IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION, VOL. 65, NO. 2, FEBRUARY 2017

PWE-Based Radar Equation to Predict Backscattering of Millimeter-Wave in a Sand and Dust Storm

Mu-Min Chiou and Jean-Fu Kiang

Abstract—A parabolic wave equation-based radar equation is proposed to compute the backscattered power of millimeter-wave in a sand and dust storm, in which the height profiles of particle-size distribution and total number density are accounted for. The conventional radar equation is also improved by incorporating the effects of earth curvature and beam divergence. The improvement of accuracy by using these two radar equations, especially when the specific attenuation is a function of height, is verified by simulations.

Index Terms—Backscattering, parabolic wave equation (PWE), radar equation, sand and dust storm (SDS).

I. INTRODUCTION

SAND and dust storm (SDS) may significantly affect radio relays and satellite communications by absorbing and scattering wave signals, which has been a subject of continuous interest to the Antennas and Propagation community [1]–[3]. In certain areas with frequent occurrence of SDSs, continuous monitoring of emergence and movement of SDSs is necessary to avoid possible hazards. For example, a dust storm was reported on March 10–11, 2009 in the Arabian Peninsula, which choked thousands of people in Riyadh and caused many car accidents due to low visibility [4].

Typical dust storms move near ground, making them difficult to detect at early stage, hence high-frequency terrestrial radars were proposed for early detection [5]. Relevant information like backscattering of millimeter-wave radar signals in SDSs was reported [6], [7], and their turbulence effects were also studied [8]. A ground-based monitoring system is more suitable than space-borne systems to collect more detailed information. Compared with the latter, fewer literatures were available about ground-based techniques suitable for monitoring regional SDSs, including the effects of earth curvature and particle-size distribution.

In the conventional radar equation, the height dependence of scatterers within a resolution volume was not considered [9], [10]. In practice, the number density and particle-size distribution in an SDS vary with height due to gravity, aggregation, melting, evaporation, and other factors [11]. As a radar beam diverges along its path, the height profile of scatterers can no longer be ignored beyond certain distance. For example, the vertical profile of reflectivity was reported as the major source of error in measurements [9].

In this paper, a parabolic wave equation (PWE)-based radar equation is proposed, which accounts for the effects of earth curvature and height profile of SDS on radar backscattering. Mie scattering is assumed to compute the backscattering of an SDS with height-dependent scatterers, and the Mie scattering is applied to calculate the absorption and backscattering effects exerted on the returned signals. The number density and size distribution of dust particles are included in this model, providing more accurate prediction than the existing methods.

This paper is organized as follows. The models of SDS and their impact on backscattering are presented in Section II. A PWE-based radar equation and an improved radar equation are proposed in Sections III and IV, respectively. Simulation results are discussed in Section V, and some conclusions are drawn in Section VI.

II. SDS MODELS AND THEIR IMPACT ON BACKSCATTERING

The total number density of dust particles is a function of height, which can be empirically represented as [12]

\[ N_t = N_{t_0} z^{-\Gamma_p} \]  

where \( z \) is the height above ground, \( N_{t_0} \) \((1/m^3)\) is the total number density at a reference height of 1 m above ground, \( \Gamma_p \) is a coefficient related to the sedimentation and frictional velocities of particles, and \( \Gamma_p = 0.29 \) [12] will be adopted in the simulations.

The average particle radius \( r_0 \) is also a function of height, which can be represented as [5]

\[ r_0 = r_{0_0} z^{-\gamma_m} \]  

where \( r_{0_0} \) is the average particle radius at 1 m above ground, and \( \gamma_m = 0.15 \) is derived by fitting the data listed in Table I [5].
The distribution of dust-particle radius $r_p$ in an SDS was fit with a log-normal distribution as [1], [5]

$$P(r_p) = \frac{1}{\sqrt{2\pi} r_p \sigma_\ell} e^{-(\ln r_p - \mu_\ell)^2/(2\sigma_\ell^2)}$$

where $\mu_\ell$ and $\sigma_\ell$ are the mean and standard deviation, respectively, of $\ln r_p$, which are related to the mean ($r_{p0}$) and standard deviation ($\sigma_p$) of the particle radius as

$$\mu_\ell = \ln \frac{r_{p0}}{\sqrt{1 + (\sigma_p/r_{p0})^2}}$$
$$\sigma_\ell = 2 \ln \sqrt{1 + (\sigma_p/r_{p0})^2}.$$\hfill (4)

In this paper, the standard deviation of dust-particle radius is assumed to follow the same height dependence as the mean dust-particle radius, namely:

$$\sigma_p = \sigma_p z^{-\gamma_\sigma}$$\hfill (5)

where $\gamma_\sigma = 0.125$ is obtained by fitting the data listed in Table 1.

