Logic Synthesis & Verification, Fall 2010

National Taiwan University

Problem Set 2

Due on 2010/10/27 before lecture

1 [Quantification and Boolean Difference]

(a) (10%) Consider the following 8 quantified Boolean formulas

$$\begin{aligned}
F_1 &= \exists x, \exists y. f(x, y, z),\\
F_2 &= \exists y, \exists x. f(x, y, z),\\
F_3 &= \exists x, \forall y. f(x, y, z),\\
F_4 &= \forall y, \exists x. f(x, y, z),\\
F_5 &= \forall x, \exists y. f(x, y, z),\\
F_6 &= \exists y, \forall x. f(x, y, z),\\
F_7 &= \forall x, \forall y. f(x, y, z),\\
F_8 &= \forall y, \forall x. f(x, y, z).
\end{aligned}$$

List possible implications among them. (Implications inferred from transitivity can be omitted.)

(b) (5%) For some Boolean function f and variable x, if $\frac{\partial f}{\partial x}$ is satisfiable, we call x a *functional support* of f. Please establish the connection between the (structural) support mentioned in the lecture and functional support.

(c) (10%) Given two Boolean functions f_1 and f_2, prove or disprove the following statements:

(1) $\frac{\partial (f_1 \oplus f_2)}{\partial x}$ equals constant 0 if both $\frac{\partial f_1}{\partial x}$ and $\frac{\partial f_2}{\partial x}$ equal constant 0.
(2) $\frac{\partial (f_1 \oplus f_2)}{\partial x}$ equals constant 0 only if both $\frac{\partial f_1}{\partial x}$ and $\frac{\partial f_2}{\partial x}$ equal constant 0.

2 [AIG and CNF]

(a) (5%) Convert the AIG of Figure 1 to a CNF ϕ_1 with intermediate variables a, b allowed.

(b) (5%) Convert the AIG of Figure 1 to a CNF ϕ_2 without having intermediate variables a, b.

(c) (5%) The CNFs ϕ_1 and ϕ_2 are certainly not functionally equivalent. Explain in what sense they are equivalent.

(d) (5%) How can we make ϕ_1 and ϕ_2 functionally equivalent by quantification?

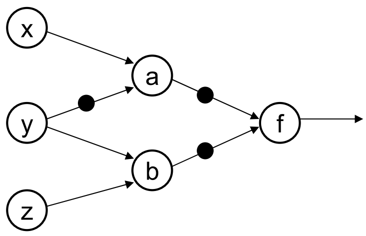

Fig. 1. AIG for CNF conversion

3 [BDD Operation]

(a) (5%) To compute $\exists x.f$ for some Boolean function f and variable x, how can we achieve it using BDD `ITE` and `COMPOSE` operations?

(b) (15%) Consider the BDD of function f as shown in Figure 2. Apply the operations in (a) to derive the BDD of $\exists c.f$. Show the steps in terms of shared BDDs.

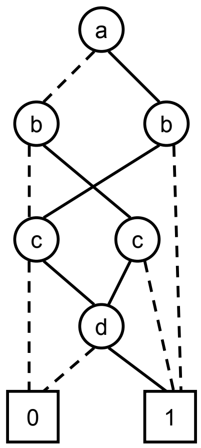

Fig. 2. BDD with variable c to be existentially quantified

4 [BDD and Functional Decomposition]

(10%) Suppose a function $f(a,b,c,d)$ can be rewritten as $h(a,b,g(c,d))$ for some functions h and g. Let the BDD of f has variable ordering c and d on top of a and b. What property the BDD must have such that $f(a,b,c,d) = h(a,b,g(c,d))$? Why?

5 [SAT Solving]

Consider SAT solving the CNF formula consisting of the following 8 clauses

$$C_1 = (a + b + c), C_2 = (a + b' + c), C_3 = (a' + c + d), C_4 = (a' + c + d'),$$
$$C_5 = (a + c' + d'), C_6 = (a' + b + c'), C_7 = (a + c' + d), C_8 = (b' + c' + d).$$

(a) (10%) Apply implication and conflict-based learning in solving the above CNF. Assume the decision order follows a, b, c, and then d; assume each variable is assigned 0 first and then 1. Whenever a conflict occurs, draw the implication graph and enumerate all possible learned clauses under the Unique Implication Point (UIP) principle. (In your implication graphs, annotate each vertex with "`variable = value@decision_level`", e.g., "$b = 0@2$", and annotate each edge with the clause that implication happens.) If there are multiple UIP learned clauses for a conflict, use the one with the UIP closest to the conflict in the implication graph.

(b) (5%) The **resolution** between two clauses $C_i = (C_i^* + x)$ and $C_j = (C_j^* + x')$ (where C_i^* and C_j^* are sub-clauses of C_i and C_j, respectively) is the process of generating their **resolvent** $(C_1^* + C_j^*)$. The resolution is often denoted as

$$\frac{(C_i^* + x) \qquad (C_j^* + x')}{(C_1^* + C_j^*)}$$

A fact is that a learned clause in SAT solving can be derived by a few resolution steps. Show how that the learned clauses of (a) can be obtained by resolution with respect to their implication graphs.

6 [Unsatisfiability and Resolution]

(10%) Prove that a CNF formula is unsatisfiable if and only if an empty clause (a clause without any literal) can be derived through resolution.