Electrostatic Fields Due to an Electrode Mounted on a Conducting Pad of Finite Extent in a Planar Stratified Medium

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Abstract—The quasi-static fields generated by an electrode mounted on a perfectly conducting pad of finite extent and embedded in a planar stratified medium are analyzed. An integral equation in the spectral domain is derived for the outflowing current density distribution on the pad-electrode surface. The method of moments is then applied to solve the integral equation. The effects of the electric properties of the stratified medium and the standoff thickness on the total electrode current are investigated. Several conductivity profiles modeling different practical measurement environments are also considered.

I. INTRODUCTION

THE ROUTINE APPLICATION of resistivity sounding measurement in geophysical exploration started with the work of Wenner in 1915 and Schlumberger in 1920 [1]-[5], both using a four-terminal electrode arrays. The dipole array [1] has also been routinely used, which is a modified version of the Wenner array.

In geophysical prospect for oil, a borehole is drilled in the earth. The borehole is filled with mud to balance the downhole pressure. The conductivity of the mud is usually higher than the conductivity of the surrounding rock formation. The mud may flush into the rock formation, generating an invaded zone behind the borehole wall. Usually the conductivity of the invaded zone forms a profile with conductivity values varying between that of the mud and the rock formation.

In this paper we will consider a canonical problem that can be employed to study the physics of electrode tools that are routinely used in prospecting the conductivity of rock formations. Fig. 1(a) shows the geometry of the problem. A perfectly conducting electrode is mounted on a metallic pad of a finite extent. The pad is assumed to be perfectly conducting and has the effect of focusing the current into the formation. The pad-electrode arrangement is pressed against the borehole wall, injecting low-frequency currents into the rock formation.

To simplify the analysis, the geophysical environment is modeled as a planar stratified lossy medium [8], [9] as shown in Fig. 1(b). The pad is modeled as a perfectly conducting rectangular plate as shown in Fig. 2, and the electrode is modeled as a small rectangular patch located in the center of a hole on the pad surface and is isolated from the pad by an insulator. The pad is embedded in a planar stratified medium of an arbitrary number of layers.

In Section II, an electrostatic formulation in the spectral domain is introduced starting from a transverse magnetic (TM) wave formulation. An integral equation in the current is then obtained by imposing the appropriate boundary condition on the pad-electrode surface. Expressions for the potential distribution in each layer are also derived.

In Section III, the method of moments is applied to solve the integral equation numerically. In Section IV, the numerical results are presented and discussed to investigate the performance of the pad-electrode as a geophysical tool. Different models representing the measurement environment are investigated. Four different types of conductivity profiles for the invaded zone are also considered.

II. PROBLEM FORMULATION

The geometrical configuration of an electrode mounted on a perfectly conducting pad buried in the ith layer of a stratified medium is shown in Fig. 1(b). The thickness and conductivity of the jth layer are hj and σj, respectively. The pad is located at z = zp.

The top view of the pad-electrode configuration is shown in Fig. 2. The pad is modeled as a rectangular plate with a square hole at the center. The electrode is modeled as a square patch located in the middle of the hole, and is electrically insulated from the pad.

We choose the coordinate system such that the centers of the pad and the electrode are at the origin, and the pad sides are parallel to the x and y axes. The widths of the pad along the x and y direction are denoted by wp and ws, respectively. Similarly, the widths of the hole are denoted by whp and whs, and those of the electrode by wep and wes.

Since in the low frequency limit, the problem reduces to an electrostatic one, we therefore consider only the TMz field components. The z component of the electric field in the ith layer can be expressed as [9], [10]

