從信號與系統到控制

單元：連續F轉換-6

傅立葉轉換 範例 - 方波函數

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單元學習目標與大綱

- 根據 傅立葉轉換 的公式與關係式
- 計算 方波函數 的 傅立葉轉換

傅立葉轉換 的 表示式

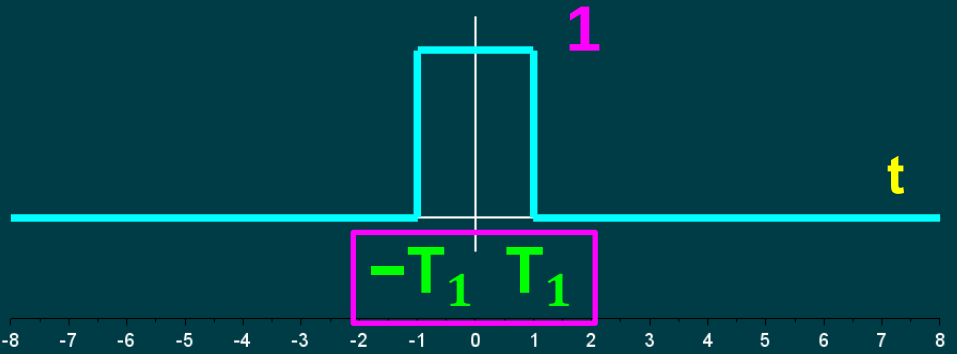
$$x(t) \xleftrightarrow{FT} X(jw)$$

$$X(jw) = \int_{-\infty}^{\infty} x(t)\, e^{-jwt}\, dt$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(jw)\, e^{jwt}\, dw$$

方波函數 的 傅立葉轉換

$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & |t| > T_1 \end{cases}$$



$$X(jw) = \int_{-\infty}^{\infty} x(t)\, e^{-j w t}\, dt$$

$$= \int_{-T_1}^{T_1} 1 \cdot e^{-j w t}\, dt = \frac{1}{(-jw)}\, e^{(-jw)t} \Big|_{-T_1}^{T_1}$$

方波函數 的 傅立葉轉換

$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & |t| > T_1 \end{cases}$$

$$X(jw) = \frac{1}{(-jw)} e^{(-jw)t} \Big|_{-T_1}^{T_1}$$

$$= \frac{1}{(-jw)} \left(e^{(-jw)T_1} - e^{(-jw)(-T_1)} \right)$$

$$= \frac{1}{(jw)} \left(e^{jwT_1} - e^{-jwT_1} \right)$$

方波函數 的 傅立葉轉換

$$X(jw) = \frac{2j}{(jw)} \cdot \frac{1}{2j}\left(e^{jwT_1} - e^{-jwT_1} \right)$$

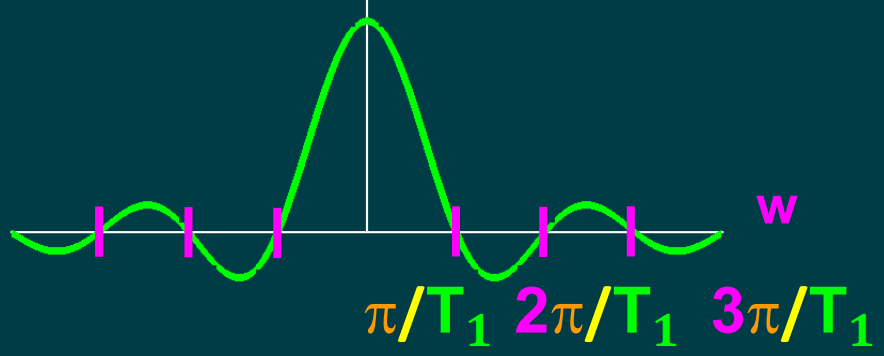
$$= \frac{2j}{(jw)} \sin(wT_1)$$

$$= \frac{2}{w} \sin(wT_1)$$

$$\sin(s) = \frac{1}{2j}\left(e^{js} - e^{-js}\right)$$

$w = 1\pi/T_1,\ 2\pi/T_1, \ldots, k\pi/T_1$

$\sin((k\pi/T_1) T_1) = \sin(k\pi)$



方波函數 的 傅立葉轉換

$$X(jw) = \frac{2}{w}\sin(wT_1)$$

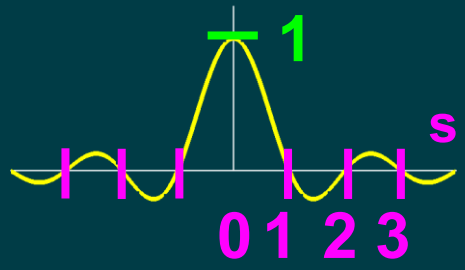
$$= \frac{2T_1}{\pi \frac{wT_1}{\pi}}\sin(\pi \frac{wT_1}{\pi})$$

