

Logic Synthesis & Verification, Fall 2011

National Taiwan University

Problem Set 1

Due on 2011/10/12 before lecture

1 [Boolean Algebra Definition]

Is $(D_n, \text{lcm}, \text{gcd}, 1, n)$ a Boolean algebra for $n = 90$? Why or why not?

2 [Boolean Algebra Properties]

Prove the following equalities using only the postulates of Boolean algebra.

- (a) $a + (b + c) = (a + b) + c$
- (b) $a \cdot b + a' \cdot c + b \cdot c = a \cdot b + a' \cdot c = (a + b) \cdot (a' + c) \cdot (b + c) = (a + b) \cdot (a' + c)$

(Please specify clearly which postulate is applied in each step of your derivation.)

3 [Boolean Function]

Show that a function $f : \mathbb{B}^n \rightarrow \mathbb{B}$ is Boolean if and only if

$$X^A f(X) = X^A f(A)$$

for all $A \in \{0, 1\}^n$. (Note that $\mathbb{B}$ can have more than two values.)

4 Alternative Views on Boolean Functions

Given a three-variable Boolean function f_1 over $\mathbb{B} = \{0, 1\}$ with

$$f_1 = (1)x'y'z' + (0)x'y'z + (1)x'yz' + (0)x'yz + (1)xy'z' + (1)xy'z + (0)xyz' + (1)xyz,$$

define a two-variable Boolean function $f_2(y, z)$ taking values over $\{0, 1, x, x'\}$ as an alternative representation of f_1 .

5 Boole's Expansion Theorem

Use Boole's expansion theorem to prove the following equalities.

- (a) $uf(u, v, w) = uf(1, v, w)$
- (b) $u' + f(u, v, w) = u' + f(1, v, w)$