\[
E_z(r) = \int_{-\infty}^{\infty} \frac{d\theta_k e^{ik_z'}}{2\pi} \left\{ \right. \\
\left. \left[ \pm e^{ik_z'(z-z_p)} + \frac{1}{1-R_{ij}R_{ij}e^{2ik_z h_j}} \left[ R_{ij}e^{ik_z'(-z-z_p-2d_{ij}-1)} - R_{ij}R_{ij}e^{2ik_z(z-z_p+2h_j)} \\
- R_{ij}e^{ik_z(z-z_p+2h_j)} + R_{ij}R_{ij}e^{ik_z'(-z-z_p+2h_j)} \right] \right\} \right. 
\]
where the plus sign applies when $z > z_p$, and the minus sign applies when $z < z_p$. $k_s = \hat{x}k_x + \hat{y}k_y$ is the transverse wave vector, $\hat{x} = \partial_x + 
abla y$ is the transverse position vector, $\hat{e}_t(k_s)$ is the spectral amplitude of $E_t(z)$. The wavenumber along the $z$ direction, $k_{tz}$, satisfies the the dispersion relation $k_{tz}^2 + k_l^2 = k_f^2$ and the radiation condition $\text{Im}(k_{tz}) > 0$. Here, $k_f^2 = \alpha^2 \mu_0 \varepsilon_0 (\varepsilon_1 + i\sigma_1/\omega_0)$, where $\varepsilon_1$ and $\sigma_1$ are the relative permittivity and conductivity of the $l$th layer, $\varepsilon_0$ and $\mu_0$ are the permittivity and permeability of free space.

In (1), $R_{ui}$ and $R_{ni}$ are the TM$_t$ reflection coefficients at the upper and lower boundaries of the $l$th layer, respectively. They can be obtained recursively as

\[
R_{ui} = \frac{R_{ui-1} + R_{ui-1} e^{2i(k_{tz} + k_f)z_{l-1}}}{1 + R_{ui-1} R_{ui-1} e^{2i(k_{tz} + k_f)z_{l-1}}}
\]

\[
R_{ni} = \frac{R_{ni+1} + R_{ni+1} e^{2i(k_{tz} + k_f)z_{l+1}}}{1 + R_{ni+1} R_{ni+1} e^{2i(k_{tz} + k_f)z_{l+1}}}
\]

where $R_{ui+1}$ is the Fresnel reflection coefficients from the $l$th layer to the $(l+1)$th layer. The explicit form of $R_{ui+1}$ is

\[
R_{ui+1} = \frac{(\varepsilon_{l+1} + i\sigma_{l+1}/\omega_0)k_{t+1} - (\varepsilon_l + i\sigma_l/\omega_0)k_{l+1}}{(\varepsilon_{l+1} + i\sigma_{l+1}/\omega_0)k_{l+1} + (\varepsilon_l + i\sigma_l/\omega_0)k_{l+1}}.
\]

In the low-frequency limit, we have $\sigma_l \gg \omega_0 \varepsilon_0$, then (3) reduces to

\[
R_{ui+1} = \frac{\sigma_{l+1} k_{l+1} - \sigma_l k_{l+1}}{\sigma_{l+1} k_{l+1} + \sigma_l k_{l+1}}.
\]

In the electrostatic limit, the frequency is identically set to zero. We thus have $k_{l+1} = k_{l+1}$ for all $l$, and (3a) reduces to

\[
R_{ui+1} = \frac{\sigma_{l+1} - \sigma_l}{\sigma_{l+1} + \sigma_l}.
\]

The electrostatic potential $V_l(z)$ in the $l$th layer can be represented in the spectral domain as

\[
V_l(z) = \int_{-\infty}^{\infty} d\bar{z} \hat{E}_t(\bar{z}) \frac{1}{k_{lt}} \hat{e}_t(k_{lt})
\]

\[
\left\{ e^{i k_{lt} z - \tau_p} - \frac{1}{1 - R_{ui} R_{ni} e^{2i k_{lt} w}} \right\}
\]

\[
\left\{ R_{ui} e^{ik_{lt} (z - \tau_p - 2i k_{lt} w)} - R_{ui} R_{ni} e^{i(k_{lt} + k_f) (z - \tau_p + 2i k_{lt} w)}
\]

\[
+ R_{ni} e^{i(k_{lt} + k_f) (z + 2i k_{lt} w)} - R_{ui} R_{ni} e^{i(k_{lt} + k_f) (z + 2i k_{lt} w)} \right\}.
\]
Using (1), the outflowing current density \( J_z(\vec{r}_s) \) on the pad-electrode surface can be obtained as

\[
J_z(\vec{r}_s) = \sigma [E_{z2}(\vec{r}_s, z = z_{p, +}) - E_{z2}(\vec{r}_s, z = z_{p, -})]
\]

where

\[
E_{z2}(\vec{r}_s, z) = \int \frac{d\vec{k}_z e^{i\vec{k}_z \cdot \vec{r}_s}}{2\sigma |k_z|} \delta(k_z) e^{i\vec{k}_z \cdot \vec{r}_s}
\]

