Wraparound patch resonators on a composite ground plane

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Indexing terms: Patch resonator, Laminated ground plane, Wraparound patch

Abstract: The effects of a laminated ground plane on the resonant frequencies of wraparound patch resonators are studied. An integral equation is formulated in terms of the surface current on the patch. The laminated ground is modelled by a transition matrix in the spectral domain. Factors analysed include substrate thickness, substrate dielectric constant and laminate conductivity. Due to ohmic losses, more power loss is incurred for certain laminate conductivities.

1 Introduction

Cylindrical rectangular patches and wraparound patches can be used as radiators on missile and airplane surfaces due to their conformability [1]. Various techniques used to study the radiation properties of these radiators on a perfect conductor ground plane have been reviewed [2]. Composite materials are widely used on airplanes and missiles to reduce their weight and radar cross-section. The effects of a composite ground plane on the resonant frequencies of a cylindrical rectangular patch have been analysed [2]. An integral equation is derived in terms of the electric current density on the patch surface where the composite ground plane is modelled by a cascaded transition matrix. The Galerkin method is then used to solve the integral equation for the resonant frequencies.

Since the resonance properties of a wraparound patch are different from those of a cylindrical rectangular patch, the technique used in [2] will be used to calculate the resonant frequencies of the wraparound patch on a composite ground plane. The major difference between this formulation and [2] is in the choice of basis functions and weighting functions. For clarity of presentation, the formulation in [2] will be briefly described in the next Section, followed by the numerical results.

2 Formulation

Fig. 1 shows the geometrical configuration of a wraparound patch resonator on a composite ground plane which consists of N laminas with the thickness of the ith lamina being t_i. The laminated ground plane extends from $\rho = \rho_N = a$ back to $\rho = \rho_0 = a - t_1 - t_2 - \dots - t_N$. A perfect conductor coating is plated at the inner surface $\rho = \rho_0$. The wraparound patch is on top of the substrate which extends from $\rho = a$ to $\rho = b$ with thickness $h = b - a$.

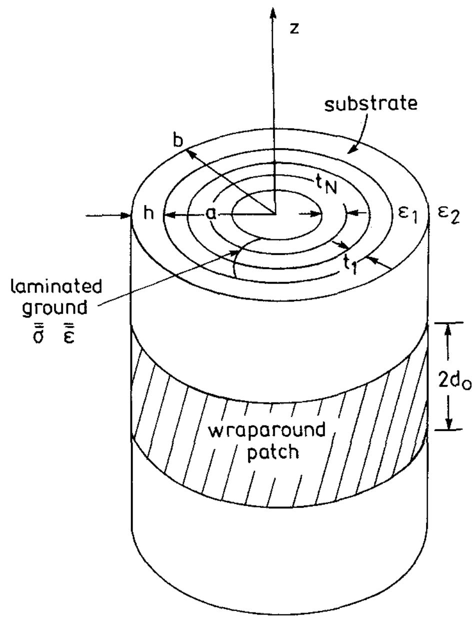

Fig. 1 *Geometrical configuration of wraparound patch on laminated composite ground plane*

Assuming the time harmonics of $e^{-i\omega t}$, each field component can be expressed as a series in the ϕ direction and an integral in the k_z domain [2]. Each lamina of the composite ground plane is made by laying fibres (usually metallic) in a matrix (like resin) to enhance its mechanical strength. The electrical properties of each lamina can be described by a permittivity tensor and a conductivity tensor [2, 3], and an isotropic permeability of μ_0 is assumed in the whole medium. The tangential field components in the spectral domain at $\rho = \rho_N$ and $\rho = \rho_0$ are related by a transition matrix $\bar{\mathbf{P}}_n(k_z; \rho_N, \rho_0)$.

Next , one imposes the boundary conditions that: (a) $E_z(\rho_0) = E_\phi(\rho_0) = 0$; (b) the tangential fields at $\rho = a$ are continuous; (c) the tangential electric fields at $\rho = b$ are continuous; and (d) the discontinuity of the tangential magnetic fields at $\rho = b$ accounts for the electric surface current on the patch. Thus, a relation is obtained between the tangential electric field at $\rho = b$ and the electric surface current on the patch.

