ACKNOWLEDGMENT

The authors would like to express their appreciation for the support by R. Staehle, S. Weinstein, and H. Price of the Pluto Express Pre-Project Office and S. Robertson from the University of Michigan.

REFERENCES

[1] C. Y. Chi and G. M. Rebeiz, "Planar microwave millimeter-wave lumped elements and coupled-line filters using micromachining techniques," *IEEE Trans. Microwave Theory Tech.*, vol. 43, no. 4, pp. 730–738, Apr. 1995.

[2] S. V. Robertson, L. P. Katehi, and G. M. Rebeiz, "W-band microshield low-pass filters," *IEEE Trans. Microwave Theory Tech.*, vol. 2, pp. 625–628, 1994

[3] T. M. Weller, L. P. Katehi, and G. M. Rebeiz, "A 250 GHz microshield bandpass filter," *IEEE Microwave Guided Wave Lett.*, vol. 5, no. 5, pp. 153–155, May 1995.

[4] T. M. Weller, L. P. Katehi, M. I. Herman, and P. D. Wamhof, "Membrane technology (MIST-T) applied to microstrip: A 33 GHz Wilkinson power divider," *IEEE Trans. Microwave Theory Tech.*, vol. 2, pp. 911–914, 1994.

[5] T. M. Weller, "Micromachined high frequency transmission lines on thin dielectric membranes," Ph.D. Dissertation, Radiation Lab., Univ. of Mich., 1995.

[6] M. Hamadallah, "Microstrip power dividers at mm-wave frequencies," *Microwave J.*, pp. 116–127, July 1988.

[7] M & I Materials Ltd., P.O. Box 136, Manchester M60 1AN, England.

[8] A. Biswas, T. Weller, and L. P. B. Katehi, "Stress determination of micromembranes using laser vibrometry," *Review Sci. Instrum.*, in press.

[9] K. Sabetfakhri, Univ. of Mich., personal communication.

[10] B. C. Wadell, *Transmission Line Design Handbook.* Boston: Artech House, 1991, pp. 95–97.

Axisymmetric Modes of Cylindrical Resonators with Cascaded Inhomogeneous Dielectrics

Jean-Fu Kiang

***Abstract*— A generic numerical scheme is developed to calculate the resonant frequency of axisymmetric modes in an inhomogeneous cylindrical dielectric resonator. The resonator consists of sections of cylindrically stratified dielectrics within a cylindrical waveguide. In each section, the TM_{0m} and TE_{0m} waveguide modes are solved by expanding the H_ϕ and E_ϕ components in terms of the eigenmodes in an empty waveguide. The fields in each section are then expanded in terms of these TM_{0m} and TE_{0m} modes. The transverse resonance technique is then applied to obtain the resonant frequencies. Comparison with literatures validates the effectiveness of this approach. Results with continuous dielectric profiles are also obtained.**

I. INTRODUCTION

Cylindrical cavities have been used to cure materials [1], to measure complex permittivity of materials [2], and as a resonator in microwave circuits [3]–[11]. In all these applications, the circular waveguide section forming the cavity contains inhomogeneous dielectrics. For the resonator application, the resonant frequencies of the dielectric loaded cavity need to be determined precisely. Several coupled dielectric rod or ring resonators can be arranged coaxially in a circular waveguide to form a bandpass filter. The resonant frequencies of the axisymmetric $\mathrm{TE}_{01\delta}$ and $\mathrm{TM}_{01\delta}$ modes have been calculated by using a mode-matching technique [3], [4]. For both modes, the resonant frequencies are below the cutoff frequency of the TE_{01} waveguide mode. In [5] and [6], the resonant frequency of nonaxisymmetric hybrid modes are calculated by using a similar technique.

Manuscript received December 18, 1995; revised May 24, 1996. This work was supported by the National Science Council, Taiwan, ROC under Contract NSC 85-2213-E005-010.

The author is with the Department of Electrical Engineering National Chung-Hsing University, Taichung, Taiwan, ROC.

Publisher Item Identifier S 0018-9480(96)06394-6.

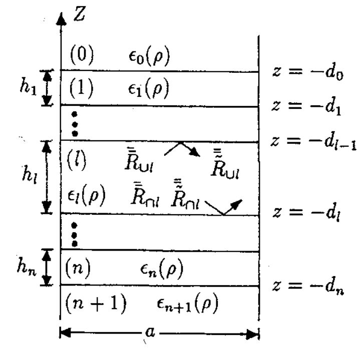


Fig. 1. Geometrical configuration of cascaded circular waveguide sections loaded with inhomogeneous dielectrics.