Expressing \( J_z(\vec{r}_s) \) in terms of the two-dimensional Fourier transform, we have

\[
J_z(\vec{r}_s) = \int d\vec{k}_z J_z(k_z) e^{i\vec{k}_z \cdot \vec{r}_s},
\]

Thus the spectral amplitude \( J_z(k_z) \) is related to \( E_z(k_z) \) by

\[
E_z(k_z) = \frac{1}{2\sigma |k_z|} J_z(k_z)
\]

The electrostatic potential at any point \( P \) in layer (I) can then be represented in terms of \( J_z(k_z) \) as

\[
V_I(\vec{r}) = \int \frac{d\vec{k}_z e^{i\vec{k}_z \cdot \vec{r}} g_I(\vec{k}_z, z, z_{p, I}) J_z(k_z)}{2\sigma |k_z|}
\]

where \( g_I(\vec{k}_z, z, z_{p, I}) \) is the scalar Green's function in the spectral domain. It can be represented in the following explicit form:

\[
g_I(\vec{k}_z, z, z') = \frac{i}{2\sigma |k_z|} \left\{ \frac{e^{ik_z(z'-z)}}{R_{U}(1)e^{2ik_{z, z_I}} - 1} \right\}
\]

By solving the integral equation (9), the outflowing current density distribution on the pad-electrode surface is obtained. Thus, the potential at any point in the layered medium can be calculated. The potential distribution in the \( m \)th layer due to the source in the \( l \)th layer can be represented in terms of the scalar Green's function \( g_m(k_z, z, z') \) as follows:

\[
V_m(\vec{r}) = \int \frac{d\vec{k}_z e^{i\vec{k}_z \cdot \vec{r}} g_m(\vec{k}_z, z, z_{p, m}) J_z(k_z)}{2\sigma |k_z|}
\]

When \( m = 1 \), \( g_m(\vec{k}_z, z, z_{p, m}) \) can be calculated by (8). When \( m \neq l \), the scalar Green's function \( g_m(k_z, z, z') \) can be calculated as follows:

**Case (i) \( z > z' \)**

\[
g_m(k_z, z, z') = \frac{i}{2\sigma |k_z|} X_{U,m}[e^{ik_z z_m} - R_{U,m}e^{ik_z (z_{m-1})}] - \frac{e^{-ik_z z'_m} - R_{U,m}e^{ik_z (z_{m-1})}}{1 - R_{U,m}e^{2ik_z z_m}}
\]

**Case (ii) \( z < z' \)**

\[
g_m(k_z, z, z') = \frac{i}{2\sigma |k_z|} X_{U,m}[e^{ik_z z_m} - R_{U,m}e^{ik_z (z_{m-1})}] - \frac{e^{-ik_z z'_m} - R_{U,m}e^{ik_z (z_{m-1})}}{1 - R_{U,m}e^{2ik_z z_m}}
\]

### III. Numerical Calculations

In this section, a method of moments is applied to solve (9) for the outflowing current distribution on the pad-electrode surface. It is difficult to find a set of global basis functions that is complete over the domain of the pad with a hole. Therefore, we use a set of local basis functions to represent the outflowing current distribution on the pad-electrode surface. The outflowing current distribution can thus be represented accurately by employing a reasonable number of these basis functions.

First, the domain of the pad-electrode is divided into small rectangular patches. The size of the patches can be adjusted to achieve the required accuracy.
where $N$ is the total number of basis functions, $a_j$ are the expansion coefficients to be determined, $r_{0j}$ is the center coordinate of the $j$th basis function. Denote the Fourier transform of $R_i(r_j)$ by $\tilde{R}_j(k_j)$, hence the Fourier transform of $J_z(r_j)$ can be represented as

$$J_z(k_j) = \sum_{j=1}^{N} a_j e^{-ik_j \cdot r_j} \tilde{R}_j(k_j).$$  \hfill (18)

Substituting (18) into (9), we get

$$\int d\phi \int_0^{\infty} \sin(k x) \sin(k y) \sum_{j=1}^{N} a_j e^{-ik \cdot r_j} \tilde{R}_j(k_j) = V_{\phi},$$

where $r_j$ on $D_{\phi}$, $z = z_p$. \hfill (19)