To model wave propagation in an atmosphere filled with dust particles, the propagation constant of a plane wave in an atmosphere can be represented as [13], [14]

$$k_{ds}(\vec{r}) = k_0 + \frac{2\pi}{k_0} N_i(\vec{r}) \int_0^{r_{p_{max}}} P(\vec{r}, r_p) f_i(r_p) dr_p$$\hfill (6)

where $k_0$ is the wavenumber in free space, $P(\vec{r}, r_p)$ (1/µm/m$^3$) is the probability density function of $r_p$, and $f_i(r_p)$ is the forward scattering amplitude of a dust particle with radius $r_p$. The radius of most dust particles is smaller than 100 µm [5], and a maximum value of $r_{p_{max}} = 150$ µm [15] is assumed in this paper.

The propagation constant of a plane wave in an atmosphere filled with dust particles can be represented as [14], [16]

$$n_{ds}(\vec{r}) = k_{ds}(\vec{r})/k_0$$\hfill (7)

$$a_{ds} = 8686 \times \text{Im}[k_{ds}] \text{ (dB/km)}.$$\hfill (8)

Fig. 1 shows the schematic to compute the backscattered power from dust particles. The transmitting antenna is located at $(0, 0, z_c)$, and a rectangular pulse of duration $\tau$ is emitted at $t = 0$. The backscattered power received at time $t_c$ is attributed to particles floating in the range $[x_c - \Delta x/2, x_c + \Delta x/2]$, where $\Delta x = c\tau/2$ and $c$ is the speed of light.

The ratio between the expectation value of the received power $P_r$ (in watt) and the transmitting power $P_t$ (in watt) can be estimated by using the conventional radar equation as [10]

$$E\{P_r/P_t\} = \eta \frac{G_a^2 \lambda^2 \Delta \Phi \Delta \phi \tau}{210 (\ln 2) \pi^2 x_c^2} \times 10^{-2} f_i^{x_c} d\epsilon_{ds}/10$$\hfill (9)

where $G_a$ is the antenna gain, $x_c$ (m) is the distance between the radar and the center of the resolution volume, and $\Delta \theta$ and $\Delta \phi$ (in radians) are the vertical and horizontal beamwidths, respectively, of the antenna radiation pattern.

Assuming the antenna points horizontally, the attenuation along either the forward or the backscattering path can be derived by using the value of $a_{ds}$ derived at the height of antenna. The reflectivity $\eta$ is defined as

$$\eta = \frac{\pi^5}{\lambda^4} |K|^2 Z$$\hfill (10)

where $K = (\epsilon_d - 1)/(\epsilon_d + 2)$, under Rayleigh scattering approximation, $\epsilon_d$ is the permittivity of dust material, $\lambda$ is the wavelength, and

$$Z = N_i \int_0^{r_{p_{max}}} P(r_p)(2r_p)^6 dr_p$$\hfill (11)

is the reflectivity factor.

Fig. 2 shows the height profiles of reflectivity $\eta$ and specific attenuation constant $a_{ds}$, assuming that the particle size follows a log-normal distribution. It is observed that both $\eta$ and $a_{ds}$ decrease significantly with height.
If the earth curvature is taken into account, the center of the radiation beam will be elevated to \( h_c = (z_i + r_e)/\cos(x/r_p) - r_e \) above ground at a distance \( x \) from the antenna site, where \( r_e \) is the earth radius. Taking \( z_i = 10 \text{ m} \) for example, the height of the beam center will be elevated to \( h_c = 12 \text{ m} \) at \( x = 5 \text{ km} \), which is 2 m higher than \( z_i \).

As shown in Fig. 2, the reflectivities at heights of 10 and 12 m are \( 1.056 \times 10^{-6} \) and \( 0.956 \times 10^{-6} \), respectively, differing by \((0.956 - 1.056)/0.956 \simeq -10\% \) or \(-0.46 \text{ dB/km} \). Similarly, the specific attenuation constants at heights of 10 and 12 m are 2.659 and 2.419 dB/km, respectively, differing by 0.24 dB/km. Such differences at \( x = 5 \text{ km} \) are attributed to the earth curvature when the conventional radar equation in (8) is used. In the next two sections, two alternative radar equations are proposed to improve the accuracy in predicting the backscattered power.