In [7], a finite integration technique (FIT) based on the integral forms of Maxwell's equations is proposed to calculate the resonant frequencies of a cavity filled with an inhomogeneous dielectric. A brief summary of mode nomenclature is also provided in [7]. In [8], a variational expression is used to calculate the resonant frequencies of axisymmetric modes where the radial variation of field components are expanded by the first-order finite element (FE) basis functions and the axial variation is expanded in terms of sinusoidals. Finite-difference method in the frequency domain [9], finite-difference time-domain (FDTD) method [10], and finite element method (FEM) [11] have also been used.

Mode-matching method proves to be efficient for many canonical resonator structures. For example, the cylindrical dielectric rod and ring in a cylindrical cavity. Usually, the eigenmodes in a stratified medium need to be solved first to represent the field distribution in the later stage. If the dielectric ring consists of many layers or if a dielectric rod has a continuous permittivity profile, conventional mode-matching method becomes tedious or impossible. For such structures, finite element method, FDTD method, and FIT method can be used at the expense of finer grids to express the fields accurately.

In this paper, we will present a generic numeric scheme to solve such problems. First, the eigenmodes in each uniform dielectric loaded waveguide section are obtained by solving a symmetric eigenvalue problem, where dielectrics with continuous profile can also be handled. Reflection matrices at the junctions of waveguide sections are defined to reduce the number of unknowns. Then the transverse resonance technique is applied to obtain the resonant frequencies of the resonators.

II. FORMULATION

Fig. 1 shows the configuration of a cylindrical resonator with radius a, which consists of several sections of circular waveguides loaded with inhomogeneous dielectrics. The permittivity in each layer is a piecewise continuous functions of ρ and is independent of ϕ and z.

Axisymmetric modes exist in such a medium, and are categorized into TM (to z) and TE (to z) modes [7]. For the TM modes, the existing field components are E_z, E_ρ, and H_ϕ. For the TE modes, the existing field components are H_z, H_ρ, and E_ϕ.

First, consider the TM modes in an infinitely long circular waveguide with an inhomogeneous dielectric profile which is uniform along the axial direction. Expand Maxwell's equations to obtain

$$E_\rho = \frac{1}{i\omega\epsilon}\frac{\partial H_\phi}{\partial z}, E_z = -\frac{1}{i\omega\epsilon}\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho H_\phi$$
$$\left(\rho\epsilon\frac{\partial}{\partial\rho}\frac{1}{\rho\epsilon}\frac{\partial}{\partial\rho}+k^2+\frac{\partial^2}{\partial z^2}\right)\rho H_\phi = 0. \quad (1)$$

Next, express H_ϕ of the nth eigenmodes by a set of basis functions $S_m(\rho)$ as

$$\begin{aligned} H_{n\phi} &= \phi_n(\rho)e^{\pm ik_{nz}z} \\ &= \sum_{m=1}^{N} b_{nm}S_m(\rho)e^{\pm ik_{nz}z} \\ &= \bar{b}_n^t\cdot\bar{S}(\rho)e^{\pm ik_{nz}z} \end{aligned} \quad (2)$$

where $\bar{b}_n^t = [b_{n1},\cdots,b_{nN}]$, and $\bar{S}^t(\rho) = [S_1(\rho),\cdots,S_N(\rho)]$. Choose the H_ϕ distribution in an empty circular waveguide as the basis functions, i.e., $S_m(\rho) = J_1(\xi_m\rho/a)$ with $J_0(\xi_m) = 0$. Substitute (2) into (1), then take the inner product of $S_p(\rho)$ with the resulting equation and use the integration by parts technique to have

$$\begin{aligned} &\sum_{m=1}^{N} b_{nm}\left[-\langle\frac{1}{\rho\epsilon}\frac{\partial}{\partial\rho}\rho S_p(\rho)\frac{\partial}{\partial\rho}\rho S_m(\rho)\rangle + \langle S_p(\rho),\omega^2\mu\rho S_m(\rho)\rangle\right] \\ &= k_{nz}^2\sum_{m=1}^{N} b_{nm}\langle S_p(\rho),\rho\epsilon^{-1}S_m(\rho)\rangle, 1\le p\le N \end{aligned} \quad (3)$$

where the inner product is defined over $0\le\rho\le a$. Hence, (3) constitutes a symmstric eigenvalue problem to be solved numerically for the propagation constant k_{nz}. The eigenvectors $\bar{b}_n$'s satisfy the orthonormality specification that $\bar{b}_p^t\cdot\bar{\bar{N}}\cdot\bar{b}_m = \delta_{pm}$ where $\bar{\bar{N}} = \langle\bar{S}(\rho),\rho\epsilon^{-1}\bar{S}^t(\rho)\rangle$. The general field in the waveguide can be expressed in terms of these eigenmodes as

$$H_\phi = \sum_{n=1}^{N}\phi_n(\rho)e^{\pm ik_{nz}z}\alpha_n = \bar{\phi}^t(\rho)\cdot e^{\pm i\bar{\bar{K}}_z z}\cdot\bar{\alpha}$$

