Reconstruction of Ionospheric Perturbation Induced by 2004 Sumatra Tsunami Using a Computerized Tomography Technique

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Abstract—A computerized tomography technique is used to reconstruct a 3-D profile of electron density perturbation in the ionosphere based on the total electron content data recorded at a planned GPS receiver network. Simulations are conducted using the data of the 2004 Sumatra event. An effective early-warning system may become feasible based on this work.

Index Terms—Computerized tomography (CT), global positioning system (GPS), gravity wave, total electron content (TEC), tsunami.

I. INTRODUCTION

A tsunami traveling over the ocean may trigger atmospheric gravity waves (AGWs) which will perturb the electron density distribution in the ionosphere. Before real tsunami waveforms were recorded recently, researchers had chosen by intuition the Airy’s wave, Laitone solitary wave, or Stokes wave to study phenomena related to AGWs under different assumptions of fluid dynamics [1], [2]. The theory of a nondispersive shallow water wave has been applied when dealing with tsunamis in an open sea, while nonlinear Boussinesq and full Navier–Stokes equations have been solved considering more complex seabeds [1], [3]. Simulation models of the 2004 Sumatra tsunami ($M_{w} = 9.3$ at 05:58:50 UT, 3.3°N, 95.8°E) have been proposed using method of splitting tsunami and finite difference bathymetry [4]–[6], in which the epicenter, seabed profile, propagation time, and sea surface displacement have been estimated on a global scale.

The ionospheric irregularities induced by the 2004 Sumatra tsunami have been widely analyzed under different assumptions [7]–[11]. The 2004 event provides an example of a traveling tsunami in the open sea, which was detected with satellite altimetry, and the associated total electron content (TEC) data were also recorded [12]. The satellite-borne altimetry in Jason-1, Topex/Poseidon, Envisat, and Geosat-Follow-On recorded profiles of sea surface beneath their respective orbital track across the Indian Ocean between 2 and 9 h after the earthquake broke out [13]–[15]. In the preparation of this paper, a complete model of tsunami-atmosphere coupling and ionospheric response has been derived. The 2004 Sumatra tsunami profile can also be reconstructed based on the Jason-1 altimetry data.

Ground-based HF Doppler radars have been used over many decades to detect vertical oscillation of lower ionospheric layers via the Doppler shift of reflected sounding signal [16]. Atmospheric waves induced by earthquake, volcano eruption, or AGWs triggered by tsunami can be detected. However, the sounding altitudes are limited to about 150 km.

Satellite-borne altimetry nowadays can measure the sea surface profile precise to a few centimeters [14]. However, measurement data can only be collected along satellite tracks, and 2-D profile is hardly available. Pending on the satellites, the data were relayed to the central station hours, sometimes even days, after the event, which is far too late to be used for real-time detection to construct an effective early-warning system [13].

Since a tsunami can induce irregularities in the ionosphere, the former can be detected over a wide geographical area by observing the perturbation of electron density in the ionosphere and displaying it in real-time 3-D image. The global positioning system (GPS) has been widely used in daily lives. A network of GPS receivers can be well planned at reasonable cost to continuously collect data which contain ionospheric information. Computerized tomography (CT) techniques have been proposed to estimate the ionospheric electron density distribution [17]–[20], 3- and even 4-D ionospheric tomography is plausible [21]. The electron density distribution in the ionosphere under normal condition can be forecast to a reasonable accuracy using the International Reference Ionosphere (IRI) model [22]; hence irregularities can be detected to certain confidence level.

Currently, many active systems serve in the fields to monitor the ionosphere. For example, the National Aeronautics and Space Administration has demonstrated a Global Differential GPS (GDGPS) system to process real-time GPS data from a global network of approximately 100 real-time GDGPS tracking sites, augmented with additional sites that are available on an hourly basis [23]. The real-time global maps of ionospheric electron density is currently produced every 5 min.