III. PWE-BASED RADAR EQUATION

Fig. 3 shows a schematic to compute the backscattered wave from a dust particle at \( \vec{r} = (x, y, z) \).

\[ \vec{u}_b = \hat{u}_b \hat{u}_f f_b(r_p) \frac{1}{r_s} \exp \left\{ -j \int_0^{r_s} k_{db} d\ell \right\} \tag{12} \]

where \( \vec{r}_s = \vec{r}' - \vec{r}, \hat{r}' = (0, 0, z), \hat{r}, \) and \( \hat{x} \) are the unit vectors in the backscattering and incident directions, respectively; \( u_f' \) is the amplitude of the forward-propagating wave, with polarization vector \( \hat{u}_f' \); and \( f_b(r_p) \) is the backscattering amplitude. Equation (5) is used to compute \( k_{db} \), with the SDS parameters characterized by (1)–(4).

The radiation pattern of the antenna is approximated with a Gaussian distribution in \( \chi \) as \( e^{-\chi^2/(\theta_0^2/\ln 4)} \), where \( \theta_0 \) is the 3 dB beamwidth. The backscattered field at the antenna site can be computed as

\[ \vec{u}_b = \hat{u}_b \hat{u}_f f_b(r_p) \frac{1}{r_s} \exp \left\{ -j \int_0^{r_s} k_{db} d\ell \right\} e^{-\chi^2/(\theta_0^2/\ln 4)} \tag{12} \]

A. Rayleigh Scattering Versus Mie Scattering

The Rayleigh scattering amplitude can be represented as [16]

\[ f_r(r_p, \theta) = k_0^2 c_0^2 \frac{e^2 - 1}{e^2 + 2} r_p^3 \cos \theta \tag{13} \]

where \( \theta \) is the angle between the incident and the scattering directions. Similarly, the Mie scattering amplitude can be represented as [16]

\[ f_m(r_p, \theta) = \frac{S(\cos \theta)}{j k_0} \tag{14} \]

with

\[ S(\cos \theta) = \sum_{\ell=1}^{\infty} \frac{2\ell + 1}{\ell(\ell + 1)} (a_\ell \tau_\ell + b_\ell \pi_\ell) \]

where \( \pi_\ell \) and \( \tau_\ell \) are \( \theta \)-dependent functions, following the recurrence relations [17]:

\[ \pi_\ell = \frac{2\ell - 1}{\ell - 1} \cos \theta \pi_{\ell - 1} - \frac{\ell - 1}{\ell - 2} \pi_{\ell - 2} \tag{15} \]

\[ \tau_\ell = \ell \cos \theta \pi_\ell - (\ell + 1) \pi_{\ell + 1} \tag{16} \]

where \( \pi_0 = 0, \pi_1 = 1, \tau_0 = 0, \) and \( \tau_1 = \cos \theta \). The Mie coefficients \( a_\ell \) and \( b_\ell \) can be expressed as [18]

\[ a_\ell = \frac{n_0^2 j_1(n_0 \zeta) j_1(\zeta) - j_0(\zeta) n_0 j_1(n_0 \zeta)}{n_0^2 j_1(n_0 \zeta) h_1^{(1)}(\zeta) - h_0^{(1)}(\zeta) n_0 j_1(n_0 \zeta)} \tag{17} \]

\[ b_\ell = \frac{j_1(n_0 \zeta) j_0(\zeta) j_1(n_0 \zeta) j_0(\zeta) - j_0(\zeta) n_0 j_1(n_0 \zeta) j_0(\zeta)}{j_1(n_0 \zeta) h_1^{(1)}(\zeta) - h_0^{(1)}(\zeta) n_0 j_1(n_0 \zeta)} \tag{18} \]

where \( n_0 = \sqrt{c_0} \) is the refractive index of dust material, \( \zeta = k_0 r_p \), and \( j_0(\beta) \) and \( h_1^{(1)}(\beta) \) are the spherical Bessel function and Hankel function, respectively, of order \( \ell \).

Fig. 4 shows the normalized Rayleigh and Mie scattering patterns, respectively, at different frequencies, of a spherical dust particle with radius 150 \( \mu \text{m} \). The relative permittivity of dust material at 100 GHz is \( e_0 = 3.5 - j1.64 \) [15]. The difference between Rayleigh and Mie scattering patterns...
becomes more obvious at higher frequencies. At 10 GHz, both radiation patterns look almost identical. At 100 GHz, the forward scattering amplitudes are $f_1 = 7.7 \times 10^{-6}$ and $f_{10} = 8.01 \times 10^{-6}$, respectively, differing by 4.56%. The Mie scattering approximation will be used in our model.