$$E_\rho = \sum_{n=1}^{N}\pm\frac{k_{nz}}{\omega\epsilon}\phi_n(\rho)e^{\pm ik_{nz}z}\alpha_n = \pm\frac{1}{\omega\epsilon}\bar{\phi}^t(\rho)\cdot\bar{\bar{K}}_z\cdot e^{\pm i\bar{\bar{K}}_z z}\cdot\bar{\alpha} \quad (4)$$

where $\bar{\phi}^t(\rho) = [\phi_1(\rho),\cdots,\phi_N(\rho)]$, $\bar{\bar{K}}_z = \text{diag.}[k_{1z},\cdots,k_{Nz}]$, and $e^{\pm i\bar{\bar{K}}_z z} = \text{diag.}[e^{\pm ik_{1z}z},\cdots,e^{\pm ik_{Nz}z}]$.

Next, use the waveguide modes to represent the fields in layer (l) of the resonator as

$$\begin{aligned} H_{l\phi} &= \bar{\phi}_l^t(\rho)\cdot\left[e^{i\bar{\bar{K}}_{lz}z_l}\cdot\bar{\alpha}_l + e^{-i\bar{\bar{K}}_{lz}z_l}\cdot\bar{\beta}_l\right] \\ E_{l\rho} &= \frac{1}{\omega\epsilon_l}\bar{\phi}_l^t(\rho)\cdot\bar{\bar{K}}_{lz}\cdot\left[e^{i\bar{\bar{K}}_{lz}z_l}\cdot\bar{\alpha}_l - e^{-i\bar{\bar{K}}_{lz}z_l}\cdot\bar{\beta}_l\right] \end{aligned} \quad (5)$$

where $z_l = z + d_l$. The first term in $H_{l\phi}$ and $E_{l\rho}$ is a wave propagating in the $+z$ direction, and the second term is a wave propagating in the $-z$ direction. Define a reflection matrix $\bar{\bar{R}}_{\cup l}$ at the upper boundary $z = -d_{l-1}$ so that the reflection matrix multiplied by the upward wave gives the downward wave, i.e., $\bar{\bar{R}}_{\cup l}\cdot e^{i\bar{\bar{K}}_{lz}h_l}\cdot\bar{\alpha}_l = e^{-i\bar{\bar{K}}_{lz}h_l}\cdot\bar{\beta}_l$. Define another reflection matrix $\bar{\bar{R}}_{\cap l}$ at the lower boundary $z = -d_l$ so that the reflection matrix multiplied by the downward wave gives the upward wave, i.e., $\bar{\bar{R}}_{\cap l}\cdot\bar{\beta}_l = \bar{\alpha}_l$. Thus, we obtain the resonance condition

$$\det.\left(\bar{\bar{I}} - \bar{\bar{R}}_{\cap l}\cdot e^{i\bar{\bar{K}}_{lz}h_l}\cdot\bar{\bar{R}}_{\cup l}\cdot e^{i\bar{\bar{K}}_{lz}h_l}\right) = 0. \quad (6)$$

The resonant frequencies are obtained by solving (6).

The fields in layer (m) with $m > l$ can be expressed as

$$\begin{aligned} H_{m\phi} &= \bar{\phi}_m^t(\rho)\cdot\left[e^{i\bar{\bar{K}}_{mz}z_m}\cdot\bar{\bar{R}}_{\cap m} + e^{-i\bar{\bar{K}}_{mz}z_m}\right]\cdot\bar{\beta}_m \\ E_{m\rho} &= \frac{1}{\omega\epsilon_m}\bar{\phi}_m^t(\rho)\cdot\bar{\bar{K}}_{mz}\cdot\left[e^{i\bar{\bar{K}}_{mz}z_m}\cdot\bar{\bar{R}}_{\cap m} - e^{-i\bar{\bar{K}}_{mz}z_m}\right]\cdot\bar{\beta}_m. \end{aligned} \quad (7)$$

By matching the boundary conditions that $H_{r\phi} = H_{(r+1)\phi}$ and $E_{r\rho} = E_{(r+1)\rho}$ at $z = -d_r$, we have

$$\begin{aligned} &\left(\bar{\bar{R}}_{\cap r}+\bar{\bar{I}}\right)\cdot\bar{\beta}_r = \bar{\bar{B}}_{r(r+1)} \\ &\quad\cdot\left[e^{i\bar{\bar{K}}_{(r+1)z}h_{r+1}}\cdot\bar{\bar{R}}_{\cap(r+1)} + e^{-i\bar{\bar{K}}_{(r+1)z}h_{r+1}}\right]\cdot\bar{\beta}_{r+1} \\ &\left(\bar{\bar{R}}_{\cap r}-\bar{\bar{I}}\right)\cdot\bar{\beta}_r = \bar{\bar{K}}_{rz}^{-1}\cdot\bar{\bar{B}}_{(r+1)r}^t\cdot\bar{\bar{K}}_{(r+1)z} \\ &\quad\cdot\left[e^{i\bar{\bar{K}}_{(r+1)z}h_{r+1}}\cdot\bar{\bar{R}}_{\cap(r+1)} - e^{-i\bar{\bar{K}}_{(r+1)z}h_{r+1}}\right]\cdot\bar{\beta}_{r+1} \end{aligned} \quad (8)$$