The Jet Propulsion Laboratory (JPL) ionospheric and atmospheric remote sensing group takes GPS data collected over a global network of ground stations to produce real-time global maps of ionospheric TEC [24]. These global maps are updated...
at 5-min intervals, which can be used for calibration to single-frequency GPS users.

Another 3-D time-dependent global ionospheric model, the global assimilative ionospheric model, has been under development at JPL and University of Southern California since 1999. The JPL group applies this model to process data from global and regional GPS networks to estimate the spatial and temporal variation of the global ionosphere on a daily basis.

Other organizations and networks relevant to ionospheric mapping include the Fusion Numerics, Inc., funded by the U.S. Air Force Research Laboratory, the U.S. National Oceanic and Atmospheric Administration Space Environment Center [25], [26], the International GPS Service ionosphere working group, the Southern California Integrated GPS Network [27]–[29], the Japanese GPS Earth Observation Network System [7], [30]–[33], the Center for Orbit Determination in Europe, the Reference Frame Sub-Commission for Europe which has been establishing and maintaining the European Terrestrial Reference System 89, and European Vertical Reference System [34], [35]. These organizations and networks provide data derived from GPS signals for a wide range of scientific applications such as monitoring of ground deformations, earthquakes, sea level, tsunami, as well as space weather forecast.

Sea-based tsunami alert systems have been deployed to measure the sea surface profile, for example, the Global Sea Level Observing System (GLOSS) [36], [37] and the Deep-ocean Assessment and Reporting of Tsunami (DART) [38], [39]. In the GLOSS, tide gauges made of tubes and floats are anchored along coasts or islands to monitor the sea level. The sea level can also be monitored with radar, sonar, or seabed pressure sensors. In the DART system, buoys and sensors are anchored onto shelf of continent or archipelago. The pressure recorder on the seabed sends its data to a buoy on the sea surface. The buoy sends the data of sea surface and the data from the pressure recorder to a satellite to be relayed to a central station. The DART system is able to detect a tsunami when its wavefront reaches the sensors. The land far away from the DART system is thus warned at early stage, but the territories near the DART system do not have enough time margin for evacuation. Not to mention that the buoys are expensive to install and maintain. Germany is working on a joint project with Indonesia to install ten of these buoys. India, Thailand, and Australia are also planning to install DART buoys along the Sunda Trench, a high-risk area of tsunami.

In this paper, we propose a practical approach of detecting a tsunami in its early phase by observing the perturbation of ionospheric electron density distribution. CT technique is applied to the GPS data collected from a well-planned receiver network to display such perturbation. The scenario of 2004 Sumatra tsunami is used as a test bench. A possible GPS receiver network is envisioned, which consists of floating units carried by three ocean currents and fixed units on the Indian subcontinent, neighboring coastlines and islands. Measured data from Jason-1 are used to reconstruct the tsunami profile which is then used in a model to estimate the perturbed electron density distribution. The TEC data over each satellite-receiver link can thus be simulated, and then used in the CT algorithm to reconstruct the perturbed electron density distribution.

II. CT Technique

The TEC over a specific path \( \ell \) is defined as

\[
\text{TEC} = \int_{\ell} \rho_{e}(s) ds
\]  

where \( \rho_{e}(s) \) is the electron density along path \( \ell \) between the given satellite and receiver, and \( s \) the coordinate along path \( \ell \).

As shown in Fig. 1(a), the region of interest is divided into \( N \) voxels of approximately rectilinear shape, the \( n \)th voxel occupies the space \( V_n \) with the center located at \( \bar{r}_n = (\theta_n, \phi_n, z_n) \), where \( \phi, \theta, \) and \( h \) stand for the longitude, latitude, and altitude, respectively. The electron density distribution \( f(\bar{r}) \) in the target region can be approximated by a set of \( N \) local basis functions \( \{b_n(\bar{r})\} \) as

\[
f(\bar{r}) \approx \sum_{n=1}^{N} x_n b_n(\bar{r})
\]

