從信號與系統到控制

單元：離散F轉換-1

從週期信號到非週期信號的特性改變

授課老師：連豊力
單元學習目標與大綱

• 複習 - 離散時間 週期方波 之 傅立葉級數

• 推演 - 不同週期的信號之 傅立葉級數差異

• 猜想 - 將 週期 變成 無限大 的 非週期信號 之 傅立葉級數
離散時間週期方波函數的傅立葉級數

\[ x[n] = \begin{cases} 
1, & -N_1 \geq n > N_1 \\
0, & \text{others in } <N> 
\end{cases} \]

\[ a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jkw_0n} \]

\[ a_k = \frac{1}{N} \left( 2N_1 + 1 \right) \]

\[ \frac{1}{N} \sin(\frac{kw_0(N_1+1/2)}{2}) \]

\[ w_0 = \frac{2\pi}{N} \]

\[ k = 0, \pm N, \pm 2N, \ldots \]

\[ k \neq 0, \pm N, \pm 2N, \ldots \]
離散時間週期方波函數的傅立葉級數

\[ x[n] = \begin{cases} 1, & -N_1 \geq n > N_1 \\ 0, & \text{others in } <N> \end{cases} \]

\[ a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jkw_0n} \]

\[ N a_k = N \frac{1}{N} (2N_1 + 1) = (2N_1 + 1) \quad k = 0, \pm N, \pm 2N, \ldots \]

\[ = N \frac{1}{N} \frac{\sin(\frac{kw_0(N_1 + 1/2)}{2})}{\sin(\frac{kw_0}{2})} = \frac{\sin(\frac{kw_0(N_1 + 1/2)}{2})}{\sin(\frac{kw_0}{2})} \quad k \neq 0, \pm N, \pm 2N, \ldots \]

\[ w_0 = \frac{2\pi}{N} \]
離散時間週期方波函數的傅立葉級數

\[ 2N_1 + 1 = 5 \]

\[ N = 10 \]

\[ x[n] \]

\[ N a_k = (2N_1 + 1) \]

\[ = \frac{\sin(kw_0(N_1 + 1/2))}{\sin(kw_0/2)} \]

\[ k = 0, \pm N, \pm 2N, \ldots \]

\[ k \neq 0, \pm N, \pm 2N, \ldots \]
離散時間週期方波函數的傅立葉級數

\[ 2N_1 + 1 = 5 \]
\[ N = 20 \]
\[ x[n] \]

\[ N a_k = \begin{cases} 
2N_1 +1 & k = 0, \pm N, \pm 2N, \ldots \\
\frac{\sin(kw_0(N_1 + 1/2))}{\sin(kw_0/2)} & k \neq 0, \pm N, \pm 2N, \ldots
\end{cases} \]
離散時間週期方波函數的傅立葉級數

\[ 2N_1 + 1 = 5 \]
\[ N = 40 \]

\[ x[n] \]

\[ a_k = \begin{cases} 
(2N_1 + 1) & \text{for } k = 0, \pm N, \pm 2N, \ldots \\
\frac{\sin \left( kw_0 \frac{(N_1 + 1/2)}{2} \right)}{\sin \left( kw_0 / 2 \right)} & \text{for } k \neq 0, \pm N, \pm 2N, \ldots 
\end{cases} \]
不同週期信號之傅立葉級數

\[ 2N_1 + 1 = 5 \]

\[ N = 10 \]

\[ x[n] \]

\[ N a_k \]

\[ N = 20 \]

\[ N = 40 \]
不同週期信號之傅立葉級數

$$w_0 = \frac{2\pi}{N}$$

$$w_0 = \frac{2\pi}{10} = \frac{\pi}{5}$$

$$w_0 = \frac{2\pi}{20} = \frac{\pi}{10}$$

$$w_0 = \frac{2\pi}{40} = \frac{\pi}{20}$$
不同週期信號之傅立葉級數

\[ w_0 = \frac{2\pi}{N} \]

1. 当 \( N = 10 \):
   \[ w_0 = \frac{2\pi}{10} = \frac{\pi}{5} \]
   \[ Nw_0 = 2\pi \]

2. 当 \( N = 20 \):
   \[ w_0 = \frac{2\pi}{20} = \frac{\pi}{10} \]
   \[ Nw_0 = 2\pi \]

3. 当 \( N = 40 \):
   \[ w_0 = \frac{2\pi}{40} = \frac{\pi}{20} \]
   \[ Nw_0 = 2\pi \]
不同週期信號之傅立葉級數

\[ w = k w_0 \]

\[ k w_0 \]

\[ N w_0 \]
不同週期信號之傅立葉級數

\[ 2N_1 + 1 = 5 \]
\[ N = 10 \]

\[ N = 20 \]

\[ N = 40 \]
不同週期信號之傅立葉級數

\[ 2N_1 + 1 = 5 \]

\[ N = 20 \]

\[ N \rightarrow \infty \]

\[ w_0 = \frac{2\pi}{N} \]

\[ w_0 \rightarrow 0 \]
参考文献


- SciLab: Open source software for numerical computation http://www.scilab.org/