$$\begin{aligned} \bar{\bar{R}}_{\cap r} = &\left\{\left[\bar{\bar{I}} - e^{i\bar{\bar{K}}_{(r+1)z}h_{r+1}}\cdot\bar{\bar{R}}_{\cap(r+1)}\cdot e^{i\bar{\bar{K}}_{(r+1)z}h_{r+1}}\right]^{-1}\cdot\left(\bar{\bar{K}}_{rz}^{-1}\cdot\bar{\bar{B}}_{(r+1)r}^t\cdot\bar{\bar{K}}_{(r+1)z}\right)^{-1}\right. \\ &\left.+\left[\bar{\bar{I}} + e^{i\bar{\bar{K}}_{(r+1)z}h_{r+1}}\cdot\bar{\bar{R}}_{\cap(r+1)}\cdot e^{i\bar{\bar{K}}_{(r+1)z}h_{r+1}}\right]^{-1}\cdot\bar{\bar{B}}_{r(r+1)}^{-1}\right\}^{-1} \\ &\cdot\left\{\left[\bar{\bar{I}} - e^{i\bar{\bar{K}}_{(r+1)z}h_{r+1}}\cdot\bar{\bar{R}}_{\cap(r+1)}\cdot e^{i\bar{\bar{K}}_{(r+1)z}h_{r+1}}\right]^{-1}\cdot\left(\bar{\bar{K}}_{rz}^{-1}\cdot\bar{\bar{B}}_{(r+1)r}^t\cdot\bar{\bar{K}}_{(r+1)z}\right)^{-1}\right. \\ &\left.-\left[\bar{\bar{I}} + e^{i\bar{\bar{K}}_{(r+1)z}h_{r+1}}\cdot\bar{\bar{R}}_{\cap(r+1)}\cdot e^{i\bar{\bar{K}}_{(r+1)z}h_{r+1}}\right]^{-1}\cdot\bar{\bar{B}}_{r(r+1)}^{-1}\right\} \end{aligned} \quad (9)$$

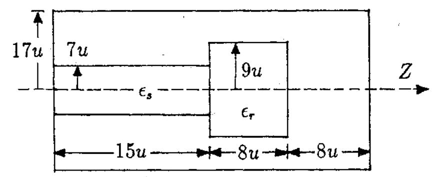
$$\begin{aligned} H_{m\phi} &= \bar{\phi}_m^t(\rho)\cdot\left[e^{i\bar{\bar{K}}_{mz}z_m} + e^{i\bar{\bar{K}}_{mz}(h_m-z_m)}\cdot\bar{\bar{R}}_{\cup m}\cdot e^{i\bar{\bar{K}}_{mz}h_m}\right]\cdot\bar{\alpha}_m \\ E_{m\rho} &= \frac{1}{\omega\epsilon_m}\bar{\phi}_m^t(\rho)\cdot\bar{\bar{K}}_{mz}\cdot\left[e^{i\bar{\bar{K}}_{mz}z_m} - e^{i\bar{\bar{K}}_{mz}(h_m-z_m)}\cdot\bar{\bar{R}}_{\cup m}\cdot e^{i\bar{\bar{K}}_{mz}h_m}\right]\cdot\bar{\alpha}_m \end{aligned} \quad (10)$$

$$\begin{aligned} &\left(\bar{\bar{I}}+\bar{\bar{R}}_{\cup r}\right)\cdot e^{i\bar{\bar{K}}_{rz}h_r}\cdot\bar{\alpha}_r = \bar{\bar{B}}_{r(r-1)}\cdot\left[\bar{\bar{I}} + e^{i\bar{\bar{K}}_{(r-1)z}h_{r-1}}\cdot\bar{\bar{R}}_{\cup(r-1)}\cdot e^{i\bar{\bar{K}}_{(r-1)z}h_{r-1}}\right]\cdot\bar{\alpha}_{r-1} \\ &\left(\bar{\bar{I}}-\bar{\bar{R}}_{\cup r}\right)\cdot e^{i\bar{\bar{K}}_{rz}h_r}\cdot\bar{\alpha}_r = \bar{\bar{K}}_{rz}^{-1}\cdot\bar{\bar{B}}_{(r-1)r}^t\cdot\bar{\bar{K}}_{(r-1)z} \\ &\cdot\left[\bar{\bar{I}} - e^{i\bar{\bar{K}}_{(r-1)z}h_{r-1}}\cdot\bar{\bar{R}}_{\cup(r-1)}\cdot e^{i\bar{\bar{K}}_{(r-1)z}h_{r-1}}\right]\cdot\bar{\alpha}_{r-1} \end{aligned} \quad (11)$$

