t) = h(t) \ast x(t) \]
摺積計算操作之交換律

\[ y[n] = x[n] \ast h[n] \quad y[n] = h[n] \ast x[n] \]
\[ y(t) = x(t) \ast h(t) \quad y(t) = h(t) \ast x(t) \]

\[ \begin{align*}
x(t), x[n] & \rightarrow \text{LTI} \rightarrow y(t), y[n] \\
h(t), h[n] & \rightarrow \text{LTI} \rightarrow y(t), y[n] \\
h(t), h[n] & \rightarrow \text{LTI} \rightarrow y(t), y[n]
\end{align*} \]
摺積計算操作之分配律

\[
x[n] \ast \left( h_1[n] + h_2[n] \right) = x[n] \ast h_1[n] + x[n] \ast h_2[n]
\]

\[
x(t) \ast \left( h_1(t) + h_2(t) \right) = x(t) \ast h_1(t) + x(t) \ast h_2(t)
\]

\[
\text{LTI} \left( h_1 + h_2 \right)
\]

\[
x(t) \ast \left( h_1 + h_2 \right) = x(t) \ast h_1 + x(t) \ast h_2
\]
摺積計算操作之分配律

\[
\begin{align*}
(x_1[n] + x_2[n]) \ast h[n] &= x_1[n] \ast h[n] + x_2[n] \ast h[n] \\
(x_1(t) + x_2(t)) \ast h(t) &= x_1(t) \ast h(t) + x_2(t) \ast h(t)
\end{align*}
\]
摺積計算操作之分配律

\[(x_1 + x_2) \ast h = x_1 \ast h + x_2 \ast h\]
摺積計算操作之結合律

\[
\begin{align*}
(a[n] \ast (b[n] \ast c[n])) & = (a[n] \ast b[n]) \ast c[n] \\
(a(t) \ast (b(t) \ast c(t))) & = (a(t) \ast b(t)) \ast c(t)
\end{align*}
\]
摺積計算操作之結合律

\[ a \ast (b \ast c) = (a \ast b) \ast c \]
交換律 分配律 結合律 與系統的關係

• 交換律 (Commutative)
  \[ x \ast h = h \ast x \]

• 分配律 (Distributive)
  \[ y \ast (x + h) = y \ast x + y \ast h \]

• 結合律 (Associative)
  \[ y \ast (x \ast h) = (y \ast x) \ast h \]
参考文献


- SciLab:
  Open source software for numerical computation
  http://www.scilab.org/