where \( x_n \) is the average electron density in \( V_n \), and

\[
b_n(\bar{r}) = \begin{cases} 
1, & \text{if } \bar{r} \text{ is inside } V_n \\
0, & \text{otherwise}
\end{cases}
\]
Assume that there are \( I \) satellites and \( J \) receivers, and the data are collected at \( M \) time instants. Let \( y_{jm}^{i} \) be the measured TEC from satellite \( i \) located at \( S_{jm}^{i} \) to receiver \( j \) located at \( R_{jm} \), at time instant \( m \), then \( y_{jm}^{i} \) is the line integral of \( f(\vec{r}) \) along path \( \ell_{jm}^{i} \) as

\[
y_{jm}^{i} = \int_{\ell_{jm}^{i}} f(\vec{r}) ds
\]

with \( 1 \leq i \leq I \), \( 1 \leq j \leq J \), and \( 1 \leq m \leq M \).

By substituting (2) into (4), we have

\[
y_{jm}^{i} \simeq \sum_{n=1}^{N} Z_{jm}^{i,n} x_{n}
\]

where

\[
Z_{jm}^{i,n} = \int_{\ell_{jm}^{i}} b_{n}(\vec{r}) ds
\]

is the path length of \( \ell_{jm}^{i} \) within voxel \( V_{n} \), as shown in Fig. 1(b). Equation (5) can be put in a matrix form as

\[
\begin{bmatrix}
\bar{Y}
\end{bmatrix} \simeq \bar{Z} \cdot \bar{X}
\]

where the expressions for \( \bar{Y} \), \( \bar{Z} \), \( \bar{X} \), \( \bar{Y}_{j}^{i} \), and \( \bar{Z}_{j}^{i} \) are shown at the bottom of the page.

### A. ART

Finding the solution to (6) is equivalent to finding the intersection of \( IJM \) hyperplanes in an \( N \)-dimensional space [17], [18]. For example, if \( N = 2 \), \( I = 1 \), \( J = 2 \), and \( M = 1 \), then (6) is reduced to

\[
\begin{align*}
Z_{11}^{11} x_{1} + Z_{12}^{11} x_{2} &= y_{1}^{11} \\
Z_{21}^{11} x_{1} + Z_{22}^{11} x_{2} &= y_{2}^{11}
\end{align*}
\]

As shown in Fig. 2(a), \( L_{1} \) and \( L_{2} \) represent the first and the second equation, respectively, in (7). Starting with an initial guess at \( X^{(0)} \), project this initial guess onto \( L_{1} \) to obtain \( \bar{X}^{(1)} \), then project \( \bar{X}^{(1)} \) onto \( L_{2} \) to obtain \( \bar{X}^{(2)} \), then project \( \bar{X}^{(2)} \) back onto \( L_{1} \), and so on, until the intersection point converges to \( \bar{X} \).

Fig. 2(b) shows the schematic to derive \( X^{(1)} \) from \( X^{(0)} \). The normal vector of \( L_{1} \) can be expressed as \( Z_{11}^{11} = \hat{x}_{1} Z_{11}^{11} + \hat{x}_{2} Z_{12}^{11} \). Let \( p_{0} \) mark the initial guess, and \( p_{1} \) be the projection of \( p_{0} \) onto \( L_{1} \). If \( A(x_{1}, x_{2}) \) is an arbitrary point on \( L_{1} \), then we have

\[
p_{0} \mid L_{1} = \frac{p_{0} \cdot A \cdot Z_{11}^{11}}{Z_{11}^{11}}, Z_{11}^{11} Z_{11}^{11} = y_{1}^{11} - X^{(0)} \cdot Z_{11}^{11} Z_{11}^{11}.
\]

\[
\bar{Y} = \begin{bmatrix} \bar{Y}_{1}^{1} & \bar{Y}_{1}^{2} & \cdots & \bar{Y}_{1}^{i} & \bar{Y}_{2}^{1} & \bar{Y}_{2}^{2} & \cdots & \bar{Y}_{2}^{j} & \bar{Y}_{3}^{1} & \bar{Y}_{3}^{2} & \cdots & \bar{Y}_{3}^{k} \end{bmatrix}^{t}
\]

