從信號與系統到控制

單元：連續F級數-2

連續時間 三角函數 的 傅立葉級數 - 直覺

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單元學習目標與大綱

- 討論 連續時間 三角函數 的 傅立葉級數
- 直接比較 函數 與 傅立葉級數的係數

連續時間三角函數

$$x(t) = 1 + \sin(w_0 t) + 2\cos(w_0 t) + \cos\left(2w_0 t + \frac{\pi}{4}\right)$$

$$= 1$$

$$+ \frac{1}{2j}\left(e^{jw_0 t} - e^{-jw_0 t}\right)$$

$$+ 2\,\frac{1}{2}\left(e^{jw_0 t} + e^{-jw_0 t}\right)$$

$$+ \frac{1}{2}\left(e^{j\left(2w_0 t + \frac{\pi}{4}\right)} + e^{-j\left(2w_0 t + \frac{\pi}{4}\right)}\right)$$

$$\cos(s) = \frac{1}{2}\left(e^{js} + e^{-js}\right)$$

$$\sin(s) = \frac{1}{2j}\left(e^{js} - e^{-js}\right)$$

連續時間三角函數

$$x(t) = 1 + \frac{1}{2j}\left(e^{jw_0t} - e^{-jw_0t}\right)$$
$$+ \left(e^{jw_0t} + e^{-jw_0t}\right)$$
$$+ \frac{1}{2}\left(e^{j\frac{\pi}{4}} e^{j2w_0t} + e^{-j\frac{\pi}{4}} e^{-j2w_0t}\right)$$
$$= 1 + \left(1 + \frac{1}{2j}\right) e^{jw_0t} + \left(1 - \frac{1}{2j}\right) e^{-jw_0t}$$
$$+ \left(\frac{1}{2} e^{j\frac{\pi}{4}}\right) e^{j2w_0t} + \left(\frac{1}{2} e^{-j\frac{\pi}{4}}\right) e^{-j2w_0t}$$

連續時間三角函數

$$x(t) = \boxed{1} + \boxed{\left(1+\frac{1}{2j}\right)} e^{jw_0t} + \boxed{\left(1-\frac{1}{2j}\right)} e^{-jw_0t}$$

$$+ \boxed{\left(\frac{1}{2}e^{j\frac{\pi}{4}}\right)} e^{j2w_0t} + \boxed{\left(\frac{1}{2}e^{-j\frac{\pi}{4}}\right)} e^{-j2w_0t})$$

$$= \boxed{a_0} + \boxed{a_1} e^{jw_0t} + \boxed{a_{-1}} e^{-jw_0t}$$

$$+ \boxed{a_2} e^{j2w_0t} + \boxed{a_{-2}} e^{-j2w_0t}$$

$$a_0 = 1$$

$$a_1 = \left(1+\frac{1}{2j}\right) \quad a_2 = \left(\frac{1}{2}e^{j\frac{\pi}{4}}\right) \quad a_{-1} = \left(1-\frac{1}{2j}\right) \quad a_{-2} = \left(\frac{1}{2}e^{-j\frac{\pi}{4}}\right)$$

連續時間三角函數

$e^{js} = \cos(s) + j\sin(s)$

$a_0 = 1$

$a_1 = (1 + \frac{1}{2j}) = 1 - \frac{1}{2}j$

$a_{-1} = (1 - \frac{1}{2j}) = 1 + \frac{1}{2}j$

$a_2 = (\frac{1}{2} e^{j\frac{\pi}{4}}) = \frac{1}{2}[\cos(\frac{\pi}{4}) + j\sin(\frac{\pi}{4})] = \frac{\sqrt{2}}{4}(1 + j)$

$a_{-2} = (\frac{1}{2} e^{-j\frac{\pi}{4}}) = \frac{1}{2}[\cos(\frac{\pi}{4}) - j\sin(\frac{\pi}{4})] = \frac{\sqrt{2}}{4}(1 - j)$

