# Logic Synthesis \& Verification, Fall 2010 <br> National Taiwan University 

## Problem Set 5

Due on 2010/12/15 before lecture

## 1 [Node Value and Elimination]

(a) Show that the value of node $j$ can be calculated as

$$
\operatorname{value}(j)=\left(\sum_{i \in F O(j)} n_{i}\right)\left(l_{j}-1\right)-l_{j}
$$

where $n_{i}$ is the number of times that literals $y_{j}$ and $y_{j}^{\prime}$ occur in the factored form of node $i$, and $l_{j}$ is the number of literals in the factored form of node $j$.
(b) Given the Boolean network of Figure 1 (assuming the node functions are expressed in the sum-of-products form), compute the values of nodes 1,2 , 3 , and 4 . After eliminating node 1 (by collapsing it into its fanout nodes), compute the new values of nodes $2,3,4$.


Fig. 1. Boolean network for node elimination.

## 2 Weak Division

Given an expression $F$ and a divisor $G$, suppose $F=G \cdot H+R$ by weak division. Prove that $H$ and $R$ are unique.

## 3 [Kernelling and Factoring]

Let $F=a e f h+a e g h+a e i+b e f h+b e g h+b e i+c d e f h+c d e g h+c d e i$.
(a) Compute $\operatorname{KERNEL}(0, F)$ with literals ordered alphabetically. Draw the kernelling tree (as in the slides) and list the kernels and their corresponding co-kernels.
(b) Compute all 2-cube divisors and 2-literal cube divisors (including those after complementation). For each 2-cube divisor, indicate whether or not it is a kernel.
(c) Apply GFACTOR on $F$ by using the largest level-0 kernels as the divisors and using weak division. (In case that there are several choices of divisors, using one of them is sufficient.)
(d) Apply GFACTOR on $F$ by using the 2-cube divisors with literals appearing most frequently in $F$ and using weak division. (In case that there are several choices of divisors, using one of them is sufficient.)

## 4 [Extraction by Rectangle Covering]

Given two algebraic expressions

$$
\begin{aligned}
& F=a c+a d+a f+b e+b c+b d+b f, \text { and } \\
& G=a c d+a e+b c d+b e,
\end{aligned}
$$

use rectangle covering to extract a common sub-expression with the largest value.