\[
\bar{Z} = \begin{bmatrix} \bar{Z}_{1}^{1} & \bar{Z}_{1}^{2} & \cdots & \bar{Z}_{1}^{i} & \bar{Z}_{2}^{1} & \bar{Z}_{2}^{2} & \cdots & \bar{Z}_{2}^{j} & \bar{Z}_{3}^{1} & \bar{Z}_{3}^{2} & \cdots & \bar{Z}_{3}^{k} \end{bmatrix}^{t}
\]

\[
\bar{X} = \begin{bmatrix} x_{1} & x_{2} & \cdots & x_{N} \end{bmatrix}^{t}
\]

\[
\bar{Y}_{j}^{i} = \begin{bmatrix} y_{j}^{11} & y_{j}^{12} & \cdots & y_{j}^{i,M} \end{bmatrix}
\]

\[
\bar{Z}_{j}^{i} = \begin{bmatrix} Z_{j1}^{11} & Z_{j1}^{12} & \cdots & Z_{j1}^{i,M} \\
Z_{j2}^{11} & Z_{j2}^{12} & \cdots & Z_{j2}^{i,M} \\
\vdots & \vdots & \ddots & \vdots \\
Z_{jN}^{11} & Z_{jN}^{12} & \cdots & Z_{jN}^{i,M} \end{bmatrix}
\]
Hence
\[ \bar{X}^{(1)} = \bar{X}^{(0)} + \frac{1}{\rho_0 p_1} = \bar{X}^{(0)} + \frac{y_1^{11} - \bar{X}^{(0)} \cdot \bar{Z}_1^{11}}{(Z_1^{11})^2 + (Z_1^{12})^2} \bar{Z}_1^{11}. \]

Extending to an \( N \)-dimensional space, the initial guess at \( \bar{X}^{(0)} \) is first projected onto the first hyperplane, represented by the first equation in (6), to obtain \( \bar{X}^{(1)} \), then \( \bar{X}^{(1)} \) is projected onto the second hyperplane to obtain \( \bar{X}^{(2)} \), and so on. The iteration process can be described as
\[
\bar{X}^{(k+1)} = \bar{X}^{(k)} + \frac{y_j^m}{\rho_k} \left( \sum_{n=1}^{N} Z_j^{nm} \bar{x}_n^{(k)} \right) Z_j^{im} \tag{9}
\]
where \( \rho_k \) is an empirical relaxation parameter which lies between 0 and 2. The iteration stops when
\[
\frac{||\bar{X}^{(k+1)} - \bar{X}^{(k)}||}{||\bar{X}^{(k)}||} \leq \delta \tag{10}
\]
with \( \delta \) a given threshold.

B. SIRT

The algorithm of simultaneous iterative reconstruction technique (SIRT) is similar to algebraic reconstruction technique (ART), except the former updates the solution \( \bar{X} \) after the deviations of the previous solution from all the hyperplanes are attained. The initial guess is first projected onto all the hyperplanes to obtain \( \bar{x}_n^{im(1)} \), one for each hyperplane. Next, take the sum of \( \bar{x}_n^{im(1)} \)s associated with all the voxels \( V_n \)'s that are penetrated by the path \( P_j^m \), divide this sum by the number of times \( p_n \) that voxel \( V_n \) is penetrated by all the paths to obtain a correction factor. The element \( x_n^{(2)} \) is then obtained by adding the correction factor to \( x_n^{(1)} \). Explicitly, the iteration process can be described as
\[
x_n^{(k+1)} = x_n^{(k)} + \frac{\rho_k}{p_n} \sum_{m=1}^{M} \sum_{j=1}^{I} \sum_{i=1}^{Q} x_j^{im(k)} \tag{11}
\]
with
\[
x_j^{im(k)} = \frac{y_j^m - \sum_{n=1}^{N} Z_j^{im} x_n^{(k)}}{\sum_{n=1}^{N} (Z_j^{im})^2} \]
or in matrix form
\[
\bar{X}^{(k+1)} = \bar{X}^{(k)} + \lambda_k \left[ \left\{ \bar{Q} \cdot \left[ \bar{Y} - \bar{Z} \cdot \bar{X}^{(k)} \right] \right\}^t \bar{Z} \cdot \bar{P} \right]^t \tag{11}
\]
with
\[
\bar{P} = \begin{bmatrix} 1/p_1 & 0 & \cdots & 0 \\ 0 & 1/p_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1/p_N \end{bmatrix}
\]
\[
\bar{Q} = \begin{bmatrix} 1/q_1 & 0 & \cdots & 0 \\ 0 & 1/q_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1/q_{IJM} \end{bmatrix}
\]
where \( p_n \) is the number of nonzero element in the \( n \)th column of \( \bar{Z} \), \( q_n \) is the square of the norm of the \( n \)th row vector of \( \bar{Z} \). Since there are \( N \) elements in \( \bar{X} \), the number of equations \( IJM \) is preferred greater than the number of unknowns \( N \). The iteration stops when the criterion in (10) is met.