When a dust particle bears charge, the coefficients $a_\ell$ and $b_\ell$ can be expressed as [19]

$$a_\ell = \left[ 1 + \frac{\ell g D_r(m_d\chi)}{m_d} + \frac{\ell}{\chi} \right] \chi j_\ell(\chi)$$

$$- \left[ 1 + \frac{\ell g D_r(m_d\chi)}{m_d} \right] \chi j_{\ell-1}(\chi) \right]$$

$$\times \left[ 1 + \frac{\ell g D_r(m_d\chi)}{m_d} + \frac{\ell}{\chi} \right] \chi h^{(1)}_\ell(\chi)$$

$$- \left[ 1 + \frac{\ell g D_r(m_d\chi)}{m_d} \right] \chi h^{(1)}_{\ell-1}(\chi) \right]^{-1}$$

$$b_\ell = \frac{[m_d D_r(m_d\chi) + (\ell/\chi)(1-g\chi/\ell)] j_\ell(\chi) - \chi j_{\ell-1}(\chi)}{[m_d D_r(m_d\chi) + (\ell/\chi)(1-g\chi/\ell)] h^{(1)}_\ell(\chi) - \chi h^{(1)}_{\ell-1}(\chi)}$$

where

$$D_r = j_{\ell-1}(m_d\chi) - \frac{\ell}{m_d}$$

$$g = \frac{\chi}{2 \omega_0^2 + \gamma_0^2} \left( 1 - \frac{j \chi}{\omega_0} \right)$$

where $\omega_0^2 = 2e\Phi/(m_e r_p^2)$ is the surface plasma frequency, $e = 1.602 \times 10^{-19}$ C, $m_e = 9.109 \times 10^{-31}$ kg, $\Phi = Q/(4\pi e_0 c_0 r_p)$ is the electrostatic potential on the surface of the charged sphere, $\sigma_e$ is the surface charge density, $Q = 4\pi r_p^2 \sigma_e$ is the total charge on the particle surface, $\gamma_0 = \kappa T/h$, $T$ (K) is the particle temperature, $\kappa = 1.38 \times 10^{-23}$ (J/K) is the Boltzmann’s constant, and $h = 1.0546 \times 10^{-34}$ (J·s) is the Plank’s constant divided by $2\pi$.

Fig. 5 shows the Mie scattering amplitude pattern $f_{me}$, with charge density of $\sigma_e = 0.05$ and 0.1 C/m², respectively. The pattern without charge, $f_{m0}$, is also shown for comparison. The particle radius is assumed to be $r_p = 50 \mu$m. The magnitude of $f_{me}$ is generally larger than that of $f_{m0}$. For example, the forward scattering magnitude without charge is $2.87 \times 10^{-7}$, and those with surface charge densities of $\sigma_e = 0.05$ and 0.1 C/m² are $3.063 \times 10^{-7}$ and $3.244 \times 10^{-7}$, respectively.

The electrification generated by wind-blown sands was measured in a wind tunnel [20]. The electric charge was found to depend mainly on the diameter of sand particles and height. The average charge-to-mass ratio of sand particles with small diameter is larger than that with large diameter. The average charge-to-mass ratio increases with height measured from the sand bed.

For example, the charge-to-mass ratio of mixed sand with various diameters is $-1120.7 \mu$C/kg at the height of 0.5 m and wind speed of 10 m/s. A sand particle with a radius of 50 µm will carry a charge of $1.431 \times 10^{-6}$ µC, where the mass density of dust particles is $2440 \text{kg/m}^3$ [1]. The corresponding surface charge density is $4.56 \times 10^{-5}$ C/m².

By observing dust particles in a plasma, the number of charges on a dust particle was on the order of $10^6$ [21], [22]. For a particle with radius of 50 µm, this amount to a surface charge density of $5.1 \times 10^{-6}$ C/m². The surface charge density of fine sediments was reported to be 0.53–1.64 C/m² [23], [24].