TABLE I

COMPARISON OF RESONANT FREQUENCIES (GHZ) COMPUTED BY USING THIS APPROACH WITH THOSE IN [7], N IS THE NUMBER OF BASIS FUNCTIONS, $\epsilon_r = 37.3$, $\epsilon_s = 3.78$, $u = 0.427$ mm

Mode	$N=10$	$N=15$	$N=20$	$N=25$	$N=30$	[7], measured	[7], computed
TE_{01}	7.02	7.02	7.02	7.02	7.02	7.037	6.943
TE_{02}	11.39	11.39	11.39	11.39	11.39	11.391	11.316
TM_{01}	9.54	9.45	9.43	9.42	9.41	9.296	9.185
TM_{02}	11.44	11.30	11.27	11.25	11.23	11.113	10.943



where $\bar{\bar{B}}_{ij}$ is defined as $\langle \rho\epsilon_i^{-1}(\rho)\bar{\phi}_i(\rho), \bar{\phi}_j^t(\rho)\rangle$. A recursive formula is thus obtained as shown in (9) at the bottom of the previous page.

The fields in layer (m) with $m < l$ can be expressed as shown in (10) at the bottom of the previous page. By matching the boundary conditions that $H_{r\phi} = H_{(r-1)\phi}$ and $E_{r\rho} = E_{(r-1)\rho}$ at $z = -d_{r-1}$, we have as shown in (11) at the bottom of the previous page. A recursive formula is thus obtained as shown in (12) at the bottom of the page.

The same procedure can be applied to the TE modes. First, consider the TE modes in an infinitely long circular waveguide with an inhomogeneous dielectric profile which is uniform along the axial direction. Expand Maxwell's equations in terms of H_z, H_ρ, and E_ϕ. Next, expand the eigenmodes of E_ϕ by a set of basis functions $\tilde{S}_m(\rho)$, and choose the E_ϕ distribution in an empty circular waveguide as the basis functions to expand E_ϕ. The eigenmodes can be obtained by solving the eigenvalue problem formed by taking the inner product of $\tilde{S}_p(\rho)$ with the equation satisfied by E_ϕ.

Next, use the waveguide modes to represent the fields in layer (l) of the resonator. Reflection matrices $\bar{\bar{R}}_{\cup l}$ and $\bar{\bar{R}}_{\cap l}$ are defined to relate the upward wave and the downward wave in layer (l). Finally, the resonant frequencies are obtained by solving the resonance condition

$$\det.\left(\bar{\bar{I}} - \bar{\bar{R}}_{\cap l} \cdot e^{i\bar{\bar{K}}_{lz}h_l} \cdot \bar{\bar{R}}_{\cup l} \cdot e^{i\bar{\bar{K}}_{lz}h_l}\right) = 0. \qquad (13)$$

Recursive formulas for $\bar{\bar{R}}_{\cup l}$ and $\bar{\bar{R}}_{\cap l}$ can be derived by matching E_ϕ and H_ρ at interfaces between contiguous layers.

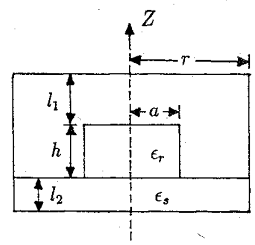
III. NUMERICAL RESULTS

First, we show the resonant frequencies of the TE_{01}, TE_{02}, TM_{01}, and TM_{02} modes of a cylindrical dielectric-loaded resonator as shown in Table I. The results from [7] are also shown for comparison.

TABLE II

COMPARISON OF RESONANT FREQUENCIES (GHZ) COMPUTED BY USING THIS APPROACH WITH THOSE IN THE REFERENCES, $N = 30$, $\epsilon_r = 38$, $\epsilon_s = 2.33$, $a = h = 4$ mm, $r = 16$ mm, $l_1 = 3$ mm, $l_2 = 0.79$ mm, [X] : [4] IN [8]

Mode	present	[8]	[x]
TE_{01}	6.82	6.82	6.73
TE_{02}	10.75	10.79	-
TE_{03}	12.02	12.09	12.10
TM_{01}	9.52	9.51	9.40
TM_{02}	13.13	13.10	-



The convergence rate for the TE_{01} and TE_{02} modes are faster than that for the TM_{01} and TM_{02} modes in this case.

