

# 從信號與系統到控制

單元：連續F級數-2

連續時間 三角函數 的 傅立葉級數 - 直覺

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# 單元學習目標與大綱

- 討論 連續時間 三角函數 的 傅立葉級數
- 直接比較 函數 與 傅立葉級數的係數

# 連續時間三角函數

$$x(t) = \boxed{1} + \boxed{\sin(w_0 t)} + \boxed{2 \cos(w_0 t)} + \boxed{\cos(2w_0 t + \frac{\pi}{4})}$$

$$= 1$$

$$+ \frac{1}{2j} ( e^{j w_0 t} - e^{-j w_0 t} )$$

$$+ 2 \frac{1}{2} ( e^{j w_0 t} + e^{-j w_0 t} )$$

$$+ \frac{1}{2} ( e^{j (2w_0 t + \frac{\pi}{4})} + e^{-j (2w_0 t + \frac{\pi}{4})} )$$

$$\cos(s) = \frac{1}{2} ( e^{js} + e^{-js} )$$

$$\sin(s) = \frac{1}{2j} ( e^{js} - e^{-js} )$$

# 連續時間三角函數

$$\begin{aligned}x(t) &= \boxed{1} + \frac{1}{2j} \left( \boxed{e^{jw_0t}} - \boxed{e^{-jw_0t}} \right) \\&\quad + \left( \boxed{e^{jw_0t}} + \boxed{e^{-jw_0t}} \right) \\&\quad + \frac{1}{2} \left( \boxed{e^{j\frac{\pi}{4}}} \boxed{e^{j2w_0t}} + \boxed{e^{-j\frac{\pi}{4}}} \boxed{e^{-j2w_0t}} \right) \\&= 1 + \left( 1 + \frac{1}{2j} \right) e^{jw_0t} + \left( 1 - \frac{1}{2j} \right) e^{-jw_0t} \\&\quad + \left( \frac{1}{2} e^{j\frac{\pi}{4}} \right) e^{j2w_0t} + \left( \frac{1}{2} e^{-j\frac{\pi}{4}} \right) e^{-j2w_0t}\end{aligned}$$

# 連續時間三角函數

$$x(t) = \boxed{1} + \left( 1 + \frac{1}{2j} \right) e^{j w_0 t} + \left( 1 - \frac{1}{2j} \right) e^{-j w_0 t} \\ + \left( \frac{1}{2} e^{j \frac{\pi}{4}} \right) e^{j 2w_0 t} + \left( \frac{1}{2} e^{-j \frac{\pi}{4}} \right) e^{-j 2w_0 t}$$

$$= \boxed{a_0} + \boxed{a_1} e^{j w_0 t} + \boxed{a_{-1}} e^{-j w_0 t} \\ + \boxed{a_2} e^{j 2w_0 t} + \boxed{a_{-2}} e^{-j 2w_0 t}$$

$$a_0 = 1$$

$$a_1 = \left( 1 + \frac{1}{2j} \right) \quad a_2 = \left( \frac{1}{2} e^{j \frac{\pi}{4}} \right)$$

$$a_{-1} = \left( 1 - \frac{1}{2j} \right) \quad a_{-2} = \left( \frac{1}{2} e^{-j \frac{\pi}{4}} \right)$$

# 連續時間三角函數

$$a_0 = 1$$

$$a_1 = \left(1 + \frac{1}{2}j\right) = 1 - \frac{1}{2}j$$

$$a_{-1} = \left(1 - \frac{1}{2}j\right) = 1 + \frac{1}{2}j$$

$$a_2 = \left(\frac{1}{2}e^{j\frac{\pi}{4}}\right) = \frac{1}{2} \left[ \cos\left(\frac{\pi}{4}\right) + j \sin\left(\frac{\pi}{4}\right) \right] = \frac{\sqrt{2}}{4} (1 + j)$$