The properties and performance of ART and SIRT algorithms, such as the choice of relaxation parameter, convergence rate, stopping criterion, error have been discussed in [17]–[21], [40]–[42]. The ART reconstructed images usually suffer from salt-and-pepper noise [42], which is due to the modification of unknowns from iteration to iteration when solving (5). When one of the equations associated with one satellite-receiver link is solved, the unknowns are modified and will affect the subsequent results when solving the next equation. The SIRT estimates the error in the same manner as the ART, except the former updates after the differences over all ray paths were calculated. The performance of these two algorithms is about the same given the same initial guess. By using relaxation technique, the image quality can usually be improved at the cost of a slower convergence rate.

![Diagram](Fig. 3. (a) Target region in scenario 1, bounded by the outer square at \( h = R + 600 \) km, the middle square at \( h = R + 100 \) km, and the inner square at \( h = R \), where \( R = 6371 \) km is the Earth radius, point \( O \) is the Earth center. (b) Voxel indices in different layers.)
Several combinations of iteration numbers and relaxation parameters were used with both ART and SIRT algorithms [17]. The quality of reconstructed images does not vary appreciably with reasonable choice of the relaxation parameter. As the relaxation parameter becomes smaller, the number of iterations increases to achieve the same image quality. Optimum choice of parameters in the presence of noise has been discussed in [41]. The stopping threshold in (10) is chosen to be 0.002, as suggested in [41].

III. Simulation of TEC Data Recorded by the Receiver Network

A. Target Region and Receiver Network in Scenario I

Scenario I is designed to study the effect of receiver number beneath the target region on the reconstructed image. Fig. 3(a) shows the target region which covers $80^\circ$ E $\leq \phi \leq 100^\circ$ E, $10^\circ$ S $\leq \theta \leq 10^\circ$ N, and 100 km $\leq h \leq 600$ km. The target region is divided into curvilinear voxels by a set of meridian planes $\{\Phi_\alpha\}$ with longitude $\phi_\alpha = \phi_0 + \alpha \Delta \phi$, a set of conical surfaces $\{\Theta_\beta\}$ with latitude $\theta_\beta = \theta_0 + \beta \Delta \theta$, and...
Fig. 7. Reconstructed profiles at 250 km in Scenario II. (a)–(f) represent results with plans DPA 1–6, respectively, in Table III.

Fig. 8. Voxels at 250 km passed through by any satellite-receiver link in Scenario II. (a)–(f) mean the same as in Fig. 7.

a set of spheres \( \{ H_\gamma \} \) with radius \( h_\gamma = h_0 + \gamma \Delta h \), where \( \phi_0 = 80^\circ \) E, \( \theta_0 = 10^\circ \) S, \( h_0 = R + 100 \) km, \( \Delta \phi = \Delta \theta = 0.5^\circ, \Delta h = 25 \) km, \( \alpha = 0, 1, \ldots, 40 \), \( \beta = 0, 1, \ldots, 40 \), and \( \gamma = 0, 1, \ldots, 20 \). Fig. 3(b) shows the indexing scheme of voxels. Each layer consists of 1600 voxels, and the total number of voxel is 32 000. The dimension of each voxel is approximately 55 km \( \times \) 55 km \( \times \) 25 km.