### B. Forward Wave Computed With PWE Method

The PWE method [25], [26] is applied to compute the forward wave. To accelerate the computation, the 3-D forward wave $u_f(x, y, z)$ is approximated by multiplying a 2-D forward wave $u_f(x, z)$ with an offset factor $C(x, y)$ as $u_f(x, y, z) = C(x, y) e^{-jk_0 s} u_f(x, z)$. Ideal 3-D and 2-D Gaussian waves can be represented as [16]

$$u_f(x, y, z) = \frac{u_0}{1 - jk_0 \ln(4)/(\pi W_0^2)}$$

$$\times \exp \left\{ -\frac{y^2 + (z - z_0)^2}{W_0^2/\ln 4 - jk_0 x/\pi} \right\}$$

$$u_f(x, z) = \frac{u_0 W_0/\sqrt{\ln 4}}{2\sqrt{\sigma}} e^{-j(\xi - 2\xi_0)/(4a)}$$

where $a = W_0^2/(8\ln 2) - jx/(2k_0)$ and $u_0$ is an arbitrary constant. The offset factor is determined as the ratio between the ideal Gaussian waves in (17) and (18) as

$$C(x, y) = \frac{2 \sqrt{W_0^2}}{8 \ln 2 - \frac{j x}{2k_0}} \left[ \frac{W_0}{\pi W_0^2} \right]^{1/4} \left[ 1 - jk_0 \ln 4 \right]^{1/4}$$

The 2-D forward wave $u_f(x, z)$ is then computed, by applying the PWE method, as [25]

$$u_f(x + \Delta x, z) = e^{-jk_0 m \Delta x/2}$$

$$\times F^{-1} \{ F\{ u_f(x, z) \} e^{j p^2/(2k_0)} \}$$

where $F$ and $F^{-1}$ represent the Fourier transform and the inverse Fourier transform, respectively; $p = k \sin \theta$ is the wavenumber in the $z$-direction, $\theta$ is the propagation angle measured from the horizon; and $m = n_{dz}^2 + 1 - 2z/r_e$ is the
modified refractive index, in which the $2z/r_c$ term accounts for the earth curvature [25].

Ground reflection can be accounted for by applying a mixed Fourier transform [27], and an absorbing layer is placed at the top boundary of the computational domain to reduce artificial reflection. In this paper, vertically polarized waves are considered, with $E_z = u_f(x, z)$.

Without loss of generality, the antenna is pointed in the horizontal direction, and the initial field is approximated as

$$u_f(0, z) = \frac{1}{W_0/\sqrt{\pi \ln 4}} \exp \left\{ \frac{(z - z_e)^2}{W_0^2/\ln 4} \right\}$$

where $W_0 = \ln 4/[k_0 \sin(\theta_0/2)]$ is the 3 dB beamwidth on the aperture at $x = 0$ [16].

C. Expectation Value of Received Power

The expectation value of the received power is computed as

$$E\{P_{pr}\} = \langle |\tilde{u}_b|^2 \rangle A_e$$

where $\langle u \rangle$ stands for the ensemble average of $u$, $A_e$ is the effective area of the receiving antenna, and the ensemble average $\langle |\tilde{u}_b|^2 \rangle$ is computed as

$$\langle |\tilde{u}_b|^2 \rangle = \frac{N_0}{\sum_{n=1}^{N_0} |\tilde{u}_{bn}|^2}$$

where $N_0$ is the total number of particles illuminated by the forward wave, and $|\tilde{u}_{bn}|$ is the backscattering amplitude from the $n$th dust particle, which is computed as

$$|\tilde{u}_{bn}|^2 = |u'_f|^2 |f_b|^2 \frac{1}{r_{sn}^2} \exp \left\{ -2j \int_0^{r_{sn}} k_{ds} d\ell \right\} \times e^{-2z_e^2/(\theta_0^2/\ln 4)}.$$

In numerical simulations, a resolution volume is divided into $Q$ layers in parallel to the ground, and $\langle |\tilde{u}_b|^2 \rangle$ is computed as

$$\langle |\tilde{u}_b|^2 \rangle = \sum_{q=1}^{Q} \sum_{n=1}^{N_q} |\tilde{u}_{bn}|^2$$

where $N_q$ is the number of dust particles in the $q$th layer. By considering the height profiles of SDS parameters, $\langle |\tilde{u}_b|^2 \rangle$ is reduced to

$$\langle |\tilde{u}_b|^2 \rangle = \frac{\Delta z \Delta x}{x^2} \sum_{q=1}^{Q} |u_f(x, z_q)|^2 \exp \left\{ -2j \int_0^{r_{eq}} k_{ds} d\ell \right\} \times N_{eq} \int_0^{r_{max}} P(z, r_p) f_b(r_p)^2 dr_p \times \int_{-\infty}^{\infty} e^{-2z_e^2/(\theta_0^2/\ln 4)} |C(x, y)|^2 dy.$$

IV. IMPROVED RADAR EQUATION

An alternative method, based on the conventional radar equation, is also proposed to account for the effect of earth curvature. Fig. 6 shows the definitions of several forward and backward attenuations, which will be used to account for the effects of earth curvature and beam divergence, where $h_c$ is the height of beam center above ground due to earth curvature.