Next, impose the boundary condition that: first, the tangential electric field vanishes on the patch surface; and secondly, no electric surface current exists outside

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of the patch, to obtain two vector integral equations with one from each condition [2]. To solve these integral equations numerically first choose a set of basis functions to represet the surface current on the patch as

$$\bar{J}_s(\phi, z) = \sum_{n,m} \bar{\mathbf{K}}_{nm}(\phi, z) \cdot \bar{A}_{nm} \tag{1}$$

where $\bar{A}_{nm}$ are the unknown coefficients, and $\bar{\mathbf{K}}_{nm}(\phi, z)$ are the basis functions. Substitute eqn. 1 into the first integral equation, then take the inner product of another set of weighting functions $\mathbf{K}'^{\dagger}_{pq}(\phi, z)$ with the resulting equation to obtain a determinantal equation to be solved for the resonant frequencies.

For the wraparound patch, the current distribution is of the travelling wave type in the ϕ-direction, and of the standing wave type in the z-direction. Note that for the cylindrical rectangular patch, the current distribution in both directions are of the standing wave type. Thus, the following basis functions and weighting functions are chosen [1]

$$\bar{\mathbf{K}}_{nm}(\phi, z) = \begin{cases} e^{in\phi}\bar{\Omega}_m(z) & |z| \le d_o \\ 0 & \text{elsewhere} \end{cases}$$

$$\bar{\mathbf{K}}'_{pq}(\phi, z) = \begin{cases} e^{ip\phi}\bar{\Omega}'_q(z) & |z| \le d_o \\ 0 & \text{elsewhere} \end{cases} \tag{2}$$

where the z-dependent functions $\bar{\Omega}_m(z)$ and $\bar{\Omega}'_q(z)$ are the same as those used in the cylindrical rectangular patch [2]. Due to the incorporation of edge condition at $z = \pm d_o$ in the basis functions $\bar{\Omega}_m(z)$, the k_z-integration in each matrix element converges more slowly than the case which does not incorporate the edge condition. If the edge condition is also considered in the weighting functions $\bar{\Omega}'_q(z)$, the convergence rate will be even worse. However, the resonant frequencies obtained using weighting functions with or without the edge condition differ only slightly. Hence, the edge condition is not incorporated in the weighting functions as a trade-off between computing time and accuracy.

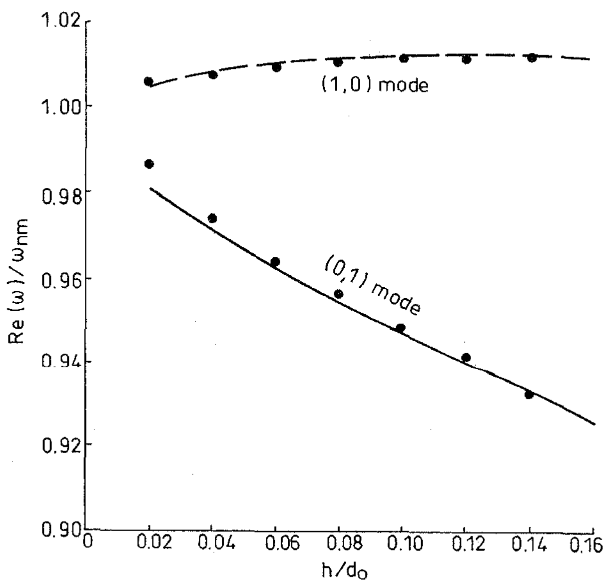

Fig. 2 *Real part of resonant frequency of wraparound patch with one lamina of solid copper ground plane*
a = 20cm, d_o = 4cm, $\varepsilon_1 = 2.3\ \varepsilon_0$, $\varepsilon_2 = \varepsilon_0$, t_1 = 3μm, $\varepsilon'_{\rho\rho} = \varepsilon'_{\phi\phi} = \varepsilon'_{zz} = \varepsilon_0$, $\sigma'_{\phi\phi} = \sigma'_{\phi\phi} = \sigma'_{zz} = 5.8 \times 10^7$ ℧/m
• results from [1]

3 Numerical results

Fig. 2 shows the real part of the resonant frequencies of the (0, 1) and (1, 0) modes with a lamina of solid copper ground plane. The resonant frequencies are normalised with respect to ω_{nm} which is the (n, m) mode resonant frequency from the magnetic wall model

$$\omega_{nm} = \frac{1}{\sqrt{\mu_o \epsilon_1}} \sqrt{\left(\frac{n}{a}\right)^2 + \left(\frac{m\pi}{2d_o}\right)^2} \tag{3}$$

The results are close to those with a perfect conductor ground plane in [1]. The imaginary part of the resonant frequency is larger than that with a perfect conductor ground plane because of the ohmic loss.