In this scenario, \( n \times n \) receivers are uniformly distributed over a square grids on the sea level right beneath the target region. A grid of 3 \( \times \) 3 receivers are marked by dots in Fig. 3(a) as an example.

B. Target Region and Receiver Network in Scenario II

In this scenario, the target region is expanded to \( 75^\circ \) E \( \leq \phi \leq 105^\circ \) E, \( 10^\circ \) S \( \leq \theta \leq 20^\circ \) N, and \( 100 \) km \( \leq h \leq 600 \) km. The dimension of voxels is the same as that in scenario I, each layer now consists of 3600 voxels, and the total number of voxels is 72 000.

Fig. 4 shows the receiver network envisioned to simulate the 2004 Sumatra event. The land-based receiver network is categorized into three sets. The first set consists of 675 ground-based receivers deployed in land, with an average distance of
about 70 km. The blank in-land areas indicate mountains. The second set consists of 248 receivers deployed along coastline with a separation of about 40 km, and another 27 receivers deployed on islands. The third set consists of 440 receivers distributed over the north Indian Ocean and the Bay of Bengal.

In the ocean, two sets of 120 receivers each are distributed over the path of the Indian Coastal Current, and a third set of 200 receivers are distributed over the path of the Indian Ocean Equatorial Counter Current.

C. Satellite Constellations

Both the existing GPS and the planned Galileo satellites [43] will be used in the simulation. The GPS constellation was originally consisted of three circular orbital planes with eight satellites in each, but was later modified to six planes with four satellites in each. Each of the six planes has an inclination angle of about 55°, and the ascending nodes of adjacent planes are separated by 60°. The orbital radius of GPS is 26 600 km; hence each satellite makes two complete revolutions in each sidereal day.

The Galileo system will provide better positioning services than the GPS at high latitudes. The constellation of the former comprises of three orbital planes with nine operational satellites in each. The three orbital planes have inclination angle of 56°, and their orbital radius is 29 593 km.

Table I lists the coordinates of GPS satellites covering the target region from UT 0257 to UT 0303, December 26, 2004. Coordinates of ten virtual Galileo satellites are also listed.

D. Ionospheric Electron Density Profile

In the simulation, the IRI, International Geomagnetic Reference Field, and Mass Spectrometer and Incoherent Scatter models are used to estimate the background parameters in the atmosphere and the background electron density profile $N_{e0}$ in the ionosphere [22], [44], [45], and the electron density perturbation $N'$, induced by the 2004 tsunami is calculated based on the models in [46].

Fig. 5 shows the wavefronts of the 2004 Sumatra tsunami. The ionospheric electron density profile above the target region shown in Fig. 4 will be reconstructed based on the simulated TEC data recorded by the receiver network. The tsunami profile in the shaded area is mapped from the data measured along track 129 of Jason 1. When the tsunami wavefronts reach the shallow water and smash the shore, the scattered waves no longer contain the spectral components to trigger AGWs, hence are not shown in the figure. Fig. 6 shows the simulated electron density profile in the ionosphere.

IV. RESULTS AND DISCUSSIONS

A. Scenario I

Assume that there are $n \times n$ receivers deployed in a uniform grid on the ocean surface right beneath the target area. The profiles of electron density perturbation at heights of $100 + 50\eta$ km with $\eta = 0, 1, \ldots, 10$ are reconstructed with $n = 3, 5, 9, 17, 33,$ and 65, respectively. To check the effectiveness of these receiver deployment plans, the number of undetected voxels that are missed by any satellite-receiver link is also watched.