$$a_{-2} = \left(\frac{1}{2}e^{-j\frac{\pi}{4}}\right) = \frac{1}{2} \left[ \cos\left(\frac{\pi}{4}\right) - j \sin\left(\frac{\pi}{4}\right) \right] = \frac{\sqrt{2}}{4} (1 - j)$$

$$e^{js} = \cos(s) + j \sin(s)$$

# 連續時間三角函數

$$a_0 = \boxed{1}$$

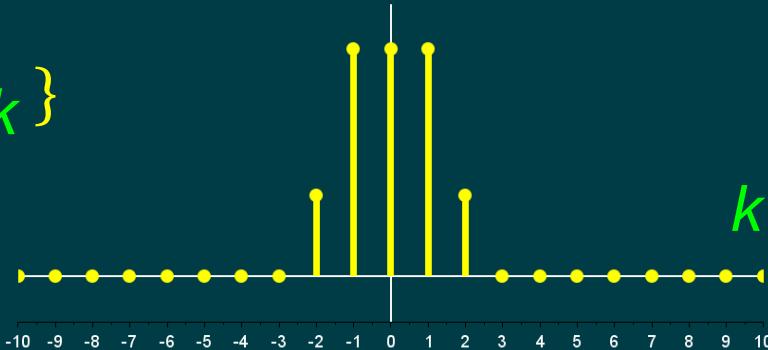
$$a_1 = 1 + j \frac{1}{2}$$

$$a_{-1} = 1 + j \frac{1}{2}$$

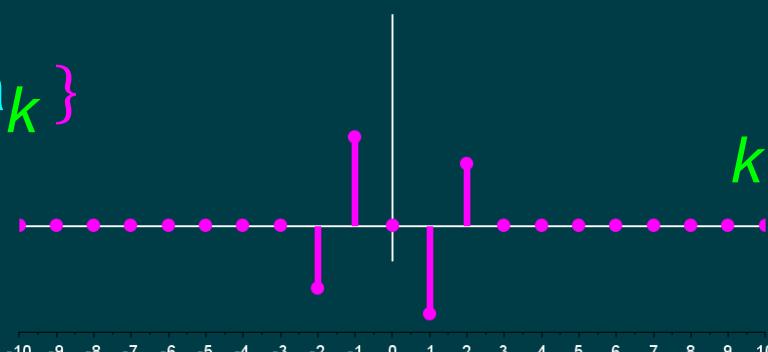
$$a_2 = \frac{\sqrt{2}}{4} + j \frac{\sqrt{2}}{4}$$

$$a_{-2} = \frac{\sqrt{2}}{4} - j \frac{\sqrt{2}}{4}$$

$$Re \{ a_k \}$$



$$Im \{ a_k \}$$



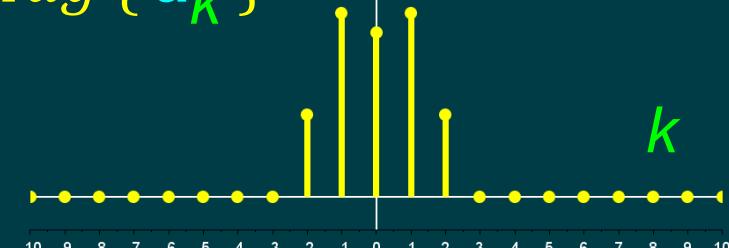
# 連續時間三角函數

$$a_0 = \boxed{1} \quad \boxed{\text{Re} + j \text{Im} = \text{Mag } e^{j \text{Ang}}} \quad \text{Mag } \{ a_k \}$$