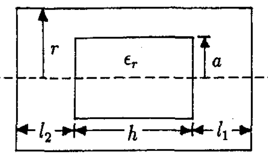
Table II shows the resonant frequencies of a dielectric resonator on top of a substrate as in a circuit board environment. Our results compare favorably with those in the literatures. Table III shows the resonant frequencies of a cylindrical dielectric resonator. The results are close to those in the literatures.

Next, we calculate the resonant frequencies of two symmetrically coupled dielectric ring resonators in a circular waveguide. The permittivity of the ring is assumed to have a parabolic profile with an extreme value ϵ_m at $\rho = (a+b)/2$. The resonant frequencies

$$\begin{aligned}\bar{\bar{R}}_{\cup r} = &\left\{\left[\bar{\bar{I}} - e^{i\bar{\bar{K}}_{(r-1)z}h_{r-1}} \cdot \bar{\bar{R}}_{\cup(r-1)} \cdot e^{i\bar{\bar{K}}_{(r-1)z}h_{r-1}}\right]^{-1} \cdot \left(\bar{\bar{K}}_{rz}^{-1} \cdot \bar{\bar{B}}_{(r-1)r}^{t} \cdot \bar{\bar{K}}_{(r-1)z}\right)^{-1}\right.\\ &\left. + \left[\bar{\bar{I}} + e^{i\bar{\bar{K}}_{(r-1)z}h_{r-1}} \cdot \bar{\bar{R}}_{\cup(r-1)} \cdot e^{i\bar{\bar{K}}_{(r-1)z}h_{r-1}}\right]^{-1} \cdot \bar{\bar{B}}_{r(r-1)}^{-1}\right\}^{-1}\\ &\cdot \left\{\left[\bar{\bar{I}} - e^{i\bar{\bar{K}}_{(r-1)z}h_{r-1}} \cdot \bar{\bar{R}}_{\cup(r-1)} \cdot e^{i\bar{\bar{K}}_{(r-1)z}h_{r-1}}\right]^{-1} \cdot \left(\bar{\bar{K}}_{rz}^{-1} \cdot \bar{\bar{B}}_{(r-1)r}^{t} \cdot \bar{\bar{K}}_{(r-1)z}\right)^{-1}\right.\\ &\left. - \left[\bar{\bar{I}} + e^{i\bar{\bar{K}}_{(r-1)z}h_{r-1}} \cdot \bar{\bar{R}}_{\cup(r-1)} \cdot e^{i\bar{\bar{K}}_{(r-1)z}h_{r-1}}\right]^{-1} \cdot \bar{\bar{B}}_{r(r-1)}^{-1}\right\} \end{aligned} \qquad (12)$$

TABLE III
COMPARISON OF RESONANT FREQUENCIES (GHZ) COMPUTED BY USING THIS APPROACH WITH THOSE IN THE REFERENCES, $N = 30$, $\epsilon_r = 35.74$, $h = 7.62$ mm, $l_1 = l_2 = 3.81$ mm, $a = 8.636$ mm, $r = 12.954$ mm, [Y] : [20] IN [8]

Mode	present	[8]	[y]	[9]
TE_{01}	3.428	3.435	3.428	3.429
TE_{02}	5.419	5.493	5.462	5.412
TM_{01}	4.572	4.601	4.551	4.542



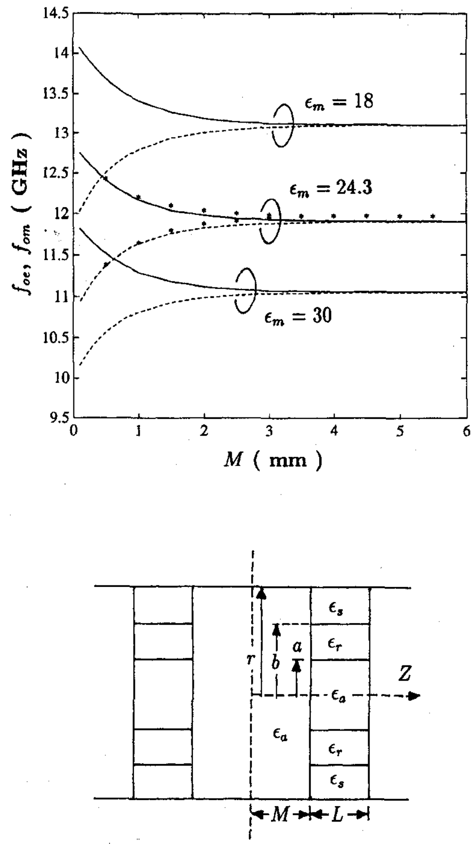


Fig. 2. Resonant frequencies of the $TE_{01\delta}$ mode of two symmetrically coupled dielectric ring resonator, $\epsilon_r(\rho) = 24.3 + 4(\epsilon_m - 24.3)(\rho - a)(b - \rho)/(b - a)^2$, $\epsilon_s = 1.031$, $\epsilon_a = 1$, $b = 2.455$ mm, $a = 0.3b$, $r = 2.39b$, $(b/L)^2 = 0.4625$. ——: electric wall in the middle. - - - -: magnetic wall in the middle (results from [3]).

are below the cutoff frequency of the circular waveguide. Due to the structure symmetry, either an electric wall or a magnetic wall can be inserted in the middle plane to form two equivalent problems. The resulting resonant frequency are denoted by f_{oe} (electric wall) and f_{om} (magnetic wall), respectively. The resonant frequencies as a function of the resonator separation are shown in Fig. 2. The results with a flat profile in the ring match well with those in [3]. The resonant frequency decreases as ϵ_m increases. The difference between f_{oe} and f_{om} increases as the two resonators move closer to each other.