Table II lists the percentage of undetected voxels and deviation of the reconstructed profile with different $n$. The reconstructed profiles with $n = 3, 5,$ and 9 are barely recognizable. With $n = 17$, rippling features of the tsunami begin to emerge, but the amplitude is not right. The reconstructed profiles with $n = 33$ and 65 show much more details, the peak of AGW-induced ionospheric irregularity is clearly observed at altitudes between 200 and 300 km. Note that the grid size of the receiver network is approximately 70 km with $n = 33$ and 35 km with $n = 65$. This simulation gives us some idea to choose a proper distance between adjacent receivers which is useful in arranging the receiver networks in scenario II.

It is also noticed that the vector $\hat{X}$ converges more quickly when $n$ is smaller, and the percentage deviation is smaller when $n$ is larger. The relaxation parameters are chosen to be $\lambda_k = 0.35$ in all the cases to accelerate the convergence rate. The IRI model is used to provide the initial guess $\hat{X}^{(0)}$.

B. Scenario II

The deployment of receivers is shown in Fig. 4. There are two options for in-land deployment, either deploy 675 receivers with grid size of about 70 km or deploy 1950 receivers with grid size of about 35 km. The coastlines and islands are fully utilized to deploy the receivers. The receivers on the ocean are distributed in three zones, marked from north to south as $A$, $B$, and $C$, respectively. The numbers of receivers in zones $A$, $B$, and $C$ are 120, 120, and 200, respectively.

First, the actual GPS ephemeris data during the Sumatra event are used to simulate the TEC data recorded by the receivers. Then, a virtual Galileo constellation is assumed to fill the gaps of GPS constellation to see how the reconstructed profile can be enhanced.

Reconstruction Using GPS Signals Only: Fig. 7(a)–(f) shows the reconstructed profiles at 250 km, using the data from receivers deployed according to deployment plan A (DPA) plans 1 to 6, respectively, as listed in Table III. The associated maps of voxels that are passed by any satellite-receiver link are also shown in Fig. 8(a)–(f), respectively. The percentage of

<table>
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<th>TABLE III</th>
<th>UNDETECTED VOXELS AND DEVIATION OF RECONSTRUCTED PROFILE UNDER DIFFERENT RECEIVER DEPLOYMENT PLANS</th>
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<tr>
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</tr>
<tr>
<td>2</td>
<td>A + B + C</td>
</tr>
<tr>
<td>3</td>
<td>CL + A + B + C</td>
</tr>
<tr>
<td>4</td>
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<td>5</td>
<td>L + CL + C</td>
</tr>
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<td>3.22</td>
<td>3.17</td>
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</table>

Rippling features of the tsunami begin to emerge, but the amplitude is not right. The reconstructed profiles with $n = 33$ and 65 show much more details, the peak of AGW-induced ionospheric irregularity is clearly observed at altitudes between 200 and 300 km. Note that the grid size of the receiver network is approximately 70 km with $n = 33$ and 35 km with $n = 65$. This simulation gives us some idea to choose a proper distance between adjacent receivers which is useful in arranging the receiver networks in scenario II.

It is also noticed that the vector $\hat{X}$ converges more quickly when $n$ is smaller, and the percentage deviation is smaller when $n$ is larger. The relaxation parameters are chosen to be $\lambda_k = 0.35$ in all the cases to accelerate the convergence rate. The IRI model is used to provide the initial guess $\hat{X}^{(0)}$. 
Fig. 9. Reconstructed profiles at 250 km in Scenario II. (a)–(f) represent results with plans DPB 1–6, respectively, in Table III.

Fig. 10. Voxels at 250 km passed through by any satellite-receiver link in Scenario II. (a)–(f) mean the same as in Fig. 9.

undetected voxels and deviation in six different plans are also listed in Table III.

In plan 1, only the receivers deployed in land, along coastlines and islands are used, but the ionospheric irregularities appear above the open sea. Most of the satellite-receiver links do not pass through the target region; hence the irregularities are barely detected.