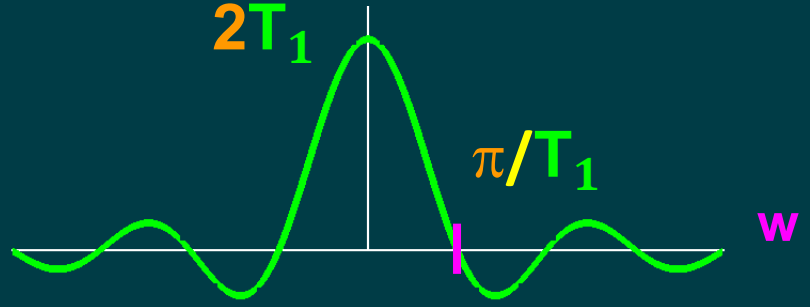
$$= 2T_1 \frac{\sin(\pi wT_1/\pi)}{\pi wT_1/\pi}$$

$$= 2T_1\,\mathrm{sinc}(wT_1/\pi)$$

$s = wT_1/\pi$

$$\mathrm{sinc}(s) = \frac{\sin(\pi s)}{\pi s}$$

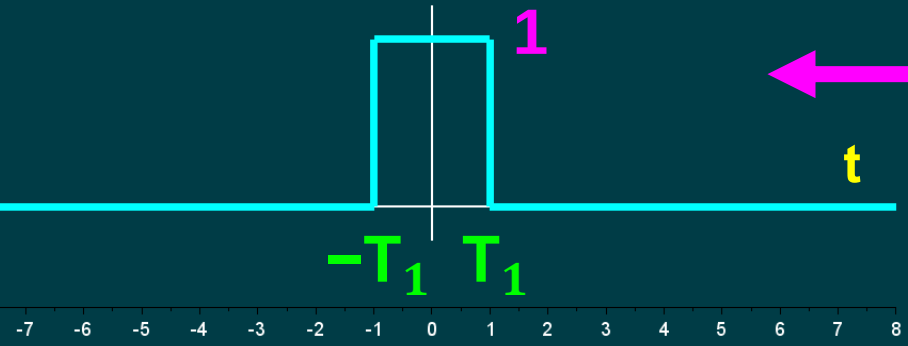




方波函數 的 傅立葉轉換

$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & |t| > T_1 \end{cases} \quad \overset{FT}{\longleftrightarrow} \quad X(jw) = \frac{2}{w}\sin(wT_1)$$

$$= 2T_1 \mathrm{sinc}(wT_1/\pi)$$



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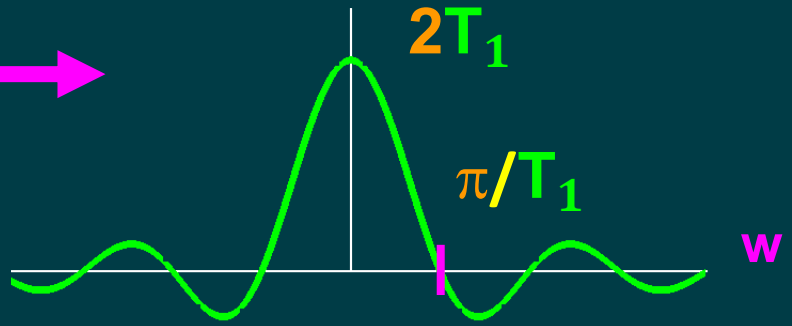
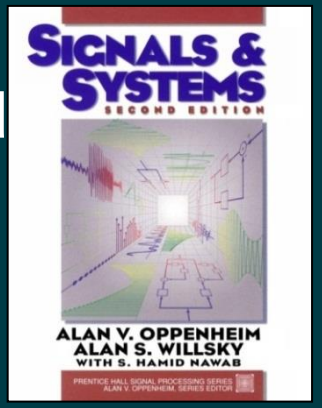


參考文獻



- Alan V. Oppenheim, Alan S. Willsky, S. Hamid
  Signals & Systems,
  Prentice Hall, 2nd Edition, 1997

- **SciLab**:
  Open source software for numerical computation
  http://www.scilab.org/