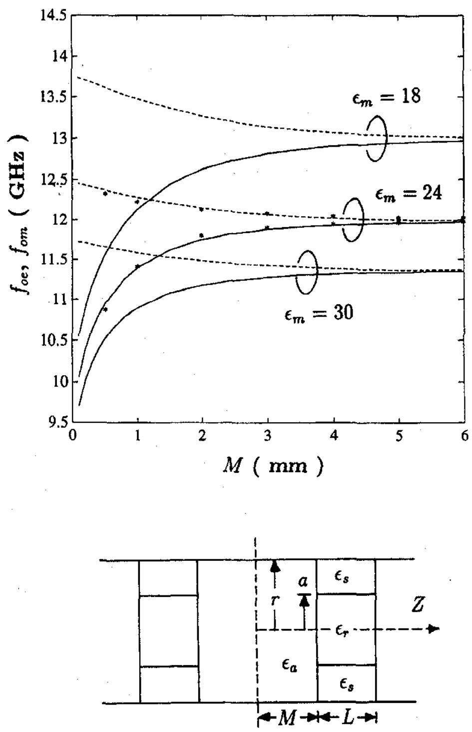


Fig. 3. Resonant frequencies of the $TM_{01\delta}$ mode of two symmetrically coupled dielectric rod resonator, $\epsilon_r(\rho) = 24 + (\epsilon_m - 24)(1 - \rho^2/a^2)$, $\epsilon_s = 1.031$, $a = 3.635$ mm, $r = 5.45$ mm, $L = 4.04$ mm. ——: electric wall in the middle. - - - -: magnetic wall in the middle (results from [4]).

Finally, we calculate the resonant frequencies of two symmetrically coupled dielectric rod resonators in a circular waveguide. The permittivity of the rod is assumed to have a parabolic profile with an extreme value ϵ_m at $\rho = 0$. The resonant frequencies are below the cutoff frequency of the circular waveguide. As shown in Fig. 3, the results with a flat profile in the rod match well with those in [4]. The resonant frequency decreases as ϵ_m increases. The difference between f_{oe} and f_{om} increases as the two resonators move closer to each other. Note that f_{oe} is lower than f_{om} in this case, and f_{oe} is higher than f_{om} for the coupled dielectric rings in the previous case.

IV. CONCLUSION

A general numeric scheme combining the eigenvalue method and the transverse resonance technique has been developed to calculate the resonant frequencies of a cylindrical resonator consisting of cascaded sections of circular waveguides loaded with inhomogeneous dielectrics. The results obtained by using this approach compare favorably with those in the literatures. The resonant frequencies with continuous dielectric profiles have also been calculated, which can not be done by using conventional mode-matching methods.

ACKNOWLEDGMENT

The author would like to thank the reviewers for their useful comments.

REFERENCES

[1] J. Jow, M. C. Hawley, M. Finzel, J. Asmussen, Jr., H.-H. Lin, and B. Manring, "Microwave processing and diagnosis of chemically reacting

materials in a single-mode cavity applicator," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, pp. 1435–1443, Dec. 1987.

[2] H. A. Buckmaster, T. H. T. van Kalleveen, H. Zaghloul, and C. H. Hansen, "9-GHz complex permittivity measurements of high-loss liquids using a variable-length reflection cavity and a dual-channel, double superheterodyne signal processing system," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, pp. 909–916, Oct. 1987.

[3] Y. Kobayashi and M. Minegishi, "Precise design of a bandpass filter using high-Q dielectric ring resonators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, pp. 1156–1160, Dec. 1987.

[4] Y. Kobayashi and M. Minegishi, "A low-loss bandpass filter using electrically coupled high-Q $TM_{01\delta}$ dielectric rod resonators," *IEEE Trans. Microwave Theory Tech.*, vol. 36, pp. 1727-1732, Dec. 1988.

[5] K. A. Zaki and C. Chen, "Coupling of nonaxially symmetric hybrid modes in dielectric resonators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, pp. 1136–1142, Dec. 1987.