Fig. 7 shows the cross section of a beam at range $x$ from the antenna. The attenuation along a path from $\vec{r}$ to $\vec{r}'$, at a given height of $z$, is computed as

$$A(z) = \int_{r}^{r'} a_{ds} d\ell.$$

By weighting $A(z)$ over the beam cross section, a forward attenuation $A_{fw}$ and a backward attenuation $A_{bw}$ are derived as

$$A_{bw} = A_{fw} = 10 \times \log_{10} \int_{z}^{z_l} 10^{-A(z)/10} \sqrt{(x\theta_0/2)^2 - (h_c - z)^2} dz$$

$$\int_{z}^{z_u} \sqrt{(x\theta_0/2)^2 - (h_c - z)^2} dz$$

where $z_l = h_c - x\theta_0/2$ and $z_u = h_c + x\theta_0/2$ are the upper and lower bounds, respectively, of the beam cross section.

A modified reflectivity is defined as

$$\eta(z) = N_f \int_0^{r_{max}} P(r_p) \sigma_b(r_p) dr_p$$
which is a function of $z$, where
\[
\sigma_b(r_p) = \frac{\lambda^2}{4\pi} \sum_{\ell=1}^{\infty} (-1)^{\ell}(2\ell + 1)(a_\ell - b_\ell)^2
\]
is the backscattering cross section of a particle with radius $r_p$, under Mie scattering approximation.

Fig. 8 shows a schematic to compute $\eta_w$. Due to the earth curvature, the beam center is shifted to $z = h_c$, and $\eta(z)$ is weighted over the beam cross section to obtain $\eta_w$ as
\[
\eta_w = \frac{\int_{z_e}^{z_u} \eta(z) \sqrt{(x/h_b)^2 - (h_c - z)^2} \, dz}{\int_{z_e}^{z_u} \sqrt{(x/h_b)^2 - (h_c - z)^2} \, dz}.
\]
Finally, the expectation value of the backscattered power at the receiver is computed as
\[
E(P_{ur}/P_t) = \eta_w \frac{G^2 \Delta \phi \Delta \phi_c \tau}{2 \log(2) \pi^2 \Delta x^2} 10^{-A_{fw}/10} 10^{-A_{bw}/10}
\]
which is called the improved radar equation.

V. SIMULATIONS AND DISCUSSION

In order to study how the height profile of SDS properties affects the received power of millimeter-waves, five scenarios, as listed in Table II, are conceived and simulated by using the two proposed radar equations. The major mechanisms of interest include forward propagation, backward propagation, and backscattering. The radar parameters are summarized in Table III [28].

The total number density at 1 m above ground is chosen as $N_{ts} = 10^7 /\text{m}^3$ to model a severe SDS, in which the visibilities at heights of 1 and 30 m are roughly 6 and 280 m, respectively. The visibility was empirically modeled as [29]
\[
V_b = \left(9.43 \times 10^{-9} / v_f \right)^{1.07} \text{(km)}
\]
where
\[
v_f = \frac{4\pi}{3} N_t \int_0^{r_{\text{max}}} P(r_p)^3 r_p \, dr_p
\]
is the fractional volume of dusts, which is a function of total number density $N_t$ and particle-size distribution $P(r_p)$.

In the first case, assume the particle-size distribution and the total number density are independent of height, with $N_t = 10^7 /\text{m}^3$. The antenna is located at $z_t = 30 \text{ m}$. The specific attenuation constant is $a_{ds} = 1.578 \text{ dB/km}$ in both forward and backward directions, by using (7); and the reflectivity is $\eta = 5.682 \times 10^{-6} /\text{m}$, by using (23). In Fig. 9 shows the power ratios $P_{pr}/P_t$ and $P_{tr}/P_t$, respectively, as functions of range. It is observed that as the range is farther than 3 km, the received powers predicted with both the conventional and the PWE-based radar equations are close to each other, with the difference less than 0.05 dB. The conventional radar equation is based on the far-field approximation [10], [30]. The received power predicted with the PWE-based radar equation is lower than that with the conventional radar equation if the range is less than the far-field boundary, $2 - D^2 / \lambda = 2.667 \text{ km}$, where $D$ is the antenna aperture size. The radiation pattern, $\exp(-\lambda^2 / (4\pi h_f \ln 4))$, used in the PWE-based radar equation tends to reduce the weighting of backscattering power contributed by resolution volumes in the near-field region.