We employ a Galerkin’s approach by using the same set of basis functions as the testing functions. Taking the inner product of the $i$th testing function with (19), we obtain

$$\sum_{j=1}^{N} Z_{ij} a_j = \beta_i$$  \hfill (20)

where

$$Z_{ij} = \int_0^{\Delta_s} \int_0^{\Delta_s} dk_x dk_y \frac{1}{k_p} \int_0^{\Delta_s} \int_0^{\Delta_s} d\phi_1 r_i(\phi_2 - \phi_1) - R_{ij}(k_j).$$  \hfill (21)

The expansion coefficients $a_j$ can be solved by inverting (20). Choosing the unit pulse function as the basis function, we have

$$R_i(r_j) = \begin{cases} 1, & -\frac{a_i}{2} \leq x \leq \frac{a_i}{2}, \quad -\frac{b_j}{2} \leq y \leq \frac{b_j}{2} \quad \text{elsewhere} \end{cases}$$  \hfill (23)

where $a_i$ and $b_j$ are the width of the $i$th patch along the $x$ and $y$ direction, respectively. The Fourier transform of $R_i(r_j)$ is thus given by

$$\tilde{R}_j(k_j) = \frac{\sin(k_x a_i/2) \sin(k_y b_j/2)}{\pi k_x k_y}.$$  \hfill (24)

Observing that $\tilde{R}_j(k_j)$ is an even function of both $k_j$ and $k_z$, the double integral in (21) over the whole $k_z - k_j$ plane can be reduced to an integral over the first quadrant as

$$Z_{ij} = 4 \int_0^{\Delta_s} dk_x \int_0^{\Delta_s} dk_y \cos k_x (x_{ij} - x_{ij}) \cos k_y (y_{ij} - y_{ij}) \cdot g_{ij}(k_j, z_p) R_{ij}(k_j) \tilde{R}(k_j).$$  \hfill (25)

where $r_{ij} = \Delta_{ij}, \tau_{ij} = \Delta_{ij}$.

From (25), it is clear that $Z_{ij} = Z_{ji}$. This property is utilized to reduce the computation time by about one half. In the electrostatic limit, the Green’s function in (8) possesses a singularity at $k_p = |k_j| = 0$, which is integrable. To perform the integration in (25), we divide the first quadrant in the $k_z - k_j$ plane into two parts. The first part is $0 \leq k_j \leq \Delta_x$, and the second part is the remaining portion of the first quadrant. The integration over the first part is given by

$$\int_0^{\Delta_x} dk_x \int_0^{\Delta_y} dk_y \cos k_x (x_{ij} - x_{ij}) \cdot \cos k_y (y_{ij} - y_{ij}) g_{ij}(k_j, z_p, y_{ij}) R_{ij}(k_j) \tilde{R}(k_j) \tilde{R}(k_j)$$

$$= \int_0^{\Delta_x} dk_x \int_0^{\Delta_y} dk_y \left\{ \cos k_x (x_{ij} - x_{ij}) \right\}$$

$$+ \gamma_0 \int_0^{\Delta_x} dk_x \int_0^{\Delta_y} dk_y \frac{1}{k_p}.$$  \hfill (26)

where $\gamma_0$ is the residue calculated by

$$\gamma_0 = \lim_{k_p \to 0} k_p \cos k_x (x_{ij} - x_{ij})$$

$$\cdot \cos k_y (y_{ij} - y_{ij}) g_{ij}(k_j, z_p, y_{ij}) R_{ij}(k_j) \tilde{R}(k_j).$$  \hfill (27)

The second integral on the right side of (26) can be performed analytically, to obtain

$$\int_0^{\Delta_x} dk_x \int_0^{\Delta_y} dk_y \frac{1}{k_p} = \Delta_x \ln \left[ \frac{\Delta_x^2 + \sqrt{1 + (\Delta_y^2)} \Delta_y^2} {\Delta_y^2 + \sqrt{1 + (\Delta_x^2)} \Delta_x^2} \right]$$

$$+ \Delta_y \ln \left[ \frac{\Delta_x^2 + \sqrt{1 + (\Delta_y^2)} \Delta_y^2} {\Delta_y^2 + \sqrt{1 + (\Delta_x^2)} \Delta_x^2} \right].$$  \hfill (28)

The choice of $\Delta_x$ and $\Delta_y$ is arbitrary, and does not affect the final value of the integral in (25). In the numerical computation of the second part of the above integral, we gradually increased the domain of integration until convergence was achieved.