The quality factor Q and the fractional bandwidth B_r are related to the resonant frequency by [4]

$$Q = \frac{\omega'}{2\omega''} \qquad B_r = \frac{1}{Q} \tag{4}$$

where ω' and ω are the real and imaginary parts, respectively, of the resonant frequency. Hence, a higher imaginary part implies a lower quality factor and a wider bandwidth. If a current probe is used to excite the patch to become an antenna, then the input impedance near the resonant frequency can be expressed as [4]

$$Z_{in} = \frac{i\omega' R/2Q}{\omega - \omega'(1 - i/2Q)} \tag{5}$$

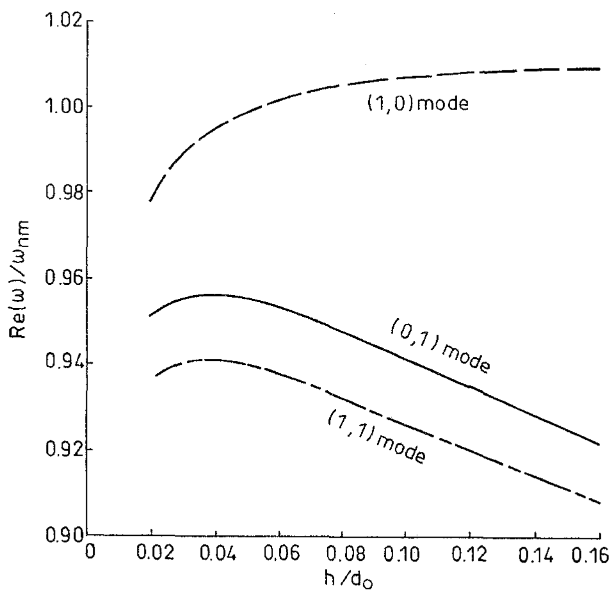

Fig. 3 *Real part of resonant frequency of wraparound patch with two-lamina G/E composite ground plane*
a = 20cm, d_o = 4cm, $\varepsilon_1 = 2.3\ \varepsilon_0$, $\varepsilon_2 = \varepsilon_0$, $t_1 = t_2$ = 25.4μm
$\alpha_1 = 0°$, $\alpha_2 = 90°$, $\varepsilon'_{\rho\rho} = \varepsilon'_{\phi\phi} = \varepsilon'_{zz} = 5.0\ \varepsilon_0$
$\sigma'_{\rho\rho} = \sigma'_{zz} = 50$ ℧/m, $\sigma'_{\phi\phi} = 4 \times 10^4$ ℧/m

where R is the input resistance, and ω is the operating frequency. Note that $\omega'(1 - i/2Q)$ in eqn. 5 is the complex resonant frequency obtained by the present approach. Figs. 3 and 4 show the resonant frequencies of the (0, 1), (1, 0) and (1, 1) modes with a two-lamina G/E composite ground plane. The effective dielectric and permittivity tensors are chosen to be the same as those in [3]. The fibre direction in the outer lamina (α_1) is perpendicular to that in the inner lamina (α_2) Comparing with the results in Fig. 2 and the associated imaginary part, it is found that the real part of resonant frequency with the laminated ground is lower when the substrate thickness approaches zero, and the imaginary parts are slightly different between the two structures. As the substrate thickness is reduced to zero, the real part does not approach that predicted by eqn. 3 which is based on the assumption of a perfect conductor ground plane. With a laminated ground, the reduction of substrate thickness causes more field to penetrate into the laminated ground, hence it affects the resonant frequency.

