Logic Synthesis & Verification, Fall 2010

National Taiwan University

Problem Set 3

Due on 2010/11/10 before lecture

1 [Incompletely Specified Functions]

(15%) Let $\mathcal{F}_i = (f_i, d_i, r_i)$ for $i = 1, 2$ be incompletely specified functions, where f_i, d_i, and r_i are the onsets, don't care sets, and offsets, respectively. Derive the following incompletely specified functions in three-tuples.

(a) $(\neg\mathcal{F}_1)$
(b) $(\mathcal{F}_1 \vee \mathcal{F}_2)$
(c) $(\mathcal{F}_1 \wedge \mathcal{F}_2)$

2 [Generalize Cofactor]

(10%) Prove or disprove the following equalities.

(a) $co(f \wedge g, h) = co(f, h) \wedge co(g, h)$
(b) $co(f, x) = f_x$

3 [Special Functions]

(25%) Prove or disprove the following statements.

(a) Any prime cover of a unate function is not necessarily a unate cover.
(b) Every prime of a unate function is an essential prime.
(c) Let f be a function symmetric on variables x and y and symmetric on variables y and z. Then f must be symmetric on variables x and z.

4 [Column Covering]

(20%) We define the *partial Max-SAT* problem as follows:

Definition 1 (partial Max-SAT). *Given a CNF formula $\Phi = C_1^h \wedge \cdots \wedge C_m^h \wedge C_1^s \wedge \cdots \wedge C_n^s$,* ***partial Max-SAT*** *is the problem of finding a truth assignment that satisfies all the* ***hard clauses*** *$C_1^h, \ldots, C_m^h$ and simultaneously satisfies the maximum number of* ***soft clauses*** *$C_1^s, \ldots, C_n^s$.*

Suppose we are given the following 0-1 matrix:

$$\begin{pmatrix} 1\,0\,1\,0\,0 \\ 0\,0\,1\,0\,1 \\ 1\,1\,0\,0\,1 \\ 0\,0\,1\,1\,0 \\ 0\,1\,0\,1\,0 \end{pmatrix}$$

Please formulate the corresponding *minimum column covering problem* (namely, selecting a minimum set of columns that covers all the rows) as a partial Max-SAT problem. What are the hard and soft clauses?

5 [Prime Generation]

(15%) Apply Unate Recursive Paradigm to generate all the prime implicants of the following cover.

$$\{a'b'c'd', a'bc'd', abc'd, abcd', abcd, ab'cd, a'bc'd, a'b'c'd\}$$

6 [Quine-McCluskey]

(15%) Let $f = w'x'y'z' + wx'z' + wxz + w'x'yz$ with don't care set $d = w'xyz' + wx'yz + w'xyz$. Use the Quine-McCluskey procedure to minimize f. Show results at each step (i.e. list the primes, give the matrix and show how the minimum cover is found).