To perform the numerical integration more efficiently, we store the computations which are repeatedly used, hence reducing the CPU time significantly. All the numerical results presented in the next section were calculated using a VAX/750.

We choose to compute the above double integral in the cartesian coordinate system rather than the polar system, because of the fact that as $k_p$ increases, one requires a finer mesh along the $\phi$ direction. This multigridding required in the polar coordinate system, makes the bookkeeping process very cumbersome.

To find the current density distribution at any point in an arbitrary layer $(m)$, we have

$$J_m(r) = -\sigma_m V_{\phi}(r), \quad m = 0, 1, 2, \ldots, t.$$  \hfill (29)

To calculate the current density in (29), we first substitute (18) and (24) into (10), then apply (29). Utilizing the following symmetrical properties of the basis functions,

$$\tilde{R}(k_z, k_j) = \tilde{R}(k_j, k_z), \quad \tilde{R}(k_z, -k_j) = \tilde{R}(k_z, k_j).$$  \hfill (30)
IV. RESULTS AND DISCUSSION

In the numerical results presented in this section, we choose the pad-electrode dimensions such that $w^p_x = w^p_y = 10$ cm, $w^p_z = 2$ cm, $w^p = 1.6$ cm. The electrode is patched by 25 unit pulse functions with equal patch size of 0.32 by 0.32 cm each. The pad surface is patched by 96 unit pulse functions with equal patch size of 1 by 1 cm.

The pad-electrode structure is first assumed to be buried in a homogeneous medium with $\sigma = 10$ S/m. The magnitude of the outflowing current distribution on the pad-electrode surface along $x = 0$ cm and $x = 4.5$ cm are displayed in Fig. 3. The edge effect is obvious near the edge of the pad and the electrode, but is not so pronounced near the edge of the hole.

Next, consider the case of a three-layer medium with both layer (0) and layer (2) as semi-infinite half-space, and the thickness of layer (1) is $h = 10$ cm. The conductivity of layer (1) is $\sigma_1 = 10$ S/m, and the conductivities of layers (0) and (2) are the same as $\sigma_0$. The pad-electrode structure is buried in the middle of layer (1) as shown in Fig. 4. The normalized total electrode current on a log scale, $\log \left( I_e/I_0 \right)$, is plotted as a function of $\log \left( \sigma_0/\sigma_1 \right)$, where $I_0$ is a reference value taken to be the total electrode current for the case of a homogeneous medium with $\sigma_0 = 10$ S/m.

Two cases are displayed with layer thickness $h = 10$ cm and $h = 2$ cm. It is observed that the total electrode current increases with $\sigma_0$. The total electrode current with $h = 2$ cm is more sensitive to the change of $\sigma_0$ than that with $h = 10$ cm. When $\sigma_0/\sigma_1 > 1$, the total electrode current is smaller for $h = 10$ cm than it is for $h = 2$ cm.

In this kind of measurement environment, the total electrode current is sensitive to both $h$ and $\sigma_0$. The total electrode current can be used to predict the conductivity $\sigma_0$ if the standoff thickness $h$ can be estimated.

In Fig. 5, the total electrode current $I_e$ is displayed as a function of the layer thickness $h$. The conductivity of layer (1) is $\sigma_1 = 10$ S/m, and the conductivity of layers (0) and (2) are the same with a value equals $\sigma_0$. Two cases with $\sigma_0 = 1$ S/m and $\sigma_0 = 10^{-3}$ S/m are displayed. It is observed that the total electrode current $I_e$ increases with layer thickness $h$. The magnitude of $I_e$ with $\sigma_0 = 1$ S/m is about 10 times larger than that with $\sigma_0 = 10^{-3}$ S/m.
In Fig. 6, the current density $J_{y}(\rho = 0, h^+/2)$ at the interface between layer (0) and layer (1) is evaluated as a function of $\log (\sigma_0/\sigma_1)$ with $\sigma_1 = 10$ S/m. The current density is normalized to the current density in a homogeneous medium at a point 1 cm right above the electrode center. When $\sigma_0/\sigma_1 < 1$, the current density is larger for $h = 2$ cm than for $h = 10$ cm.