連續時間三角函數

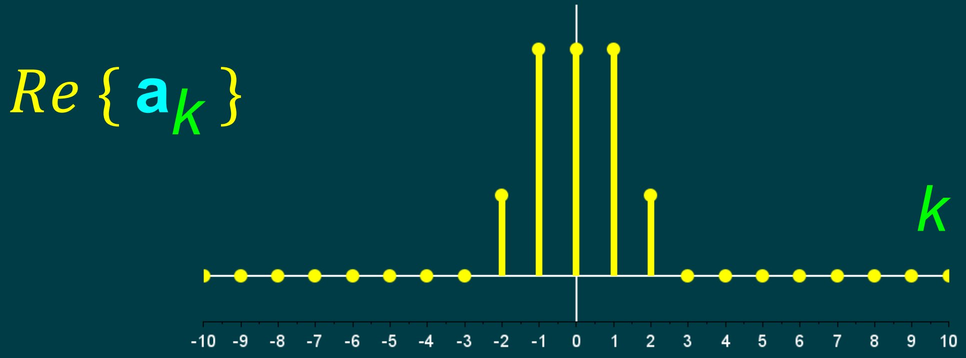
$$a_0 = 1$$

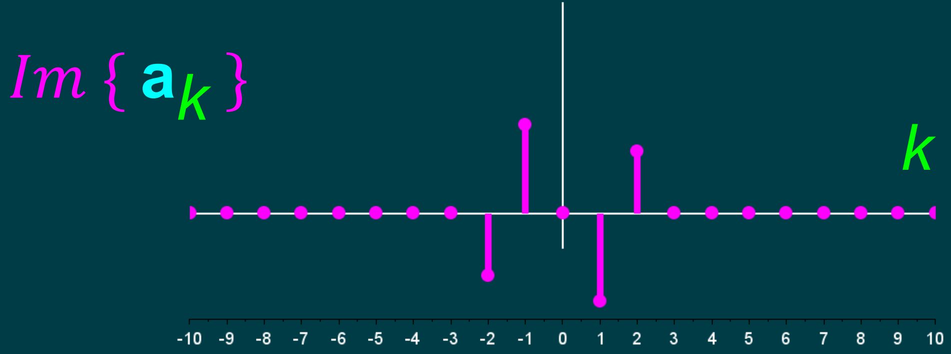
$$a_1 = 1 - j\frac{1}{2}$$

$$a_{-1} = 1 + j\frac{1}{2}$$

$$a_2 = \frac{\sqrt{2}}{4} + j\frac{\sqrt{2}}{4}$$

$$a_{-2} = \frac{\sqrt{2}}{4} - j\frac{\sqrt{2}}{4}$$





連續時間三角函數

$$a_0 = \boxed{1}$$

$$\boxed{Re + j\,Im = Mag\; e^{j\,Ang}}$$

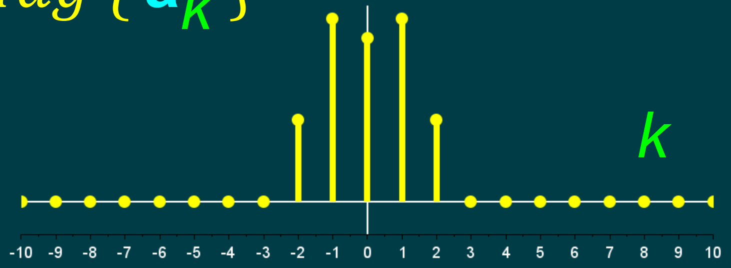
$$a_1 = 1 - j\frac{1}{2} = 1.12\, e^{j\,(-0.46)}$$

$$a_{-1} = 1 + j\frac{1}{2} = 1.12\, e^{j\,(0.46)}$$

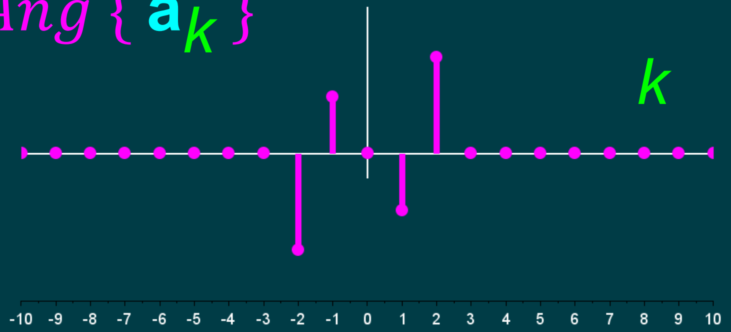
$$a_2 = \frac{\sqrt{2}}{4} + j\frac{\sqrt{2}}{4} = 0.50\, e^{j\,(0.79)}$$

$$a_{-2} = \frac{\sqrt{2}}{4} - j\frac{\sqrt{2}}{4} = 0.50\, e^{j\,(-0.79)}$$

$Mag\{a_k\}$



$Ang\{a_k\}$

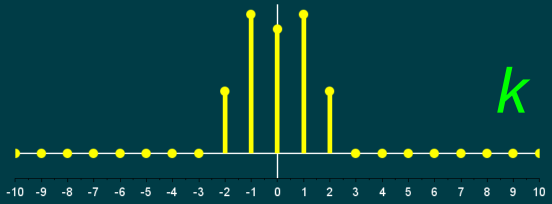


連續時間三角函數

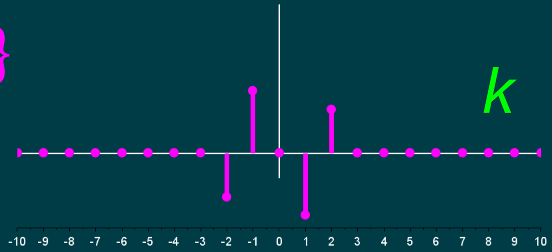
$$x(t) = 1 + \sin(w_0 t) + 2\cos(w_0 t) + \cos(2w_0 t + \frac{\pi}{4})$$

$Re\{a_k\}$

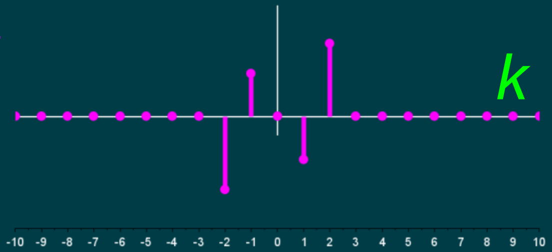
$Mag\{a_k\}$



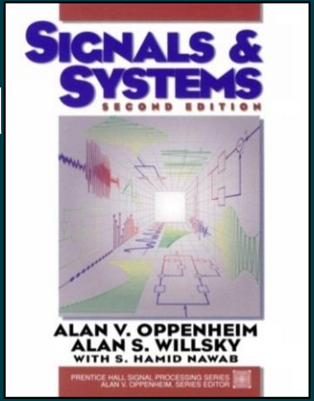
$Im\{a_k\}$



$Ang\{a_k\}$



參考文獻



- Alan V. Oppenheim, Alan S. Willsky, S. Hamid
  Signals & Systems,
  Prentice Hall, 2nd Edition, 1997

- SciLab:
  Open source software for numerical computation
  http://www.scilab.org/