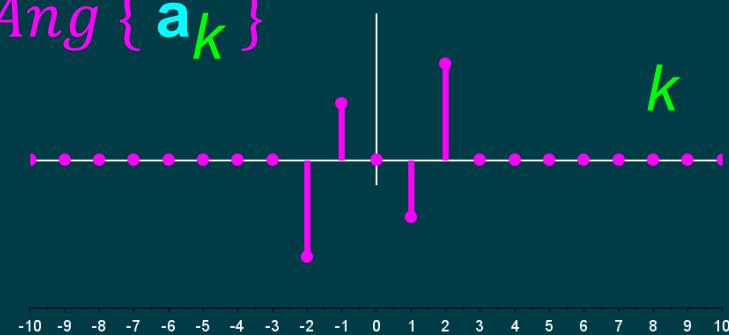
$$a_1 = 1 - j \frac{1}{2} = \boxed{1.12} e^{j (-0.46)}$$

$$a_{-1} = 1 + j \frac{1}{2} = \boxed{1.12} e^{j (0.46)}$$

$$a_2 = \frac{\sqrt{2}}{4} + j \frac{\sqrt{2}}{4} = \boxed{0.50} e^{j (0.79)}$$

$$a_{-2} = \frac{\sqrt{2}}{4} - j \frac{\sqrt{2}}{4} = \boxed{0.50} e^{j (-0.79)}$$


A plot showing the magnitude of discrete-time signal components  $a_k$  versus index  $k$ . The x-axis ranges from -10 to 10. Vertical bars represent the magnitude at each integer value of  $k$ . The values are non-zero only for  $k = -2, -1, 0, 1, 2$ , with a maximum magnitude of approximately 1.12.

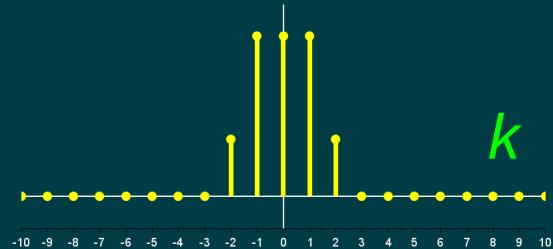


A plot showing the angle of discrete-time signal components  $a_k$  versus index  $k$ . The x-axis ranges from -10 to 10. Vertical bars represent the angle at each integer value of  $k$ . The angles are non-zero only for  $k = -2, -1, 0, 1, 2$ , with a maximum angle of approximately 0.79 radians.

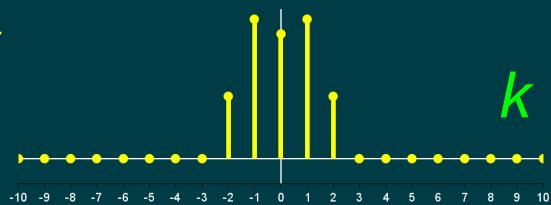
# 連續時間三角函數

$$x(t) = 1 + \sin(w_0 t) + 2 \cos(w_0 t) + \cos(2w_0 t + \frac{\pi}{4})$$

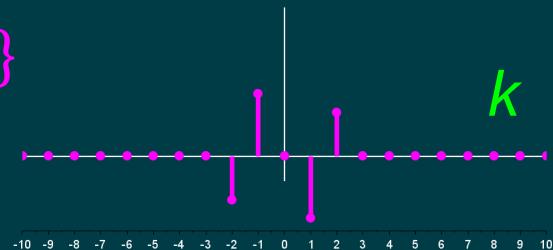
$Re \{ a_k \}$



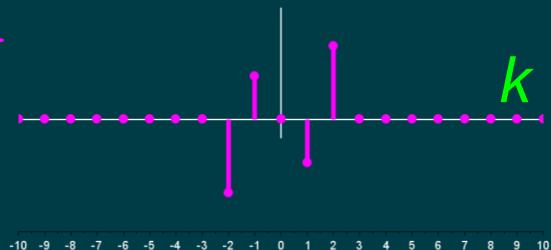
$Mag \{ a_k \}$



$Im \{ a_k \}$



$Ang \{ a_k \}$



# 參考文獻

- Alan V. Oppenheim, Alan S. Willsky, S. Hamid  
**Signals & Systems**,  
Prentice Hall, 2nd Edition, 1997
- **SciLab:**  
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