In Fig. 7, the electric field strength $E_{y}(\rho = 0, h^+/2)$ at the interface between layer (0) and layer (1) is displayed as a function of layer thickness $h$. The conductivity of layer (1) is $10^{-3}$ S/m, the conductivities of layers (0) and (2) are the same as $\sigma_0$. Two cases with $\sigma_0 = 1$ S/m and $\sigma_0 = 10^{-3}$ S/m are calculated. The field strength decreases as the layer thickness $h$ increases. The field strength is about four times larger for $\sigma_0 = 1$ S/m than for $\sigma_0 = 10^{-3}$ S/m.

In Fig. 8, layer (0) is a conductive half-space of a high conductivity $\sigma_0 = 10^{6}$ S/m, layer (1) is a resistive layer with $\sigma_1 = 10^{-3}$ S/m and $h_1 = 2$ cm. The insulator in layer (1) and the
good conductor in layer (0) have the effect of focusing the current flow into the rock formation. Layer (2) is a standoff layer with \( \sigma_2 = 10 \text{ S/m} \) and variable thickness \( h_2 \). The pad-electrode is located at the interface between layer (1) and layer (2). The region below layer (2) possesses different possible conductivity profiles, and can be modeled by a multilayered medium. The origin of the coordinate system is chosen at the interface between the standoff layer and the conductivity profile, right below the electrode center. The conductivity profiles used in this paper include step profile, linear profile, parabolic profile, and inversion profile. For a step profile,

\[
s_0(z) = \begin{cases} 
\sigma_0, & z \leq 0 \\
\frac{\sigma_2 - \sigma_0}{D} z + \sigma_0, & -D \leq z \leq 0 \\
\sigma_0, & z < -D 
\end{cases}
\]  

(34)

where \( \sigma_0 \) is a constant conductivity.

For a linear profile,

\[
s_0(z) = \begin{cases} 
\frac{(\sigma_2 - \sigma_0)(1 + z/D)}{D}, & -D \leq z \leq 0 \\
\sigma_0, & z < -D 
\end{cases}
\]  

(35)

where \( D \) is the depth of the conductivity profile, \( \sigma_0 \) is the
good conductor, \( \sigma_g = 10^5 \text{ mho/m} \)

insulator, \( \sigma_i = 10^{-8} \text{ mho/m} \)

pad-electrode standoff, \( \sigma_z = 10 \text{ mho/m} \)

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Fig. 8. Configuration of a pad-electrode in a profiled layered medium, \( w_x = w_y = 10 \text{ cm} \), \( w_z = w_y = 2 \text{ cm} \), \( w_{\phi} = w_{\phi} = 1.6 \text{ cm} \), \( V_0 = V_1 = 1 \text{ V} \).

(a) Step profile. (b) Linear profile. (c) Parabolic profile. (d) Inversion profile.

background conductivity in the region \( z < -D \) which is homogeneous by assumption.

For a parabolic profile,

\[
\sigma_p(z) = \begin{cases} 
\sigma_b & \quad -D \leq z \leq 0 \\
\sigma_k + (\sigma_0 - \sigma_k)\left[1 - \frac{z}{D}\right]^2 & \quad z < -D
\end{cases}
\]  

(36)

where the definition of \( D \) and \( \sigma_k \) is the same as that for the linear profile.

For an inversion profile,

\[
\sigma_i(z) = \begin{cases} 
\sigma_k & \quad -D \leq z \leq 0 \\
\sigma_b & \quad z < -D
\end{cases}
\]  

(37)

where \( \sigma_k \) is the conductivity of the inversion layer and \( \sigma_b \) is the conductivity of the background medium. Note that \( \sigma_k < \sigma_b \).

For both the linear profile and the parabolic profile, a multilayered medium is used to model the conductivity profile. The region \(-D \leq z \leq 0\) is first divided into a multilayered medium with the same thickness for each layer. The conductivity of each layer assumes the value calculated from either (35) or (36) at the middle of that layer.

