# Logic Synthesis & Verification, Fall 2011

**National Taiwan University**

## Problem Set 2

Due on 2011/10/26 before lecture

## 1 [Cofactor and QBF]

(a) (6%) Given two arbitrary Boolean functions $f$ and $g$ and a Boolean variable $v$, prove that $(\neg f)_v = \neg(f_v)$ and $(f \ \langle op \rangle \ g)_v = (f_v) \ \langle op \rangle \ (g_v)$ for $\langle op \rangle = \{\vee, \oplus\}$.

(b) (12%) Prove or disprove the following implications:

$$\forall x.(f(x,y) \vee g(x,y)) \Rightarrow (\forall x.f(x,y)) \vee (\forall x.g(x,y)) \tag{1}$$

$$\forall x.(f(x,y) \vee g(x,y)) \Leftarrow (\forall x.f(x,y)) \vee (\forall x.g(x,y)) \tag{2}$$

$$\exists x.(f(x,y) \vee g(x,y)) \Rightarrow (\exists x.f(x,y)) \vee (\exists x.g(x,y)) \tag{3}$$

$$\exists x.(f(x,y) \vee g(x,y)) \Leftarrow (\exists x.f(x,y)) \vee (\exists x.g(x,y)) \tag{4}$$

(c) (12%) Prove or disprove the following statement:
For any quantified formula $\exists x.\phi(x, y_1, \ldots, y_n)$, there always exists some function $f(y_1, \ldots, y_n)$ such that $\exists x.\phi(x, y_1, \ldots, y_n) = \phi(f(y_1, \ldots, y_n), y_1, \ldots, y_n)$.

## 2 [BDD Operation]

Let $f = ab(\neg c + d) + (a\neg b + \neg ab)(c\neg d + \neg cd)$.

(a) (6%) Draw the ROBDD with complemented edges of $f$ under variable ordering $a < b < c < d$ (with $a$ on top).

(b) (6%) Draw the ROBDDs with complemented edges of $f_c$ and $f_{\neg c}$ as shared ROBDDs along with that of $f$.

(c) (6%) Apply the ITE operation on the above ROBDDs to compute $\forall c.f$.

## 3 [AIG and CNF]

(a) (5%) Represent $x \oplus y \oplus z$ in CNF.

(b) (5%) Draw the AIG of the parity function $x \oplus y \oplus z$, and convert the AIG to a CNF formula (with intermediate variables allowed).

(c) (5%) The CNF formulas of (a) and (b) are certainly not functionally equivalent. Explain in what sense they are equivalent.

(d) (5%) How can we make the formulas of (a) and (b) functionally equivalent by quantification?

(e) (5%) We know representing a parity function in CNF is exponential in the number of variables and converting circuit to CNF is linear in circuit size. Is there any contradiction? Why or why not?

## 4 [SAT Solving]

(a) (6%) Write a CNF formula stating the pigeon-hole problem: *There are $n$ holes and $n+1$ pigeons. Every hole accommodates at most one pigeon and every pigeon must be in some hole.*

(b) (6%) Use MiniSat (`http://minisat.se/`) to solve the pigeon-hole problem for $n = 2, 4, 6$. (Note that the formulas should be in the DIMACS format `http://www.satcompetition.org/2009/format-benchmarks2009.html`.) What are the runtimes you get? Do you expect the solver is scalable on this problem? Why or why not?

## 5 [SAT Solving]

Consider SAT solving the CNF formula consisting of the following 8 clauses

$$C_1 = (a + b + c), C_2 = (a + b' + c), C_3 = (a' + c + d), C_4 = (a' + c + d'),$$
$$C_5 = (a + c' + d'), C_6 = (a' + b + c'), C_7 = (a + c' + d), C_8 = (b' + c' + d).$$

(a) (10%) Apply implication and conflict-based learning in solving the above CNF formula. Assume the decision order follows $a$, $b$, $c$, and then $d$; assume each variable is assigned 0 first and then 1. Whenever a conflict occurs, draw the implication graph and enumerate all possible learned clauses under the Unique Implication Point (UIP) principle. (In your implication graphs, annotate each vertex with "`variable = value@decision_level`", e.g., "$b = 0@2$", and annotate each edge with the clause that implication happens.) If there are multiple UIP learned clauses for a conflict, use the one with the UIP closest to the conflict in the implication graph.

(b) (5%) The **resolution** between two clauses $C_i = (C_i^* + x)$ and $C_j = (C_j^* + x')$ (where $C_i^*$ and $C_j^*$ are sub-clauses of $C_i$ and $C_j$, respectively) is the process of generating their **resolvent** $(C_1^* + C_j^*)$. The resolution is often denoted as

$$\frac{(C_i^* + x) \qquad (C_j^* + x')}{(C_1^* + C_j^*)}$$

A fact is that a learned clause in SAT solving can be derived by a few resolution steps. Show how that the learned clauses of (a) can be obtained by resolution with respect to their implication graphs.