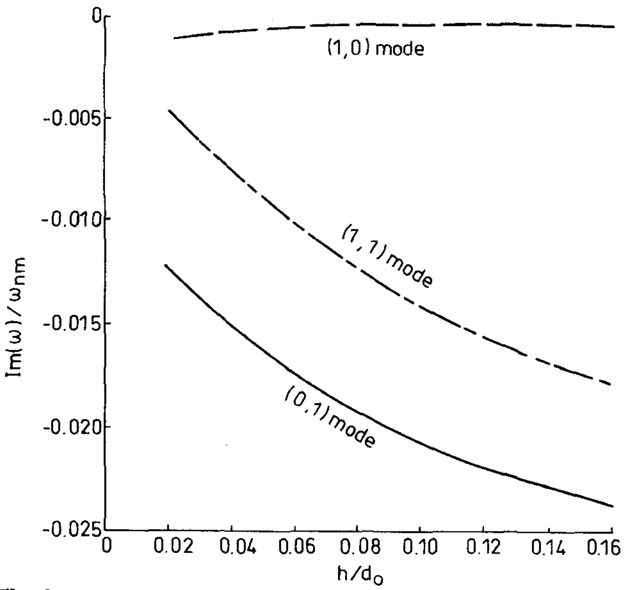

Fig. 4 *Imaginary part of resonant frequency of wraparound patch with two-lamina G/E composite ground plane*
$a = 20$cm, $d_o = 4$cm, $\varepsilon_1 = 2.3\,\varepsilon_0$, $\varepsilon_2 = \varepsilon_0$, $t_1 = t_2 = 25.4\mu$m
$\alpha_1 = 0°$, $\alpha_2 = 90°$, $\varepsilon'_{\rho\rho} = \varepsilon'_{\phi\phi} = \varepsilon'_{zz} = 5.0\,\varepsilon_0$
$\sigma'_{\rho\rho} = \sigma'_{zz} = 50$ ℧/m, $\sigma'_{\phi\phi} = 4 \times 10^4$ ℧/m

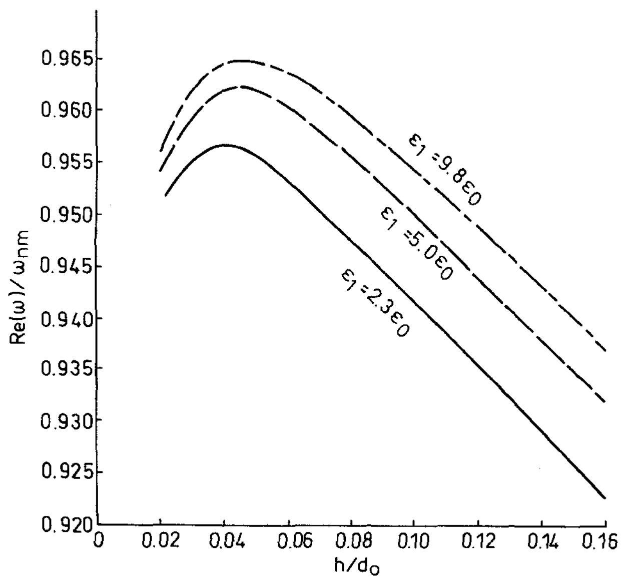

Fig. 5 *Real part of (0, 1) mode resonant frequency of wraparound patch with two-lamina G/E composite ground plane*
$a = 20$cm, $d_o = 4$cm, $\varepsilon_2 = \varepsilon_0$, $t_1 = t_2 = 25.4\mu$m
$\alpha_1 = 0°$, $\alpha_2 = 90°$, $\varepsilon'_{\rho\rho} = \varepsilon'_{\phi\phi} = \varepsilon'_{zz} = 5.0\,\varepsilon_0$
$\sigma'_{\rho\rho} = \sigma'_{zz} = 50$ ℧/m, $\sigma'_{\phi\phi} = 4 \times 10^4$ ℧/m

Fig. 5 shows the effect of the substrate dielectric constant on the resonant frequencies. When the substrate thickness approaches zero, the real part of the resonant frequency deviates from the prediction given by eqn. 3 especially when the substrate permittivity is lower. With a thin substrate, the field in the substrate is more focused and the field penetrating into the laminated ground is less obvious when the substrate permittivity is higher. This also predicts that the imaginary part is larger with a lower substrate permittivity at thin substrate thickness.