[6] S.-W. Chen and K. A. Zaki, "Dielectric ring resonators loaded in waveguide and on substrate," *IEEE Trans. Microwave Theory Tech.*, vol. 39, pp. 2069-2076, Dec. 1991.

[7] J. E. Lebaric and D. Kajfez, "Analysis of dielectric resonator cavities using the finite integration technique," *IEEE Trans. Microwave Theory Tech.*, vol. 37, pp. 1740–1748, Nov. 1989.

[8] M. M. Taheri and D. M.-Syahkal, "Accurate determination of modes in dielectric-loaded cylindrical cavities using a one-dimensional finite element method," *IEEE Trans. Microwave Theory Tech.*, vol. 37, pp. 1536–1541, Oct. 1989.

[9] C.-C. Su and J.-M. Guan, "Finite-difference analysis of dielectric-loaded cavities using the simultaneous iteration of the power method with the Chebyshev acceleration technique," *IEEE Trans. Microwave Theory Tech.*, vol. 42, pp. 1998–2006, Oct. 1994.

[10] A. Navarro, M. J. Nunez, and E. Martin, "Study of TE_0 and TM_0 modes in dielectric resonators by a finite difference time-domain method coupled with the discrete Fourier transform," *IEEE Trans. Microwave Theory Tech.*, vol. 39, pp. 14–17, Jan. 1991.

[11] J.-F. Lee, G. M. Wilkins, and R. Mittra, "Finite-element analysis of axisymmetric cavity resonator using a hybrid edge element technique," *IEEE Trans. Microwave Theory Tech.*, vol. 41, pp. 1981–1987, Nov. 1993.

Precision Broadband Wavemeter for Millimeter and Submillimeter Range

Y. A. Dryagin, V. V. Parshin, A. F. Krupnov, N. Gopalsami, and A. C. Raptis

***Abstract*— A precise, broadband, Fabry–Perot wavemeter has been designed and built to measure wavelengths in the millimeter and submillimeter range. The design of the wavemeter is novel in that it enhances the fundamental mode over a wide band and permits determination of the exact longitudinal index of the mode. With the use of an exact mode number in wavelength calculations, high measurement accuracies, to the extent permissible by the quality factor of the resonator, can be obtained. The wavemeter was tested by measuring well-known spectral lines of the OCS molecule in the frequency range of 72–607 GHz. Measurement of 24 OCS lines demonstrated an accuracy of better than 2×10^{-5} in relative units and 0.87×10^{-5} in rms units for frequency/wavelength. A discussion of further development and automation of the wavemeter is included.**

I. Introduction

In short-wave millimeter and submillimeter regions, open resonators of the Fabry–Perot type are analogs to closed cavities of the centimeter and millimeter wave regions [1]. They are based on concepts associated with optical frequencies and so are called quasi-optical Fabry–Perot resonators. The most common resonator employs a curved mirror at one end and a flat mirror at the other end. Stable Gaussian-beam resonances of the TEM_{mnq} type can be supported by these open resonators [2]. High quality factors on the order of 10^5 are routinely possible, which enable sharp resonances and high measurement accuracy of resonance locations.

Even so, the conventional method of measuring wavelength leads to diminished accuracy. It consists of tuning the high-Q quasi-optical Fabry–Perot resonator to two consecutive resonances (two consecutive longitudinal modes) and measuring the difference between the corresponding positions of a movable mirror. The difference is equal to half of the wavelength, with necessary diffraction corrections. The procedure is subject to two main sources of error.

1) The measured wavelength is the small difference between the two large distances (on the order of 100 mm) between the mirrors at the qth and $(q+1)$th longitudinal modes. The relative accuracy of measuring each resonance position is on the order of $1/Q = 10^{-5}$, but the relative accuracy of the difference in distance is on the order of $q/Q = 10^{-3}$.
2) If the oscillator whose radiation wavelength is to be measured drifts by 10^{-4} during the time the resonator is tuned from one mode to the another, the error in the wavelength measurement will be $q \times 10^{-4} = 10^{-2}$.

Manuscript received December 29, 1995; revised May 24, 1996. This work was supported in part by the U.S. Department of Energy, New Independent States-Industrial Partnering Program, under Contract W-31-109-ENG-38; Russian Fund for Fundamental Studies by Grant N 94-02-05424, a Ministry of Science and Technical Politics of Russia in the frame of the State Program on Fundamental Metrology; and by Grant R8I000 from the International Science Foundation and joint Grant R8I300 from the International Science Foundation and the Government of Russia.

Y. A. Dryagin, V. V. Parshin, and A. F. Krupnov are with the Institute of Applied Physics, Russian Academy of Sciences, 603600 Nizhnii Novgorod GSP-120, Russia.

N. Gopalsami and A. C. Raptis are with the Energy Technology Division, Argonne National Laboratory, Argonne, IL USA.

Publisher Item Identifier S 0018-9480(96)06395-8.