In the second case, the height profile of specific attenuation constant in the forward direction ($\alpha_f$) is considered. The specific attenuation constant in the backward direction is fixed as a constant, $a_b = 2.797 \text{ dB/km}$, and the reflectivity is chosen to be $\eta = 1.041 \times 10^{-5} /\text{m}$; both are derived at the height of 10 m.

Fig. 10 shows the power ratio $P_{pr}/P_t$ and $P_{tr}/P_{ur}$, respectively, as functions of range. It is observed that the received power predicted with the PWE-based radar equation is 0.165 dB higher than that with the conventional radar equation. Because the former considers the effect of earth curvature, the beam center is elevated by 2 m as the wave propagates to $x = 5 \text{ km}$, hence the attenuation becomes lower.
In case 2, \( \eta \) of 10 m; height profile of \( \alpha_f \) is considered, \( N_{ts} = 10^7 / m^3 \). --- ---: \( P_{pr} / P_r \).

--- ---: \( P_{pr} / P_{ur} \).

In this case, \( A_{bw} \) is reduced to \( A_b \) and \( \eta_w \) is reduced to \( \eta \) in (25). Fig. 10 also shows that the received power, from the resolution volume at \( x = 5 \) km, predicted with the PWE-based radar equation is 0.25 dB lower than that predicted with the improved radar equation.

Before analyzing the difference between \( P_{pr} \) and \( P_{ur} \) at \( x = 5 \) km, a propagation factor (PF) is defined as [31]

\[
PF = 20 \log_{10} |u| + 10 \log_{10} x + 10 \log_{10} \lambda \quad \text{(dB)}
\]  

(26)

which is the relative field strength with respect to that in free space. Fig. 11 shows the height profile of PFs in an SDS (PF_{ds}) and in free space (PF_{0}), respectively. The beam center in free space is elevated to 12 m at \( x = 5 \) km due to the earth curvature, while that in an SDS is elevated to 12.6 m, with an additional 0.6 m attributed to the diffraction effects due to the height profile of \( N_t \). The elevation of beam center results in a larger backscattering angle \( \chi \), leading to a lower received power due to the radiation pattern \( e^{-x^2/(\theta_0^2/\ln 4)} \).

Fig. 11 shows that the 3 dB beamwidths at \( x = 5 \) km are 8.45 and 8.75 m in an SDS and in free space, respectively. The cross section of wave beam in the SDS is smaller than that in free space, further reducing the backscattered power from dust particles.

--- ---: \( \eta_w \).

--- ---: \( \eta_w \).

In the third case, the height profile of specific attenuation constant in the backward direction (\( \alpha_b \)) is considered. The specific attenuation constant in the forward direction is set to \( \alpha_f = 2.797 \) dB/km, and the reflectivity is set to \( \eta = 1.041 \times 10^{-5} /m \), both are derived at the height of 10 m.

Fig. 12 shows the power ratios \( P_{pr} / P_r \) and \( P_{pr} / P_{ur} \), respectively, with respect to range. In this case, \( A_{fw} \) is reduced to \( A_f \) and \( \eta_w \) is reduced to \( \eta \) in (25). It is observed that the prediction with the PWE-based radar equation is 0.41 dB higher than that with the conventional radar equation at \( x = 5 \) km because the earth curvature is considered in the former. Similar to the second case, the beam center is elevated by 2 m at \( x = 5 \) km, implying less attenuation.

The ratio \( P_{pr} / P_{ur} \) is smaller than 0.1 dB when \( x > 2.7 \) km and is \(-0.02 \) dB at \( x = 5 \) km. The improved radar equation includes the beam-divergence effect via a 3 dB beamwidth, while the PWE-based radar equation computes the height distribution of forward wave, hence is expected to be more accurate in predicting the backscattering power from a resolution volume.

In the fourth case, the height profile of reflectivity (\( \eta \)) is considered, while the specific attenuation constants are set to \( \alpha_f = \alpha_b = 2.797 \) dB/km, which are derived at the height of 10 m. Fig. 13 shows the power ratios \( P_{pr} / P_r \) and \( P_{pr} / P_{ur} \), respectively, with respect to range. In this case, \( A_{fw} \) is reduced to \( A_f \) and \( A_{bw} \) is reduced to \( A_b \) in (25). It is observed
that the prediction with the PWE-based radar equation is 0.33 dB lower than that with the conventional radar equation because the earth curvature is accounted for in the former. At $x = 5$ km, the beam center is elevated to 12 m, hence the reflectivity incurred to the forward wave is smaller than that at $z = 10$ m, as shown in Fig. 2. Thus, the backscattered power is lower than that obtained under a constant reflectivity. The PWE-based radar equation and the improved radar equation predict similar received power. The ratio $P_{pr}/P_{ur}$ is smaller than 0.05 dB at $x > 2$ km and is 0.04 dB at $x = 5$ km.