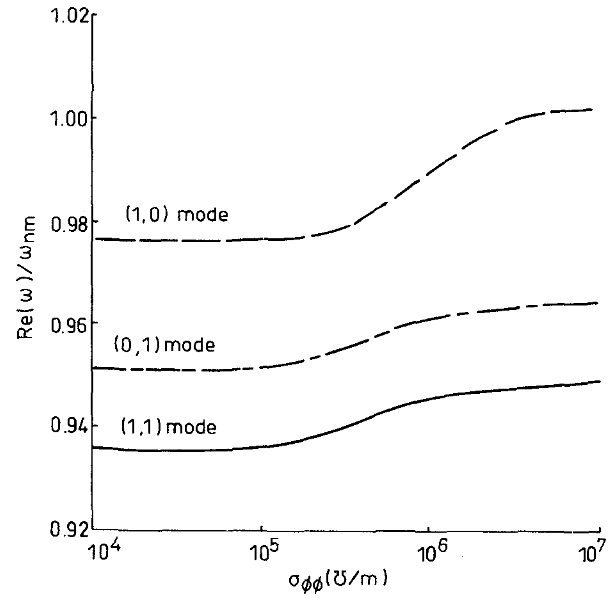

Fig. 6 *Real part of resonant frequency of wraparound patch with two-lamina G/E composite ground plane*
$h/d_o = 0.02$
$a = 20$cm, $d_o = 4$cm, $\varepsilon_1 = 2.3\,\varepsilon_0$, $\varepsilon_2 = \varepsilon_0$, $t_1 = t_2 = 25.4\mu$m
$\alpha_1 = 0°$, $\alpha_2 = 90°$, $\varepsilon'_{\rho\rho} = \varepsilon'_{\phi\phi} = \varepsilon'_{zz} = 5.0\,\varepsilon_0$
$\sigma'_{\rho\rho} = \sigma'_{zz} = 50$ ℧/m

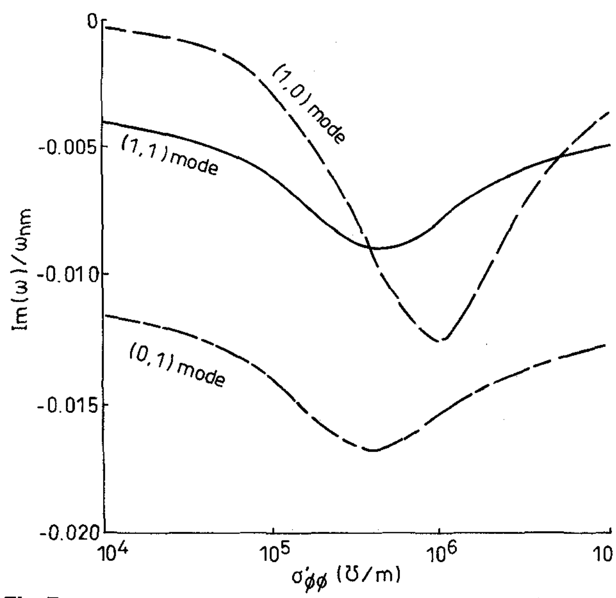

Fig. 7 *Imaginary part of resonant frequency of wraparound patch with two-lamina G/E composite ground plane*
$h/d_o = 0.02$
$a = 20$cm, $d_o = 4$cm, $\varepsilon_1 = 2.3\,\varepsilon_0$, $\varepsilon_2 = \varepsilon_0$, $t_1 = t_2 = 25.4\mu$m
$\alpha_1 = 0°$, $\alpha_2 = 90°$, $\varepsilon'_{\rho\rho} = \varepsilon'_{\phi\phi} = \varepsilon'_{zz} = 5.0\,\varepsilon_0$
$\sigma'_{\rho\rho} = \sigma'_{zz} = 50$ ℧/m

The effect of changing the fibre conductivity $\sigma'_{\phi\phi}$ in each lamina is now considered. As shown in Figs. 6 and 7, the real part of the resonant frequency increases with $\sigma'_{\phi\phi}$. At low $\sigma'_{\phi\phi}$, the fields tend to penetrate into the laminate, and the perfect conductor underneath the laminate works as the ground plane. As $\sigma'_{\phi\phi}$ increases, ohmic loss incurred in the laminate raises the imaginary part of the resonant frequency. As $\sigma'_{\phi\phi}$ is increased further, the laminated ground plane becomes a better conductor for grounding, and the imaginary part decreases.

4 Conclusions

The integral equation formulation used to study the resonance properties of a cylindrical rectangular patch resonator on a composite ground plane has been modified to analyse the wraparound patch on the same composite ground. The laminated ground is modelled by a cascaded transition matrix in the spectral domain. Galerkin's method is applied to solve the integral equation for the resonant frequencies. The effects of substrate thickness, substrate dielectric constant and laminate conductivity are analysed. It is observed that the real part of the resonant frequencies decreases when the fibre conductivity of the laminated ground is reduced. The imaginary part of the resonant frequency increases in the middle range of the fibre conductivity, this is because the inner coating works as a perfect ground at low fibre conductivity, and the laminate itself works as a good ground at high conductivity. Between these two limits, the ohmic loss of the laminate incurs a higher imaginary part of the resonant frequency.

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6 References

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