In plan 2, the receivers involved are distributed on the Bay of Bengal and part of the Indian Ocean; hence many satellite-receiver links pass through the target region. Salient features of circular arc perturbed by the tsunami-induced AGWs are observable, and the peak perturbation appears around 200 to 250 km of altitude. The results with plan 3 is similar to that with plan 2. The extra 275 receivers provide more data; hence the reconstructed profile becomes clearer, and the deviation from the original profile is also reduced.

In plan 4, the receivers on the Indian Ocean are not involved; hence only the irregularity features above the bay area is reconstructed. In plan 5, the receivers on the Bay of Bengal are replaced by those on the Indian Ocean; hence only the irregularity features above the ocean is reconstructed.
In plan 6, the data from all the receivers are utilized, and the deviation becomes the smallest among all six plans. However, the reconstructed profile is only marginally improved over those in plans 2 and 3.

Note that in order for the land-based receivers to record data related to an open-sea tsunami, the GPS satellites must be close to the horizon, which is not a favored position to receive satellite signal. It is also found that increasing the number of receivers in land to 1950 has very limited effect on the reconstructed profile. Apparently, putting the receivers under the target region is the best strategy.

**Reconstruction Using GPS and Galileo Signals:**

Fig. 9(a)–(f) shows the reconstructed profiles at 250 km, using the data from receivers deployed according to deployment plan B (DPB) plans 1 to 6, respectively, as listed in Table III. The associated maps of voxels that are passed by any satellite-receiver link are also shown in Fig. 10(a)–(f), respectively. The percentage of undetected voxels and deviation in six different plans are also listed in Table III.

With the additional information provided by the Galileo system, the profile out of plan 1 shows some features which are undetected with DPA plan 1. The reconstructed profile in each of the other plan also looks better than their counterpart DPA plan.

As a summary, the quality of reconstructed profiles can be improved more significantly by increasing the number of receivers beneath the target region as found in scenario I, or by increasing the number of satellites as found in scenario II. Plan 2 or 3 turns out to be more desirable, with or without the Galileo signals.

The three circulating currents in the Indian Ocean might be used to deliver the receivers to the ocean. The ocean currents in this area flow at an average speed of 1.26 km/h [47], a floating receiver will take about 75 days to migrate across the Indian Ocean from 77°E to 97°E.

Three-dimensional ionospheric images can be reconstructed in nearly real time using tomography techniques. The tsunami-induced AGWs cover a large geographical area and propagate from the ocean surface upward. These AGWs will induce a distinct distortion pattern of concentric arcs which lags behind the tsunami wavefront due to the propagation delay from the ocean surface to the ionosphere. In comparison, the ionospheric disturbances induced by solar radiation or other sources from the upper atmosphere usually take the form of random bubbles and spread out in all direction.

The Jason-1 detected the tsunami-induced ionospheric irregularities at the outbreak of the 2004 tsunami event and recorded a 1-D TEC profile along its track 129 over the Indian Ocean. The tsunami-induced ionospheric irregularities can be recognized by comparing with the ionospheric profile on regular days before and after the event. A nearly real-time 3-D tomography image can reveal more details of the ionospheric irregularities and help recognizing the occurrence of tsunami.

The epicenter of the 2004 Sumatra event appears about 1700 km away from Sri Lanka, and there was a marginal time of 134 min before the first wavefront smashes the shore. The proposed scheme might be able to serve as an early-warning system to avoid heavy casualties and property damages.

**V. CONCLUSION**

Several deployment plans of GPS receivers are proposed to reconstruct 3-D profiles of electron density perturbation in the ionosphere in nearly real time. In summary, a free-floating receiver network based on plans 2 and 3 with only GPS signals can reconstruct a fair profile to exhibit the main features of tsunami-induced irregularities. With the additional Galileo signals, the reconstructed profiles can be further improved. Apparently, plan 2 or 3 is more desirable, with or without the Galileo signals. A land-based receiver network can help reduce the probability of false alarms. The 2004 Sumatra tsunami is used as a test bench for this proposed technique, based on which a possible early-warning system can be implemented.

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