Logic Synthesis & Verification, Fall 2010

National Taiwan University

Problem Set 1

Due on 2010/10/6 before lecture

1 [Deduction from Axioms]

Apply *only* the five postulates (axioms) of Boolean algebra to show that

$$a + a' \cdot b = a + b$$

holds in all Boolean algebra $(\mathbb{B}, +, \cdot, 0, 1)$, for $a, b \in \mathbb{B}$.
(Specify clearly in your solution which postulate is applied in each step.)

2 [Relation over Boolean Algebra]

Define the relation $\leq$ in a Boolean algebra with carrier $\mathbb{B}$ as follows

$$a \leq b \text{ if and only if } a \cdot b' = 0$$

for all $a, b \in \mathbb{B}$, where b' is the inverse element of b. Prove that the following properties hold for all $a, b, c \in \mathbb{B}$:

(a) $a \cdot b \leq a \leq a + c$
(b) $a \leq b$ and $a \leq c$ if and only if $a \leq b \cdot c$

3 [Boolean Functions]

Let $f(x, y)$ be a Boolean function for $\mathbb{B} = \{0, 1, a, a'\}$ with the following partial function table.

x	0	0	0	0	1	1	1	1	a	a	a	a	a'	a'	a'	a'
y	0	1	a	a'	0	1	a	a'	0	1	a	a'	0	1	a	a'
f	0	1			a	1										

(a) How many Boolean functions are consistent with the above function table? Please explain.
(b) Please complete the above function table and list all possibilities if more than one.

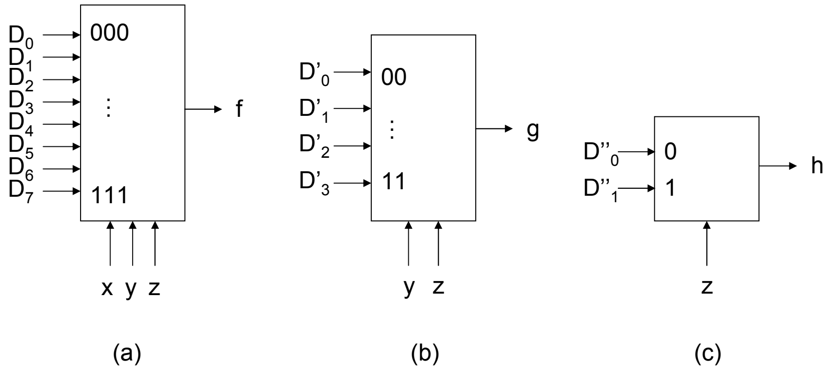

Fig. 1. Mux implementation of Boolean functions

4 Boolean Algebra Application

Let Boolean function $f(x, y, z) = xy' + x'y + y'z'$ for $\mathbb{B} = \{0, 1\}$.

(a) Consider the multiplexor implementation of f in Figure 1 (a). What are the values of D_i?

(b) Consider implementing $f(x, y, z)$ by another Boolean function $g(y, z)$ using the multiplexor of Figure 1 (b).
- What is the new Boolean algebra? Please define the five-tuple $(\mathbb{B}, +, \cdot, \underline{0}, \underline{1})$.
- What are the possible values of variables y and z? Why the multiplexor assumes y and z have only values $\{0, 1\}$?
- Please explain in what sense $f(x, y, z)$ and $g(y, z)$ can be equivalent. What should the values D'_i be?

(c) Consider implementing $f(x, y, z)$ by yet another Boolean function $h(z)$ using the multiplexor of Figure 1 (c). What is the new Boolean algebra? Please define the the five-tuple $(\mathbb{B}, +, \cdot, \underline{0}, \underline{1})$. What are the values D''_i?