The total electrode current of the pad-electrode in a step conductivity profile is displayed in Fig. 9 as a function of the standoff thickness \( h_2 \). The total electrode current is plotted on a log scale normalized to \( I_0 \) which is a reference electrode current with \( \sigma_b = 10 \text{ S/m} \) in Fig. 8(a). The thickness of the standoff layer ranges from 0 to 2 cm, and \( \sigma_b \) ranges from 0.1 to 100 S/m. It is observed that the total electrode current is almost a linear function of \( \sigma_0 \) at any standoff thickness. Furthermore, the total electrode current is insensitive to the variation of \( h_2 \).

The total electrode current for the step conductivity profile is also plotted on a normal scale in Fig. 10 to compare with the other conductivity profiles. Three cases of background conductivities are used: \( \sigma_b = 0.1 \text{ S/m} \), \( \sigma_b = 1.0 \text{ S/m} \), and \( \sigma_b = 5.0 \text{ S/m} \). The total electrode current exhibits a minimum as a function of \( h_2 \).

Next, the total electrode current of the pad-electrode in the
function of the standoff thickness \( h_2 \). The current is plotted on a normal scale. The thickness of the standoff layer ranges from 0 to 2 cm. Two \( \sigma_0 \) of 0.1 S/m and 5 S/m, and two profile depths of \( D = 4 \) and 10 cm are chosen.

Similar to the result for the linear profile, the total electrode current for a parabolic profile is insensitive to the variation of \( h_2 \). The total electrode current is mainly affected by the background conductivity \( \sigma_0 \) and the profile depth \( D \). Increasing either \( D \) or \( \sigma_0 \) will increase the total electrode current.

The total electrode current is larger in Fig. 12 compared with that in Fig. 11 having the same background conductivity and profile depth. For both linear profile and parabolic profile, the total electrode current increases as \( h_2 \) increases.

For both the linear and parabolic profiles, the background conductivity \( \sigma_0 \) can be retrieved from the total electrode current if the profile is linear or parabolic, and if the depth can be estimated independently.

Finally, consider the inversion profile with \( D = 1 \) cm or \( D = 3 \) cm. Five combinations of \((\sigma_0, \sigma_b)\) are used, these are \((\sigma_0, \sigma_b) = (0.05, 0.1), (0.1, 1.0), (0.5, 1.0), (1.0, 5.0), \) and \((2.5, 5.0)\), all in S/m. The total electrode current versus the standoff thickness \( h_2 \) is plotted in Fig. 13(a) for \( D = 1 \) cm, and in Fig. 13(b) for \( D = 3 \) cm.

It is observed that increasing either \( \sigma_0 \) or \( \sigma_b \) will increase the electrode current, and increasing the profile depth \( D \) will decrease the electrode current especially when \( \sigma_0 \) or \( \sigma_b \) is large. The total electrode current increases when \( h_2 \) is either increased or decreased. This is similar to the results for the step profile.

Comparing Fig. 10 with Figs. 11 and 12, it is observed that when \( \sigma_0 \) is increased, the increasing rate of the total electrode current for the step profile is larger than that for the linear profile or the parabolic profile. The increasing rate is larger when the profile depth is smaller. This can be explained as follows. Since the linear profile and the parabolic profile have a transition region between the standoff layer and the background medium, the effect of \( \sigma_0 \) to the total electrode current is shielded by the transition region. On the contrary, the background medium in the step profile is right below the standoff layer, hence it affects the total electrode current more directly.

V. CONCLUSION

An electro-quasistatic approach in the spectral domain is derived to analyze a pad-electrode antenna used in the applications of low frequency geophysical probing. An integral equation is derived, which is then solved by the method of moments. A set of unit pulse functions are used to represent the outflowing current distribution on the pad-electrode surface. The potential distribution in the layered medium can be calculated in terms of the outflowing current distribution.

Numerical results for a symmetrical three-layer medium is presented. The effects of layer thickness and conductivity is studied. A more practical measurement environment with four different kinds of conductivity profiles are then investigated. The effects of standoff thickness, background conductivity, and profile depth are studied. Numerical results reveal that the total electrode current is sensitive to the conductivity of the
formation medium, but is insensitive to the standoff thickness. Thus, it can be used to retrieve the background conductivity for different types of conductivity profiles, which is very useful in geophysical probing.

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REFERENCES


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