Fig. 14 shows the power ratios $P_{pr}/P_{r}$ and $P_{pr}/P_{ur}$ with respect to range, with all three mechanisms taken into account. The prediction with the PWE-based radar equation is 0.68 dB higher than that with the conventional radar equation. The beam center is elevated by 2 m due to the earth curvature, and both the reflectivity and specific attenuation constant decrease with height. As in case 4, the received power tends to decrease due to smaller reflectivity. As in cases 2 and 3, the received power tends to increase due to smaller attenuation. In this case, the combined effects of attenuation and reflectivity result in a higher received power predicted with the PWE-based radar equation than with the conventional radar equation.

which is the path integral of $a_{ds}$ from the antenna to the scattering particle, as depicted in Fig. 3. The value of $A(z)$ decreases from 13.9 to 12.25 dB as $z$ changes from 11 to 17 m. When the beam center is elevated, as shown in Fig. 11, the wave propagating above the beam center is stronger than that below it because the attenuation $A(z)$ decreases with height.

The backscattered wave predicted with the PWE-based radar equation is stronger above $z = 12$ m than below it, and the wave propagating at higher altitudes will suffer less attenuation, hence contributing stronger received power. As a result, the PWE-based radar equation predicts a received power 0.18 dB higher than the improved radar equation does, as shown in Fig. 14. The power ratios $P_{pr}/P_{r}$ and $P_{pr}/P_{ur}$ at $x = 5$ km in cases 2 to 5 are summarized in Table IV. The PWE-based radar equation accounts for both the diffraction effect and beam elevation due to the earth curvature, and is expected to be capable of provide more accurate prediction than the improved radar equation.

Fig. 16 shows three different height profiles of total number density associated with different $\Gamma_{p}$s but having the same value of $N_t$ at $z_t = 10$ m. Larger $\Gamma_{p}$ makes $N_t$ decrease faster with height, according to (1).
Fig. 17 shows the power ratio $P_{pr}/P_{ur}$ under the three $N_t$ profiles shown in Fig. 16, respectively. The values of $P_{pr}/P_{ur}$ at $x = 5$ km are 0.12, 0.18, and 0.28 dB with $\Gamma_p = 0.15$, 0.29, and 0.6, respectively. The ratio $P_{pr}/P_{ur}$ increases as the $N_t$ profile decreases more rapidly with height.

Note that the power ratio $P_r/P_t$ is proportional to $\eta$, which in turns is proportional to the total number density $N_t$, according to (9), (10), and (23). Hence, errors of 0.28 and 0.18 dB in $P_r$ at $x = 5$ km will lead to errors of $N_t$ estimation about 6.6% and 4.2%, respectively.

VI. CONCLUSION

A PWE-based radar equation is proposed to predict the backscattering power from a resolution volume in an SDS. The PWE method is used to compute the forward propagating wave in an SDS, including the effects of diffraction, earth curvature and the height profiles of SDS parameters. The backscattering power is computed under Mie scattering approximation. The conventional radar equation is also improved by including the effects of earth curvature and beam divergence. Five scenarios are conceived to investigate the effects of height profiles of forward and backward specific attenuation constants and reflectivity, respectively, on the backscattering power from resolution volumes at different ranges. The simulation results with three different radar equations are compared and the differences are explained by using the height profiles of total number density and particle-size distribution of SDSs. The results suggest that the PWE-based radar equation and the improved radar equation can be used to retrieve more accurate height profiles of SDS parameters.

REFERENCES


Mu-Min Chiou was born in Pintung, Taiwan, in 1986. He received the B.S. and Ph.D. degrees in electrical engineering from National Taiwan University, Taipei, Taiwan, in 2009 and 2016, respectively. He has been with Media Tek, Hsinchu, Taiwan, since 2015.

Jean-Fu Kiang received the Ph.D. degree in electrical engineering from the Massachusetts Institute of Technology, Cambridge, MA, USA, in 1989. He has been a Professor with the Department of Electrical Engineering, Graduate Institute of Communication Engineering, National Taiwan University, Taipei, Taiwan, since 1999. He has applied different ideas, theories, and methods to explore various electromagnetic phenomena and possible applications, including antennas, phased arrays, wave propagation, inverse imaging, signal processing.